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## Queue length estimation and signal timing based on floating vehicle data

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**Abstract** With the development of urbanization, the urban road network becomes more complex and the traffic volume is increasing. Traditional data acquisition methods such as coil and bayonet have many disadvantages such as difficult laying, high maintenance cost and missing data. With the rise of floating cars, a large amount of track data is collected. This paper integrates the intersection queuing model based on the track data and the intersection start-up wave model, estimates the queuing length of each phase of the intersection, and optimizes the timing of each phase of the intersection by using the traffic wave theory, and establishes a single intersection signal timing model based on the track data. The model uses the actual data of the intersection to model the intersection. The simulation results show that compared with the original timing scheme and Webster timing method, the model can reduce the intersection delay by 7% and 43% respectively, which verifies the effectiveness of the model.

**Keywords** traffic engineering; Floating car; Queue length; Traffic wave; Timing optimization

### 1. Introduction

With the rapid development of Internet technology, traffic data collection methods are becoming more and more diversified. Compared with the traditional coil and bayonet data, the track data has many collection methods, such as GPS track data, vehicle image recognition, video traffic flow detection and so on. Trajectory data has become a research hotspot in the field of transportation because it is easy to collect and can provide more accurate traffic conditions. At present, the research on trajectory data mainly focuses on the estimation of traffic parameters, such as flow [1-2], queue length [3-4], intersection delay [5], and travel time [6]. A large number of literatures have studied and solved the estimation of queue length and traffic from the aspects of probability theory and statistics. Comert and Cetin [7-8] considered the penetration rate of floating cars and the distribution of queue length, and concluded that the queue length at the intersection can be estimated only by the position of the last floating car. By considering the time when floating cars join the queue, the research was extended to the spatial and temporal dimensions. Li et al. [9] formulated the dynamics of queue length as the state transition process, and used Kalman filter to estimate the queue length cycle by cycle. Zhao et al. [10] summarized and innovated the methods for estimating the queue length and traffic flow at the intersection, and put forward a variety of methods for estimating the queue length and traffic flow at the intersection.

Sun Hui et al. [11] considered the impact of traffic wave on the queuing length of vehicles at intersections, and improved Webster algorithm for the shortest period of signal control at intersections by using traffic wave theory. Based on the traffic wave, Wang dianhai et al. [12] studied the queue dissipation process of vehicles at



the intersection and its impact on the upstream and downstream intersections. There are also many studies on the estimation method of intersection queue length based on the combination of trajectory data and traffic wave theory [13-14]. Wang et al. [15] also proposed a traffic wave theoretical model based on the combination of coil data and trajectory data for estimating the queue length at intersections. Qu Zhaowei et al. [16] proposed the kinematic model of starting wave at signalized intersection.

The traditional intersection timing method mainly consists of Webster algorithm and variations of various optimization algorithms. Researchers define different objective functions and select genetic algorithm [17], annealing algorithm [18], fuzzy control [19] and other methods to optimize the objective functions to further obtain the optimized intersection signal timing scheme. Yao Zhihong et al. [20] used dynamic programming to optimize the timing of intersection signals. Wang et al. [21] used the game theory to optimize the intersection signal timing. Similarly, new methods such as reinforcement learning and deep reinforcement learning are also applied to signal optimization of intersections. Ma Shoufeng et al. [22] first proposed a single intersection reinforcement learning control method by combining agent with Q-learning learning method. Zhao Xiaohu [23] combined Q-learning learning algorithm with BP neural network to determine the phase switching sequence according to environmental changes, and used fuzzy controller to determine the phase sequence to realize mixed control of traffic signals.

It can be seen from the above documents that there are many signal timing methods at single intersections, but the timing scheme based on algorithm optimization lacks data support, and real-time signal control based on reinforcement learning is difficult to land. In this paper, based on the estimation method of queue length and the theory of traffic start-up wave, an intersection signal timing model based on trajectory data is established. Compared with the above methods, this paper uses the track data as the original data of timing, takes the data as the algorithm driver, mines the information of the track data, and applies it to the signal timing, laying a foundation for the intersection timing based on the track data.

## 2. Construction of intersection signal timing model based on trajectory data

The track data generally contains the position, speed and other information of the vehicle at a certain time, and the relevant information of the intersection can be obtained by processing the track data. At the same time, on the premise of uniform mixing of floating cars and ordinary vehicles, the queue length of the floating car queue can be estimated by the position of floating cars through the methods of statistics and probability theory, that is, the blocking length caused by the intersection signal light. In the traffic wave theory, when the green light of the intersection is turned on, the starting wave will be generated from the intersection to its upstream intersection. When the starting wave reaches the location of the queue vehicles, the vehicles will start moving to the downstream.

The construction of this model is mainly divided into two steps: the estimation of the queue length of each phase at the intersection based on the trajectory data and the signal timing of each phase considering the queue length based on the start-up wave theory. Sample the track data in each cycle of the intersection, estimate the maximum queue length of each phase, and combine the start-up wave principle, the minimum green light duration of each phase is when all the vehicles in the queue pass through the intersection, and then correct the green light duration of each phase to obtain the timing information of each phase of the intersection. See Figure 1.1 for the flow chart of this algorithm.



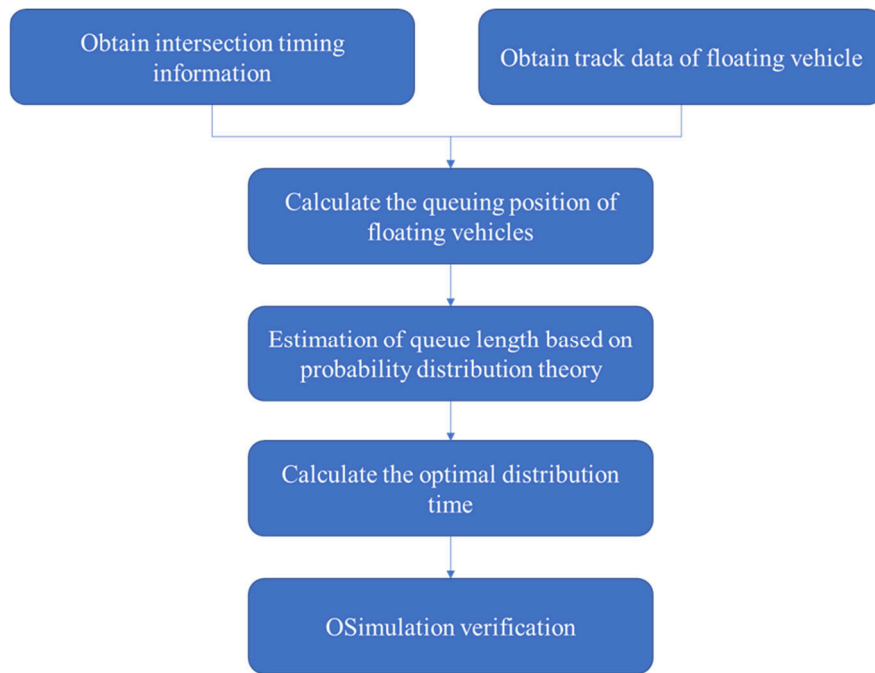


Figure 1.1: Algorithm flow chart

**2. Estimation of phase queue length at intersection**

When the vehicle stops at the intersection due to the signal light, the position, speed and other information of the floating vehicle can be collected, and then the queuing status of the intersection can be obtained. The vehicles queued at the intersection include floating vehicles and ordinary vehicles. In this paper, it is assumed that the floating vehicles and ordinary vehicles are evenly mixed on the studied Road, and the trajectory data is sampled and extracted as the trajectory of floating vehicles according to a certain proportion. See Figure 2.1, In the figure, Q represents the actual queue, Q represents the observable queue, red vehicles represent floating vehicles, and yellow vehicles represent ordinary vehicles.

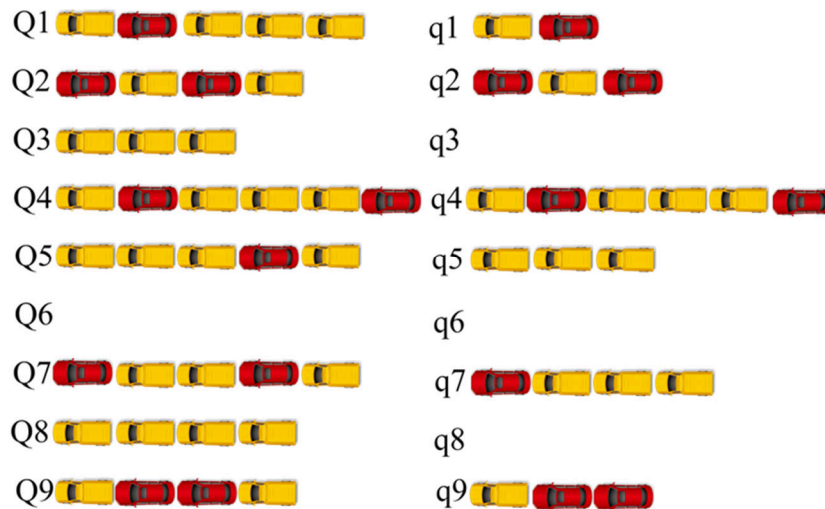


Figure 2.1: Observation process diagram

Since the track data records the position information of the floating vehicle, the observable queue length  $Q$  of the last vehicle can be obtained according to the position and vehicle spacing of the last recorded vehicle  $_{obs}$ . The number of queuing vehicles behind the last floating vehicle is the hidden queuing length  $Q_{hid}$  (Table 2.1).

**Table 2.1:** Relevant symbols

Symbol	Meaning
$Q_i$	Phase queue length of the $i$ th cycle
$N_i$	Total number of phase floating vehicles in cycle $i$
$S_i$	Position of floating car in cycle $i$
$S_{i1}$	Position of the first floating car in cycle $i$
$q_i$	Position of the last floating car in cycle $i$
$L_{max}$	Maximum queue length
$Q_{obs}$	Total length of observable queue
$Q_{hid}$	Total hidden queue length
$p$	Proportion of floating vehicles in all vehicles at the intersection

**2.1 Observation queue length  $Q_{obs}$  estimation**

According to the historical track data of floating vehicles, intersection timing and parking line position information, the stop vehicle position  $S_i$  in each cycle of the intersection can be calculated.

$$S_i = \frac{|p_i - p_l|}{l_v} \tag{1}$$

Where:  $S_i$  is the position of the floating car in the  $i$ th cycle;  $p_i$  is the distance between the floating vehicle and the intersection center point,  $m$ ;  $p_l$  is an example of the distance between the vehicle stop line of this phase and the intersection center point,  $m$ ;  $l_v$  is the average vehicle length, taking  $l_v=7m_0$ .

Since the functions of ordinary vehicles and floating vehicles are the same, there is only a difference between whether the vehicle data is easy to obtain. Therefore, it can be assumed that ordinary vehicles and floating vehicles are evenly mixed on the road, that is, the probability of whether a vehicle is a floating vehicle is constant. Based on the above assumptions, according to the knowledge of probability theory, the observable queue length  $Q_i$  in the period can be estimated according to the position  $S_{i1}$  of the first vehicle and the total number  $n_i$  of floating cars in the period I.

$$E(Q_i|N_i = n_i) = E(S_i|N_i = n_i)(n_i + 1) - 1 \tag{2}$$

See formula (3) ~ (9) for the proof process.

For  $k, n \in N$  and  $n \geq k$ , there are

$$C_n^k = \frac{n!}{k!(n-k)!} \tag{3}$$

$$A_n^k = \frac{n!}{(n-k)!} \tag{4}$$

For simplicity,  $Q_i, N_i, S_i, T_i, n_i, s_i, t_i$  is denoted by  $Q, N, S, T, n, s, t$ , respectively.

$$E(S|N = n, Q = l) = \frac{l+1}{n+1} \tag{5}$$

$$E(Q|N = n) = E(S|N = n)(n + 1) - 1 \quad (6)$$

When  $n \geq 1$ , it is proved as follows.

$$\begin{aligned} E(S|N = n, Q = l) &= \sum_{j=1}^{l-n+1} P(S = j|N = n, Q = l)j \\ &= \sum_{j=1}^{l-n+1} \frac{n C_{l-n}^{j-1} A_{j-1}^{j-1} A_{l-j}^{l-j}}{A_l^l} = \frac{n}{A_l^n} \sum_{k=0}^{l-n} A_{n+k-1}^{n-1} (l+1) - \frac{n}{A_l^n} \sum_{k=0}^{l-n} A_{n+k-1}^{n-1} (n+k) \\ &= (l+1) \sum_{k=0}^{l-n} \frac{(n+k-1)!(l-n)!n!}{k!l!(n-1)!} - \frac{n}{A_l^n} \sum_{k=0}^{l-n} A_{n+k}^n = \frac{l+1}{n+1} \end{aligned} \quad (7)$$

Based on the above derivation

$$\begin{aligned} E(S|N = n) &= \sum_{j=1}^{L_{max}} P(S = j|N = n)j \\ &= \sum_{j=1}^{L_{max}} \sum_{l=j+n-1}^{L_{max}} P(S = j|N = n, Q = l)P(Q = l|N = n)j \\ &= \frac{1}{n+1} (E(Q|N = n) + 1) \end{aligned} \quad (8)$$

$$\text{Equation (8) is } E(Q|N = n) = E(S|N = n)(n + 1) - 1 \quad (9)$$

According to formula (2), the number of floating vehicles  $n_i$  in each cycle is given, and the total observable queue length  $Q_{obs}$  can be calculated by summing up

$$\begin{aligned} Q_{obs} &= \sum_{i; n_i=0} (E(S_i|N_i = n_i)(n_i + 1) - 1) \\ &= \sum_{i; n_i=0} (s_{i_1}(n_i + 1) - 1) \end{aligned} \quad (10)$$

The average queue length  $\overline{q_{obs}}$  of the observable queue length  $q_{obs}$  of each phase can be obtained by calculating the observable queue length  $Q_{obs}$  in the total period and finding the average value

$$\overline{q_{obs}} = \frac{Q_{obs}}{N}$$

## 2.2. Hidden queue length $Q_{hid}$ estimation

### 2.2.1 Estimation of the hidden queue length $Q_{hid}$ when the floating vehicle proportion is high

When the proportion of floating cars on the road is high (proportion  $P \geq 20\%$ ), the probability of floating cars in the queue of each phase of each cycle at the intersection is high, and the length and number of hidden queues are small, which can be calculated according to the symmetry of the positions of floating cars in the queue. As shown in Figure 2.2, the queue in the  $k$ th cycle is opposite to the queue in the  $j$ th cycle. This means that the



number of vehicles after the last floating car in the  $j$ th cycle is equal to the number of vehicles before the first floating car in the  $k$ th cycle. Due to symmetry, the occurrence probability of these 2 cohorts was the same. Therefore, even if the number of vehicles behind the last floating car in one cycle is unknown, as long as the sample size is sufficient, the number of vehicles in front of the first floating car in another cycle can be used to compensate for the lost number. Therefore, the total hidden queue length  $Q_{hid}$  can also be calculated by the position  $S_{i_1}$  of the first track vehicle in each cycle, that is

$$Q_{hid} = \sum_i (S_{i_1} - 1) \tag{12}$$



Figure 2.2:  $Q_{hid}$  distribution diagram

**2.2.2 Estimation of the hidden queue length  $Q_{hid}$  when the floating vehicle ratio is low**

When the proportion of floating cars on the road is low, the probability that floating cars are not included in the queue of each phase in each cycle of the intersection increases, and the prediction accuracy of the estimation method of predicting the hidden queue length  $Q_{hid}$  according to the parking symmetry of the floating car intersection will rapidly decrease. Therefore, the calculation method of hidden queue length  $Q_{hid}$  based on Bayesian theorem is derived.

The hidden queue does not contain floating cars, so the length of the hidden queue can be expressed as

$$\sum_{i;n_i=0} E(P_i|q_i) = \sum_{i;n_i=0} \sum_l^{L_{max}} \frac{p(P_i=l)p(q_i|P_i=l)}{\sum_{j=0}^{L_{max}} p(P_i=l)p(q_i|P_i=j)} l = \sum_{i;n_i=0} \frac{pE(C_l)}{\sum_{j=0}^{L_{max}} pE(C_l)(1-p)^{j-l}} l \tag{13}$$

Equation (13) is based on the Bayes theorem, and it is solved when the number of floating cars is  $q_i$ , the expected value of queue length  $P_i$ . Where:  $P_i$  is the hidden queue length of the  $i$ th cycle;  $l$  is the value of the queue length in the hidden queue;  $p$  is the proportion of floating cars on the road section;  $E(C_j)$  is the expectation of the number of cycles with the hidden queue length  $j$ . Therefore, the hidden queue length  $Q_{hid}$  can be expressed as

$$Q_{hid}(p) = \sum_{i;n_i=0} \sum_l^{L_{max}} \frac{\bar{C}_l}{\sum_{j=0}^{L_{max}} p\bar{C}_j(1-p)^{j-l}} \tag{14}$$

Where:  $\bar{C}_l$  is the expected number of cycles of the hidden queue with queue length  $l$ . With reference to the method proposed in the paper of Zhao y et al. [15], the floating vehicle ratio  $p$  can be estimated, and thus  $Q_{hid}$  can be calculated.

After calculating the hidden queue length  $Q_{hid}$  in the total period and finding the average value, the average queue length  $\bar{q}_{hid}$  of the hidden queue length  $q_{hid}$  of each phase can be obtained.

$$\bar{q}_{hid} = \frac{Q_{hid}}{N} \tag{15}$$

### 3. Starting wave model and phase timing optimization of intersection

#### 3.1 Starting wave model of intersection

The traffic wave theory applies the basic principle of fluid mechanics to simulate the continuity of fluid and establish the continuity equation of vehicle flow. The density change of vehicle flow is modeled as the fluctuation of water wave and then the traffic wave is abstracted. When the intersection is released by the green light, the traffic density of this phase will change, thus generating the start-up wave. According to the derivation in Qu Zhaowei et al. [31], the calculation formula for the propagation speed of the start-up wave can be obtained. See formula (16).

$$u_w = -\frac{u}{hk_j u - 1} \quad (16)$$

Where:  $u_w$  is the start-up wave velocity, m/s, and the negative value represents that its propagation direction is from the intersection to the upstream intersection.  $h$  is the saturated headway, and  $h \approx 1.5\text{s/veh}$  is obtained through track data statistics;  $k_j$  is traffic congestion density, unit: veh/km;  $u$  is the maximum speed of vehicles passing through the intersection. In this paper,  $u=30\text{km/h}$  is set.

#### 3.2 Phase timing optimization

In the traffic wave theory, the vehicles queuing at the intersection can be divided into two steps: first, the green light starts, the start-up wave forms and propagates to the upstream intersection at a certain speed, and then the vehicles receiving the start-up wave start to accelerate and start to pass through the intersection. Therefore, to ensure that all waiting vehicles pass through the intersection at one time, the phase green light time can be calculated as shown in formula (17)

$$T_j = -L_j/u_w + T_v + T_m \quad (17)$$

Where:  $T_j$  is the green light time of phase  $j$ ;  $L_j$  is the queue length of phase  $j$ ;  $u_w$  is the starting wave velocity;  $T_v$  is the time taken by the vehicle at the end of the queue from starting to passing through the stop line;  $T_m$  is the phase duration correction parameter, taking  $T_m=3\text{s}$ . The calculation of  $T_v$  is shown in formula (18)

$$T_v = \begin{cases} \sqrt{2L_j/a} & L_j < l_m \\ \frac{(L_j - l_m)}{u} + \sqrt{2l_m/a} & L_j \geq l_m \end{cases} \quad (18)$$

Where:  $a$  is the acceleration;  $l_m = u^2/2a$  is the distance traveled by the vehicle when accelerating uniformly to the maximum speed.

### 4. Simulation verification and inspection

#### 4.1 Simulation construction of intersection

Sumo software is used to model the intersection, and the intersection flow data refers to the actual data of Liuquan road and Liantong road in Zhangdian District, Zibo City. Let the vehicle acceleration  $a=2.5\text{m/s}^2$ , the maximum vehicle speed  $u=40\text{km/h}$ , and the vehicle length  $l_v=5\text{m}$ , parking space  $l_g=2\text{m}$ . The intersection simulation and lane distribution are shown in Figure 4.1, and the intersection flow and signal timing are shown in table 4.1.



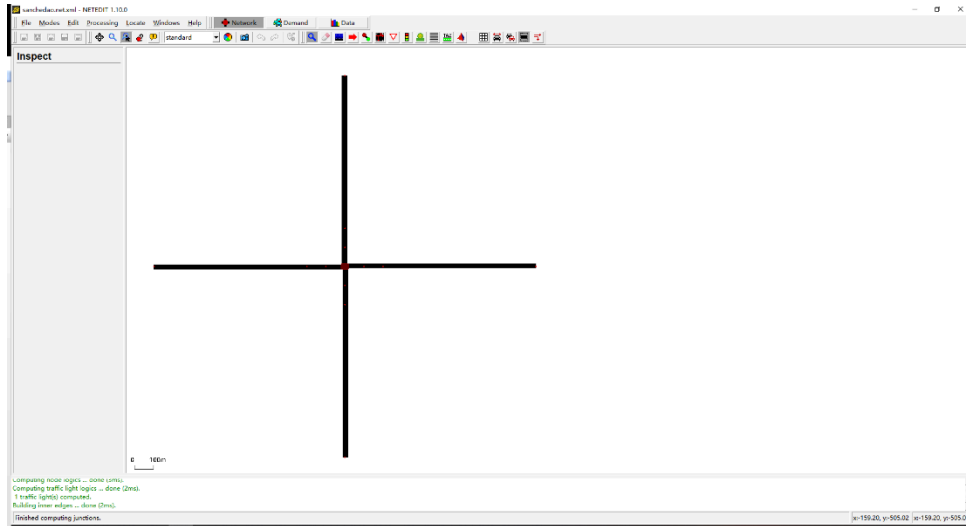


Figure 4.1: Intersection simulation diagram

Table 4.1: Flow input and timing in each direction

Phase	Traffic volume	Green light duration
	759	53
	640	
	494	34
	351	

4.2 Estimation of phase queue length

Based on the intersection simulation data, a certain proportion of vehicle collection track data is extracted for analysis, and the queue length of each phase is estimated. The average absolute percentage error (MAPE) is selected as the accuracy index of queue length estimation. See formula (19) for MAPE calculation formula

$$MAPE = \frac{1}{N} \sum_{i=1}^N \frac{|q_i - q_{il}|}{q_i} \times 100\% \tag{19}$$

In this paper, the queue length within 10 ~ 50 cycles after the simulation starts is selected for estimation. The vehicle positions in each cycle of the intersection are calculated according to formula (1), and the queue length is estimated according to formula (2) ~ (15) at different sampling rates. See table 4.2 for the estimation results of queue length under different proportions of floating vehicles.

Table 4.2: Estimation of queue length

	phase				MAPE
	South (straight)	North (straight)	South (left)	North (left)	
Actual	82.19	78.47	25.1	17.85	0
P=20%	76.62	73.15	22.15	15.75	6%
P=15%	88.64	84.63	28.05	19.95	21%
P=10%	77.42	73.92	31	22.05	25%
P=5%	63.13	60.27	33.45	23.8	35%



It can be seen from table 4.2 that the average *MAPE* of queue length under different occupancy rates is about 21.75%. When the proportion of floating cars decreases, the *MAPE* shows an upward trend. See Figure 4.2 for the histogram of estimated queue length and actual statistical value.

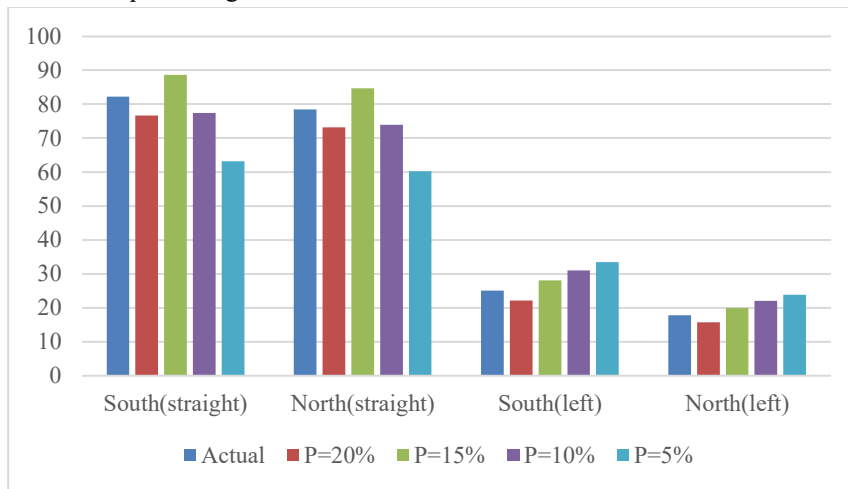


Figure 4.2: Histogram of estimated and actual queue length

### 4.3. Timing calculation verification

According to the above results, the time required for each phase is calculated by equations (16) ~ (18), and the simulation calculation delay is compared with the matching time of the original intersection and the Webster signal. See table 4.3 for phase information and intersection delay.

Table 4.3: Comparison of timing schemes

	Phase		Cycle	Delays
	↓ ↑	↙ ↘		
Original scheme	53	34	180	139.44
Webster	43	29	150	227.57
<i>P</i> =20%	58	22	168	125.28
<i>P</i> =15%	64	20	170	127.72
<i>P</i> =10%	61	24	164	131.33
<i>P</i> =5%	56	29	163	133.45

It can be seen from table 4.3 that the average delay of the intersection at the four sampling rates is about 129.44s, which is 10s lower than the original timing scheme of the intersection. The model in this paper effectively reduces the delay time of the intersection by 7%. However, in the traditional Webster timing scheme, the delay increases by 88.13s, accounting for 63%, because the intersection saturation is too high.

### 6. Concluding remarks

In this paper, a signal timing model based on the track data is proposed. The queue length of each phase is estimated by analyzing the track data, and then the phase signal duration is optimized by combining the start-up wave model of the intersection queue. Compared with the original timing scheme, the average delay reduction of 7% is achieved, which confirms the effectiveness of the model in this paper. When estimating the hidden queue length  $Q_{hid}$ , the *MAPE* fluctuates greatly under different sampling rates, and the estimation accuracy of

the queue length needs to be improved. In this paper, there is a phenomenon that the sampling rate is different and the value of *MAPE* fluctuates greatly, but the final timing effect is equivalent. The reason is that the phase duration calculated by the start-up wave theory is the minimum time required for the phase queue to dissipate, so as to avoid the secondary parking phenomenon at the intersection due to the estimation accuracy of the phase queue length. In this paper, the phase length correction parameter  $T_m$  is added to the calculation to correct the influence of the queuing estimation accuracy on the timing. In this paper,  $T_m=3s$  is selected, but the accurate selection of  $T_m$  value needs further research. In this paper, the occupancy rate of floating cars is set to be 5%, 10%, 15% and 20% respectively. It can be seen from the data results that when the occupancy rate of floating cars is increased, the queuing estimation accuracy will be improved. Therefore, more accurate phase timing information can be obtained and the service level of the intersection can be improved.

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