



Study on the Longest Sandcastles Based on Cellular Automaton and Modeling of Water-Sand Mixture

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Abstract This article mainly considers three aspects of sandcastle foundation shape and water-sand mix ratio to study the factors that affect the stability of sandcastle. The Cellular Automaton Probabilistic Model, Mixture Idealization Model is established. From the micro aspects, the sandcastle foundation and water-sand mix ratio analysis are performed. We analyzes that sandcastles that the optimal ratio of sand to water is 4.184g of water mixed with 100.000g of sand with the annular Concave slope could exist longer.

Keywords Geometric Figure, Ratio of Sand-Water, Cellular Automaton

Introduction

Every country has its own unique beach and there are ten world-renowned beaches. Not only are visitors willing to enjoy her sunshine on the beach or surf and play in the sea in fabulous bikinis. Furthermore, there is a popular casual game—building sandcastles. Meanwhile, beaches are the incubators of splendid works. Some replicas are created with unremitting endeavors. For children and adults who enjoy themselves on the beach. The ‘standing time’ of sandcastles also matters when they are building sandcastles with imaginations.

Since people will build sandcastles on the beach, all of them need to build a foundation first during this process. Then make decorations, reinforces, cutting and shaping. However, it is found that the rising and falling tides day and night and the ceaseless waves will continuously impact the foundation of sandcastles. Under the circumstance of the same distance from the sea and the same scale of construction, the castle with different structures is subject to different degrees of wave erosion. In other words, the way you build is important. The more solid foundations, the more impacts the sand castle can withstand. When building sandcastles, the geometric constructions, water and sand ratio, seawater properties, velocity of water, different volume and surface of sand, etc, should be taken into account, so as to increase the strength of the castle as much as possible.

Cellular Automaton Probabilistic Model

In the microscopic probabilistic simulation of particles, we use cellular automaton to simulate the state of sand particles and discuss the statics stability.

In the cellular automaton simulation, the sand particles in the previous grid fall vertically into the grid immediately below and are stably piled up from the bottom up. In order to better simulate the shape, we stipulate that once more than three sand particles appear on the top, the sand particles that are not on the bottom and have



gaps in the surrounding mesh will randomly move one grid to the left and right. For a more visual description we use a 7x7 grid diagram. [Fig.2]

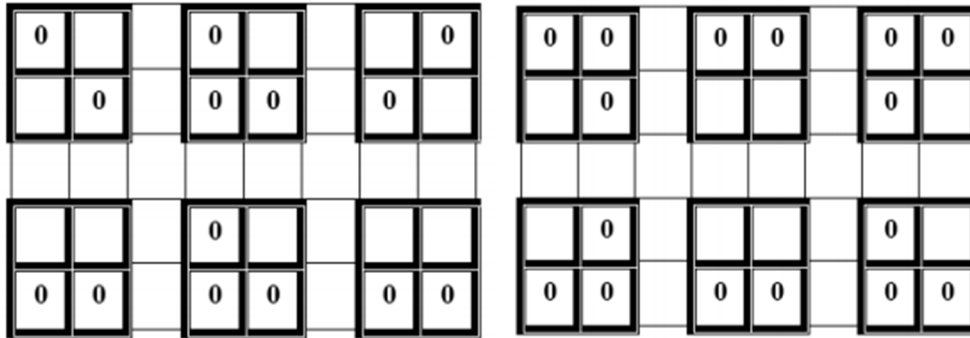


Figure 1: A 2x2 grid diagram of falling sand simulated by cellular automaton

We assume that each grain of sand falls from the center one by one. It is not difficult to see that if the mesh is large enough, the obtained sand accumulation cross section should be a function image similar to the normal distribution. As we all know, the image of normal distribution function is inward- concave, so if the sandbag foundation is the geometric shape of Concave, there is enough space for sand particles to move at random. But Linear geometry creates a lot of random space, increasing the likelihood that the sandcastle’s foundations will collapse. [Fig. 1]

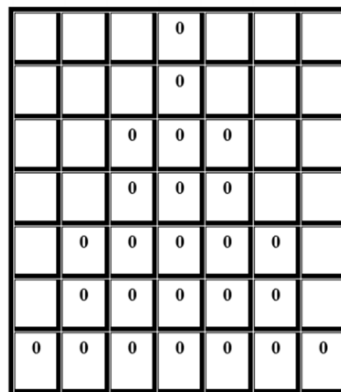


Figure 2: A 7x7 grid diagram of falling sand simulated by cellular automaton

Therefore, the construction of the three dimensional figure can take the shape of the annular concave surface. The three views and the three-dimensional figure are as follows:

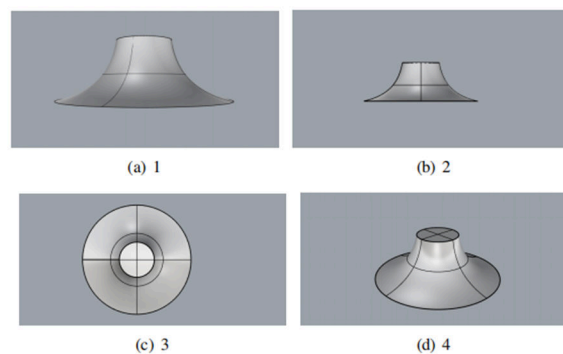


Figure 3: Three views and the three-dimensional figure

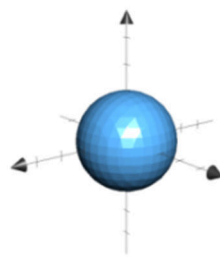
Modeling of Water-Sand Mixture

The Effect of Water on the Sand

Particle-liquid-vapor equilibrium of a three-phase system contained in a closed cell based on a thermodynamic-based original humidification method. Quantitative results on the relationship between pile stability and humidity are reported. For the first time, the effect of the wetting properties of a fluid on solids has been studied, thus providing insight into the macroscopic importance of local spatial liquid distribution [1].

It is well known that as the liquid content increases, the granular soil decreases [2]. That is to say, when the water content of the sandcastle foundation exceeds a certain limit, the collapse is faster and the duration is not long. The more seawater, the lower the adhesion between sand and soil. Therefore, more water is in the ratio of water to sand, sand is more likely to collapse, the sea itself is uncontrollable, and to ensure that the water is thick enough, all we have to do is to make the sand as wet as possible.

Water is good for sand grain bonding, and the optimal ratio of water to sand mixture will make the sand castle foundation more stable and better able to withstand tidal waves. According to the basic shape of sand grains, spherical particles are used as the research object in this paper, and the factors affecting the stability of the mixture are discussed at the microscopic level, as shown in the following table and Fig. 4.



Simulation Parameters

Particle shape	Spherical particle
Particle size/mm	1.000 (radius)
Particle density/kg·m ⁻³	2500

Figure 4: Simulated sand

To facilitate the calculation, we assume that the spherical particles of hydration are evenly distributed, as shown in Fig. 5. Under the action of water molecules, spherical particles will gather together, water forms a liquid bridge between two spherical particles, producing capillary force and thus playing an adhesive role, it is shown in Fig. 5 [3] too.

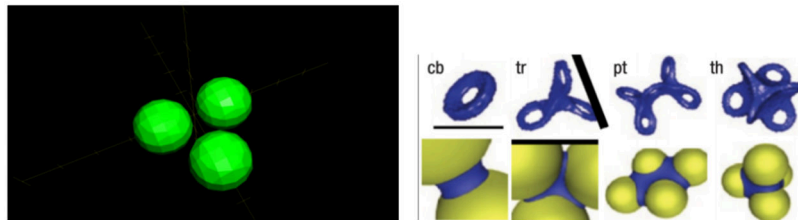


Figure 5: Spherical Particles

Modeling and Calculation

We know that when the gap between spherical particles changes, the corresponding shape of the liquid bridge changes, according to the curvature formula.

$$\rho = \frac{1}{K} = \frac{(1 + y'^2)^{3/2}}{|y''|}$$

It can be seen that when the shape of the liquid bridge changes, the corresponding function of the liquid bridge will also change, and the curvature will also change. Based on Laplace equation



$$\Delta P = \gamma \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

ΔP is the pressure difference between the two sides of the liquid surface, γ is the coefficient of surface tension of the liquid, R_1 and R_2 is the radius of curvature. The change of curvature affects the change of Laplace's equation, that is, the pressure difference between the two sides of the liquid surface changes, and the liquid bridging force changes accordingly, thus affecting the stability of the mixture. In this case, the Laplace pressure in a capillary bridges, p , is easy derived assuming a toroidal shape of the liquid surface [3]:

$$p_{cb}(\beta, \theta) = -\frac{\gamma}{R} \left[\frac{\cos(\beta + \theta)}{1 - \cos \beta} - \frac{1}{\sin \beta} \right]$$

γ is the surface tension of the liquid,

β is the angle of liquid, as shown in Fig. 6 [3]

According to the elastic response generated by "Hertzian Contact", it can be seen that two spherical particles will deform after contact, resulting in changes in curvature. It is an ideal model that two spherical particles contact with a point without deformation. In the actual situation, two spherical particles will not be able to contact with one point due to friction and other factors. To facilitate calculation, we assume that there is no friction between the two spherical particles and that they intersect with one point without deformation.

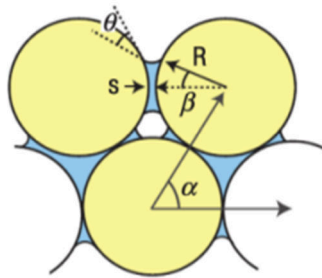


Figure 6: Triangular structure of ideal sand in microcosm [3]

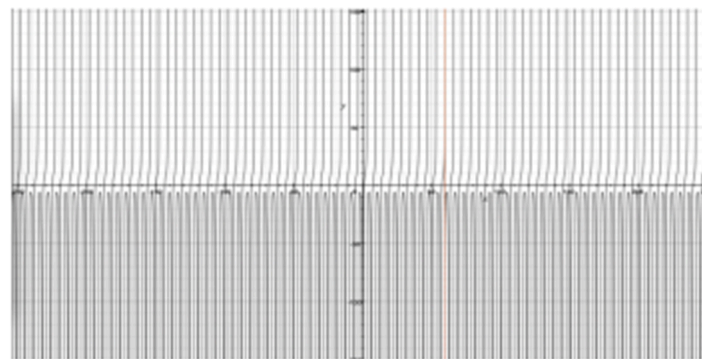


Figure 7: Image of $p_{cb}(\beta, \theta)$ [3] ($\theta = 0$)

The density of spherical particles directly affects the curvature of the bridge and the surface pressure of the liquid. As can be seen from the Fig. 7, the red line, when the spherical particles are most closely arranged, so when the β infinitely close to 60° , the maximum, now it's completely soaked, $\theta = 0^\circ$ [5]. That is, in the idealized model, when three spherical particles are in two phases, the liquid bridging force is the largest, that is, the model is the most stable. Having determined the ordering and spacing of spherical particles, we now calculate the percentage. In the case that spherical particles and water are evenly distributed, for the sake of calculation, we assume that they form a cube with ten balls on each side. We can get that the length of the side of the cube is 0.02m. We have $(10^3 + 93)$ spherical particles.



$$\frac{m_{\text{water}}}{m_{\text{sand}}} = \frac{1000 \times \left(0.02^3 - \frac{4}{3} \pi \times 0.001^3 \times (10^3 + 9^3) \right)}{2500 \times \frac{4}{3} \pi \times 0.001^3 \times (10^3 + 9^3)} \times 100\% \approx 4.184\%$$

The density of water is $1000\text{kg} \cdot \text{m}^{-3}$ and the density of sand is $2500\text{kg} \cdot \text{m}^{-3}$

When the fine granular material is mixed with the liquid, the surface tension of the liquid and the bridging force of the liquid make the fine particles aggregate to form different shapes. For the water-sand mixture, the ratio of 4.184% (the ratio of water and sand is 4.184 /100g) was prepared, and the obtained water-sand mixture had the highest stability, the least easy to be affected by external forces, and the best durability. The model selected in this paper is spherical particles, which can be extended to cubes, rotating bodies, polyhedra, or other irregular geometric shapes.

From the microscopic point of view, the liquid bridging force and water tension in the sand crevices have a strong adhesion to the fine particles. In everyday life, few people make a mixture of water and sand before building a sandcastle, because such a precise ratio is difficult to achieve by human perception. But if you can configure the mixture according to the optimal water-sand mixing ratio, then your sandcastle must be the most stable. In the same location from the beach, using the same materials, without the aid of any tools, the sand castle made with the mixture of water and sand will last longer and be less susceptible to the tidal wave and the sea breeze.

Material Factors and Strategies

The "Universal Binding Constraint" between the soil particles allowed the soil to allow plant roots to grow inside and retain water, fertilizer and air [4]. There is no such constraint between sand grains. If there is such "universal binding constraint" between sand grains, the soilization of desert is possible. In short, it's 'water' + 'sand' + 'binder'. Such an approach has been applied to desert farming. Inspired, we can also add such materials to make the sandcastle itself more stable.

When the adhesive turns the loose sand into a more stable "clay", there is no need to worry about the structure re-shaping in rainy days. By comparing the variation curves of safety coefficient of sand and clay slopes under different initial conditions, it can be seen that the influence degree of initial conditions on the stability of different soil slopes is different. It is unreasonable to assume the same maximum pore water pressure as the initial condition for the unsaturated seepage analysis of soil slope with different permeability coefficients under the same climatic conditions.

In the above analysis, under the same perennial average rainfall conditions, the steady seepage field corresponding to the sandy soil and the clay slope is similar to the initial maximum pore water pressure of -45 kPa and -25 kPa , respectively. The reason is that the water storage capacity of different types of soils is different, and the clay will be wettest than the sandy soil under the same climatic conditions. In addition, taking the steady state seepage field corresponding to the annual average rainfall as the initial condition for the unsaturated seepage analysis is reasonable and more in line with the actual situation [5].

Conclusions

When the fine granular material is mixed with the liquid, the surface tension of the liquid and the bridging force of the liquid make the fine particles aggregate to form different shapes. For the water-sand mixture, the ratio of 4.184% (the ratio of water and sand is 4.184 /100g) was prepared, and the obtained water-sand mixture had the highest stability, the least easy to be affected by external forces, and the best durability. Sandcastles that the optimal ratio of sand to water is 4.184g of water mixed with 100.000g of sand with the annular Concave slope could exist longer.



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