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Research Article

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Time Dilation due to Curved Space
- Why time is delayed when space curves by Gravity -

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Abstract In General Relativity, gravity changes the advance of time. Therefore, on a planet with a low gravitational potential, time advances more slowly than in outer space with a high gravitational potential. In this paper, since gravity curves space (space-time), it is shown that the curvature of space directly changes the advance of time, that is, the larger the curvature of space, the slower the time, and the delay of time. Since the radius of curvature is short, the advance of time is also small, resulting in a time delay. The mechanism of time delay due to the curved space has been clarified.

Keywords Gravitation; curvature; General Relativity; space-time; continuum mechanics; acceleration field; curved space; flat space

1. Introduction

According to the theory of relativity, there are two-time delays: the time of a moving object is delayed, and the flow of time is delayed by gravity in the vicinity of a celestial body with strong gravity (e.g., black hole). Especially in General Relativity, gravity distorts space (space-time) and changes the ticking of time. Therefore, on a planet with a low gravitational potential, the time progresses smaller and the time advances more slowly than in outer space with a high gravitational potential.

The reason why gravity delays time is well understood in mathematical explanations, but the picture as a mechanism of why gravity delays the advance of time is not clear. This is because the mechanism of gravity itself is not well understood. In General Relativity, gravity is the curvature of space, but the mechanism of why the force of gravity is generated when space curves has not been clarified.

Why is the time delayed by gravity? Gravity is the curvature of space, so it is the same matter as why time is delayed when space curves.

The mechanism of this gravity has been elucidated as one proposal by applying continuum mechanics and General Relativity to the space as a continuum [1,2,3,4,5,6,7,8].

In Chapter 2, the mechanism of acceleration generation and gravity due to the curvature of space are briefly introduced [3,4], and in Chapter 3, the time delay due to the curvature of space is discussed in detail.

2. Mechanism of Acceleration Generation and Gravity due to the Curvature of Space

2.1. Brief Concept of Pressure Field for Gravitational Acceleration

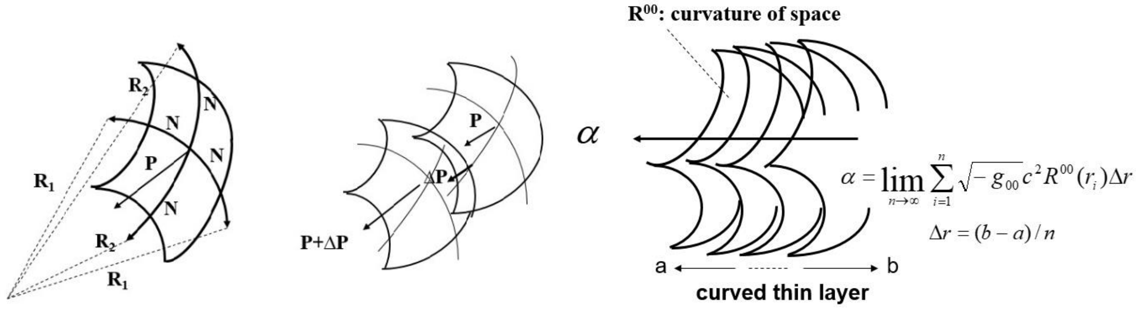
Applying the mechanical structure of space to General Relativity, a brief summary of the causes of gravity is given [3-6].

Given a priori assumption that space as a vacuum has a physical fine structure like an infinite continuum, it enables us to apply a continuum mechanics to the so-called “vacuum” of space [9-15].

That is, space can be considered as a kind of transparent elastic field: space as a vacuum performs the motions of deformation such as expansion, contraction, elongation, torsion and bending. The latest expanding universe theories (Friedmann, de Sitter, inflationary cosmological model) support this assumption. Space can be regarded as an elastic body like rubber. This conveniently coincides with the precondition of a mechanical structure of space.

General Relativity implies that space is curved by the existence of energy (mass energy or electromagnetic energy and etc.). General Relativity is based on Riemannian geometry. If we admit this space curvature, space is assumed as an elastic body. According to continuum mechanics, the elastic body has the property of the motion of deformation such as expansion, contraction, elongation, torsion and bending.

Figure 1 shows the fundamental principle of curved space. If space curves, as shown in Figure 1 (a), then an inward normal stress "$-P$" is generated. This normal stress, i.e., surface force serves as a sort of pressure field.



(a) **(b)**

Figure 1: Curvature of Space: (a) curvature of space plays a significant role. If space curves, then inward stress (surface force) "P" is generated $\Longrightarrow$ *A sort of pressure field; (b) a large number of curved thin layers form the unidirectional surface force, i.e., acceleration field* α .

Figure 1 (b) shows that when a large number of curved thin film layers are integrated, unidirectional surface forces are integrated to form a pressure field, i.e., an acceleration field. It can be used to calculate the gravitational acceleration.

$$-P = N \cdot (2R^{00})^{1/2} = N \cdot (1/R_1 + 1/R_2) \tag{1}$$

where N is the line stress, R_1, R_2 are the radius of principal curvature of curved surface, and R^{00} is the major component of spatial curvature.

A large number of curved thin layers form the unidirectional surface force, i.e., acceleration field. Accordingly, the spatial curvature R^{00} produces the acceleration field α .

The fundamental three-dimensional space structure is determined by quadratic surface structure.

It is now understood that the membrane force on the curved surface and each principal curvature generates the normal stress "$-P$" with its direction normal to the curved surface as a surface force. The normal stress "$-P$" acts towards the inside of the surface as shown in Figure 1 (a).

A thin-layer of curved surface will take into consideration within a spherical space having a radius of R and the principal radii of curvature that are equal to the radius (R_1=R_2=R). Since the membrane force N (serving as the line stress) can be assumed to have a constant value, Eq.(1) indicates that the curvature R^{00} generates the inward normal stress P of the curved surface. The inwardly directed normal stress serves as a pressure field.

Here, we give an account of curvature R^{00} in advance. The solution of metric tensor $g^{\mu\nu}$ is found by gravitational field equation as the following:

$$R^{\mu\nu} - \frac{1}{2} \cdot g^{\mu\nu} R = -\frac{8\pi G}{c^4} \cdot T^{\mu\nu} \tag{2}$$

where $R^{\mu\nu}$ is the Ricci tensor, R is the scalar curvature, G is the gravitational constant, c is the velocity of light, $T^{\mu\nu}$ is the energy momentum tensor.

Furthermore, we have the following relation for scalar curvature R :

$$R = R^{\alpha}{}_{\alpha} = g^{\alpha\beta} R_{\alpha\beta},\ R^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} R_{\alpha\beta},\ R_{\alpha\beta} = R^{j}{}_{\alpha j\beta} = g^{ij} R_{i\alpha j\beta} \quad (3)$$

Ricci tensor $R_{\mu\nu}$ is also represented by:

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\alpha,\nu} - \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} + \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} \quad (= R_{\nu\mu}) \quad (4)$$

where $\Gamma^{i}{}_{jk}$ is Riemannian connection coefficient.

If the curvature of space is very small, the term of higher order than the second can be neglected, and Ricci tensor becomes:

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\alpha,\nu} - \Gamma^{\alpha}_{\mu\nu,\alpha} \quad (5)$$

The major curvature of Ricci tensor ($\mu = \nu = 0$) is calculated as follows:

$$R^{00} = g^{00} g^{00} R_{00} = -1 \times -1 \times R_{00} = R_{00} \quad (6)$$

As previously mentioned, Riemannian geometry is a geometry that deals with a curved Riemann space, therefore a Riemann curvature tensor is the principal quantity. All components of Riemann curvature tensor are zero for flat space and non-zero for curved space. If an only non-zero component of Riemann curvature tensor exists, the space is not flat space but curved space. Therefore, the curvature of space plays a significant role.

A large number of curved thin layers form the unidirectional acceleration field (Figure 1 (b)). Accordingly, the spatial curvature R^{00} produces the acceleration field α.

2.2. Brief Concept of Gravity

Let's think about soap bubbles.

The pressure P due to the membrane force on the surface of a soap bubble of radius R is directed inward. The membrane force on the surface of the soap bubble corresponds to N in the Figure 1 (a).

$$-P = N \cdot (1/R_1 + 1/R_2) = N \cdot (1/R + 1/R) = 2N/R \quad (7)$$

This pressure "P" keeps the soap bubbles from breaking due to the expansion force of the internal air.

Figure 2 shows the surface force P toward the center of the soap bubble. The surface of the soap bubble extends due to surface tension (line stress *N*) to maintain the shape of the soap bubble, but it is known from continuum mechanics that the surface force P toward the center of the soap bubble is working. If this is applied to the space as it is, the space curved in a spherical shape applies pressure toward the center of the sphere. This is the principle of gravity generation.

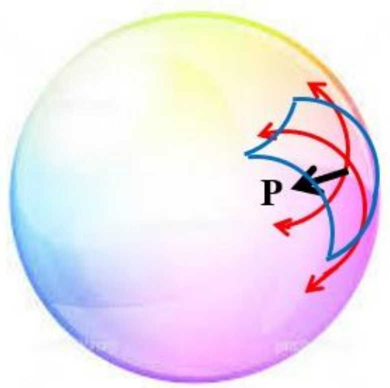


Figure 2: Surface force toward the center of the soap bubble

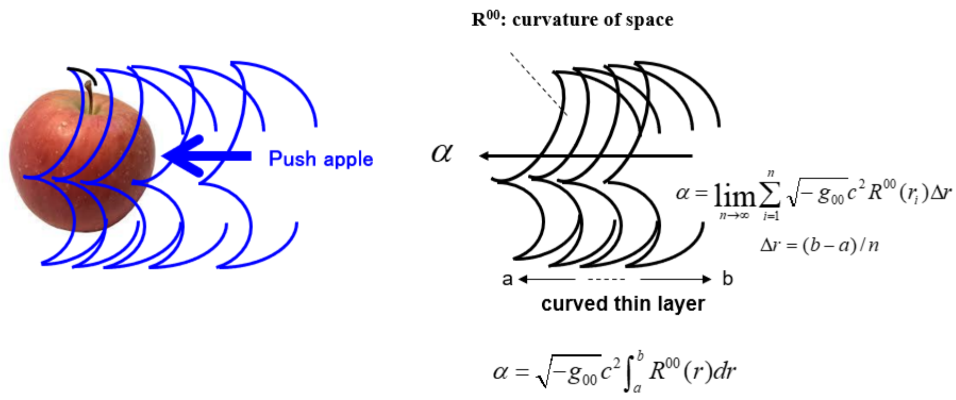


Figure 3: Surface forces of multiple curved thin film layers push apple

Figure 3 shows the surface force due to the accumulation of many curved surfaces pushes the apple.

A large number of curved thin layers form the unidirectional acceleration field (Figure 1 (b)). Accordingly, the spatial curvature R^{00} produces the acceleration field α .

$$\alpha = \sqrt{-g_{00}c^2}\int_a^b R^{00}(r)dr \tag{8}$$

where α : acceleration (m/s^2), g_{00} : time component of metric tensor, a-b: range of curved space (m), *r*: direction of radius of curvature, *c*: velocity of light, R^{00} : major component of spatial curvature(1/m^2).

Eq.(8) indicates that the acceleration field α is produced in curved space. The intensity of acceleration produced in curved space is proportional to the product of spatial curvature R^{00} and the length of curved region (i.e., *a* to *b*). See reference [3,4,5,6] for details of the derivation process of this equation.

Figure 4 shows the principle that an apple is pushed from a curved space around the Earth and falls to the Earth. As shown in Figure 4, the gravitational field around the Earth is multiply covered by concentric or spherical curved spaces centered on the Earth. Considering the case of the Earth, the curvature of space is spherically symmetric about the Earth and is fixed to the Earth, so the Earth itself cannot move due to the curvature of the space generated by the Earth.

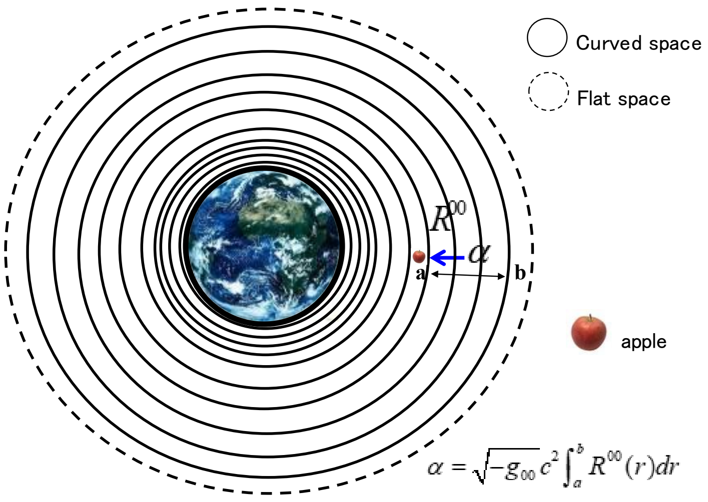


Figure 4: Why apples fall

However, as shown in Figure 4, the apple on the Earth is independently in the curved spatial region of the Earth. Since the apple exists in the curved spatial region from the curved spatial layer at the apple's position to the curved spatial layer at the distant position, the apple is pushed by the generated curved space (i.e., pressure) and falls. That is, referring to Figure 4, a sort of graduated pressure field is generated by the curved range from an arbitrary point “a” in curved space to a point “b” (the point at which space is absent of curvature, i.e., flat space of curvature "0"). Then apple moves directly towards the center of the Earth, that is, the apple falls. Falling acceleration of apple in curved space is proportional to both the value of spatial curvature and the size of curved space.

The above equation Eq.(8) gives the physical meaning of mechanism of gravity.

The mass (apples) on the Earth will not be pulled by the Earth and fall, but will be pushed and fall in the direction of the Earth due to the pressure of the field in the curved space area around the Earth. Although the spatial curvature at the surface of the Earth is very small value, i.e., $1.71\times10^{-23}(1/m^2)$, it is enough value to produce 1G (9.8 m/s^2) acceleration.

Gravity can be explained as a pressure field induced by the curvature of space.

The following Chapter 3 introduces Time Dilation due to Curved Space.

3. Time Dilation due to Curved Space

3.1. Time Dilation in the Gravitational Field

As is well known, according to Special Relativity, the time interval of moving things is delayed. When the time interval of a stationary person is Δt, the speed of light is c, and the speed of the moving body is v, the time interval $\Delta t'$ of the moving body is obtained

$$\Delta t' = \sqrt{1-\left(\frac{v}{c}\right)^2}\,\Delta t \tag{9}$$

For example, in the case of $\Delta t' = 0.436\Delta t$, where Δt elapses for 1 second, whereas $\Delta t'$ elapses only 0.436 seconds, that is, the elapse of time of the moving body becomes shorter and the time is delayed.

In addition, according to General Relativity, space-time is distorted (or curved) in the gravitational field, and the progress of time slows down.

For example, let r_g is the Schwarzschild radius, r is the distance from the center of the Earth, Δt is the time interval at a high position (flat space without gravity) far from the Earth, and $\Delta t'$ is the time interval at the distance r from the center of the Earth. The time interval $\Delta t'$ is given by the following equation.

$$\Delta t' = \sqrt{1-\frac{2GM}{c^2 r}}\,\Delta t = \sqrt{1-\frac{r_g}{r}}\,\Delta t \tag{10}$$

Concerning the derivation of Eq.(10), see APPENDIX A: Derivation of Time Dilation Equation in the Gravitational Field.

As is known well in General Relativity, the closer to the center of the Earth, the more rapidly the time delay occurs. On the contrary, the farther away from the Earth, the less time is delayed and the more it can be ignored. Here, r_g is the Schwarzschild radius, and as you can see from the equation, $r_g = 2GM/c^2$ (G: gravitational constant, M: mass of celestial body), the Schwarzschild radius is determined only by the mass, and the larger the mass, the larger it becomes. For example, the Schwarzschild radius of the sun is about 3 km, and the Schwarzschild radius of the earth is about 0.9 cm.

As mentioned above, in General Relativity, gravity changes the advance of time. Therefore, on a planet with a low gravitational potential, time advances more slowly than in outer space with a high gravitational potential. In this paper, since gravity curves space (space-time), it is shown that the curving space directly changes the advance of time, that is, the larger the curvature of space, the slower the time and the delay of time.

For example, let the time represented by a clock placed at y = 0 on the surface of the Earth be the standard time t. The time that lags below standard time or advances above standard time is expressed as (1 + ky) t with a coefficient k.

While the clock at the reference point of y = 0 advances by 1 second, the clock at the point 1 meter lower (y = −1) advances only (1−k) seconds. Conversely, at an altitude of 1 meter higher, the clock at (y = + 1) advances at the same time by (1 + k) seconds.

Since the time interval Δt' is short than Δt, that is, the lapse of time Δt' is slow, hence the time Δt' will be delayed. Here, Δt' - time interval affected by gravity; Δt - time interval uninfluenced by gravity (in a reference frame in an infinite distance from the Earth).
Gravitational time dilation is a change in the lapse of time caused by gravitational field which is described as a curving of space-time. The theory predicts that the closer an observer is to a source of gravity and the greater its mass, the slower time passes.

3.2. Time Dilation due to the Curvature of Space

Let $\alpha', \Delta t'$ be the acceleration and time interval in the curved space region with a large curvature $R^{00}\,'$, and $\alpha, \Delta t$ be the acceleration and time interval in the curved space region with a small curvature R^{00}.
Now, the strain rate of space is a value peculiar to space that is a continuum, not light. The strain rate of space, which is a unique property of space, takes a constant value "c".

$$c = \alpha \Delta t = \alpha' \Delta t' \qquad (11)$$

Consider an area of adjacent curved space (a to b; $\Delta L = b - a$), since the curvature R^{00} can be regarded as approximately constant in an area of adjacent curved space, from Eq.(8),

$$\alpha = \sqrt{-g_{00}}c^2 \int_a^b R^{00}(r)dr \approx \sqrt{-g_{00}}c^2 R^{00}\left[r\right]_a^b = \sqrt{-g_{00}}c^2 R^{00}(b-a) = \sqrt{-g_{00}}c^2 R^{00}\Delta L \qquad (12)$$

Then, using Eq.(11) and Eq.(12), we obtain:

$$\frac{\Delta t'}{\Delta t} = \frac{\alpha}{\alpha'} = \frac{\sqrt{-g_{00}}c^2 R^{00}\Delta L}{\sqrt{-g_{00}}c^2 R^{00}\,'\Delta L} = \frac{R^{00}}{R^{00}\,'} \qquad (13)$$

Accordingly, we get: $$\Delta t' = \frac{R^{00}}{R^{00}\,'}\Delta t, \quad \Delta t' < \Delta t \quad (R^{00}\,' > R^{00}) \qquad (14)$$

It can be seen that the larger the curvature of space, the smaller the time interval.
This will be described below as an intuitive picture using figures. Using the radius of curvature (s), which is the inverse number of the curvature:
Radius of curvature s:

$$s = \frac{1}{R^{00}}, \quad s = ct \quad \text{then,} \quad c = \frac{s'}{\Delta t'} = \frac{s}{\Delta t} \qquad (15)$$

Accordingly, $\frac{s'}{s} = \frac{\Delta t'}{\Delta t}$, then, $\Delta t' = \frac{s'}{s}\Delta t$ (16)

When the curvature of space is large, the radius of curvature s' is small, and when the curvature of space is small, the radius of curvature s is large. Accordingly, the propagation time of the strain rate "c" in space is short for a small radius of curvature s' and long for a large radius of curvature s.
As shown in Figure 5, the radius of curvature is small (large curvature; large gravity), and the time interval $\Delta t'$ of the time traveled in space is small, therefore the time is to be delayed.

Since the radius of curvature is short, the passage of time is also short, resulting in a time delay.
Even at the same height "h" from the black hole and the Earth, the curvature at the height "h" of the black hole is large and the radius of curvature is short, so the time interval in which time advances is shorter than the small curvature of space at the height "h" of the Earth.

Here, the strain rate of space is the propagation speed of a change in which a flat space transitions to a curved space and the curved space returns to a flat space, and its propagation speed is the same as the speed of light. As to the elastic strain wave traveling toward the center of the Earth, it is the same value as the speed of light, but it is not a photon traveling in space.

Concerning the elastic wave in continuum of space, we discuss briefly the propagation of longitudinal waves in an infinite space. In general, when a dynamic force is applied to an object, the stress and strain caused by it travel through the object as waves. This is called a stress wave. The stress wave in the elastic body is an elastic wave. Elastic waves are deformation waves that propagate through elastic bodies, and are also called elastic stress waves and elastic strain waves. The wave will be propagated with constant velocity given by:

$$c = \sqrt{\frac{\mu}{\rho}} \tag{17}$$

The strain rate, that is, propagation velocity "c" of distorted wave (strain wave) or stress wave are expressed in terms of μ (shear modulus or modulus of rigidity; Lamés' constant) and ρ (density).

The density and rigidity of the space as a continuum are undecided, but the distorted wave propagating in the space must exist.

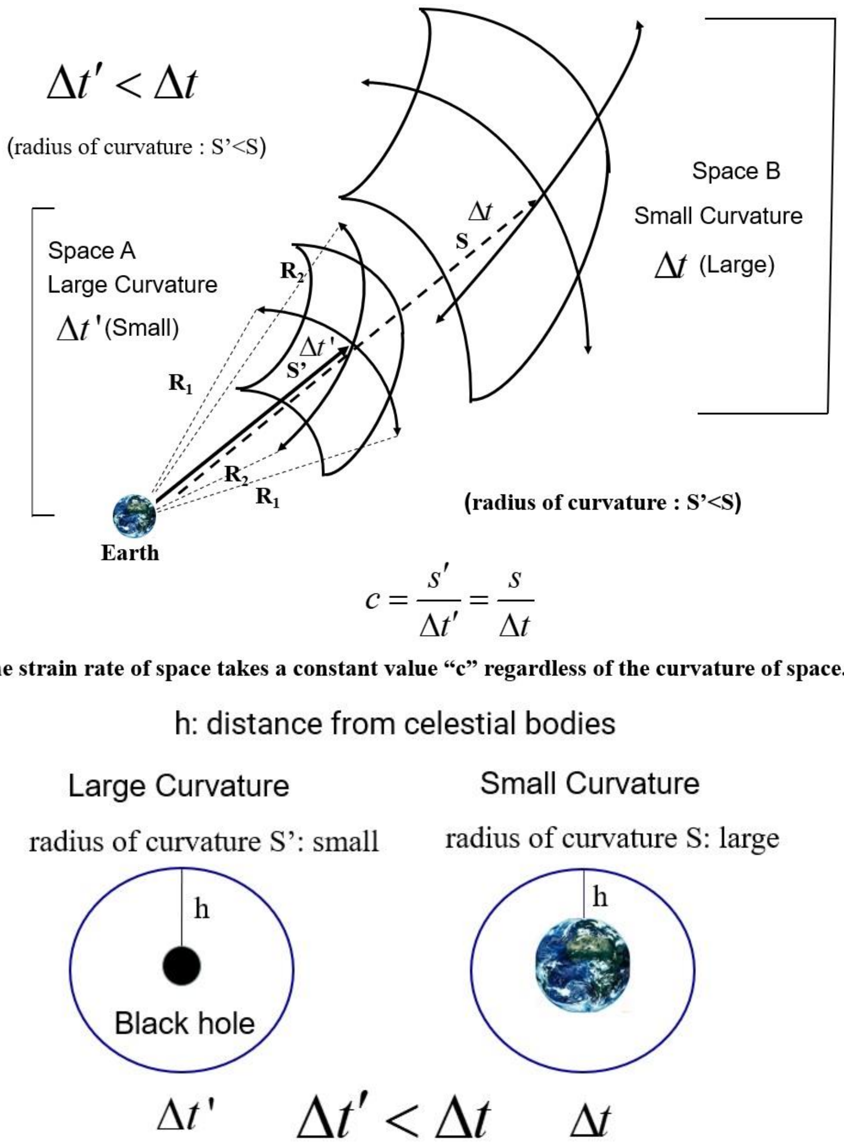


Figure 5: Time dilation due to the curvature of space.

As shown in Figure 6, if the Earth (M) were to disappear instantly, the curved surface of space close to the Earth would return to the flat surface. Because an external action causing curvature (i.e., mass energy) disappears. The change from a curved surface to a flat surface would advance the position r of the apple "m" at the speed of light (i.e., the strain rate of space-time). The propagation velocity of the change from flat space to curved space and the propagation velocity of change from curved space to flat space are both the same, i.e., the velocity of light.

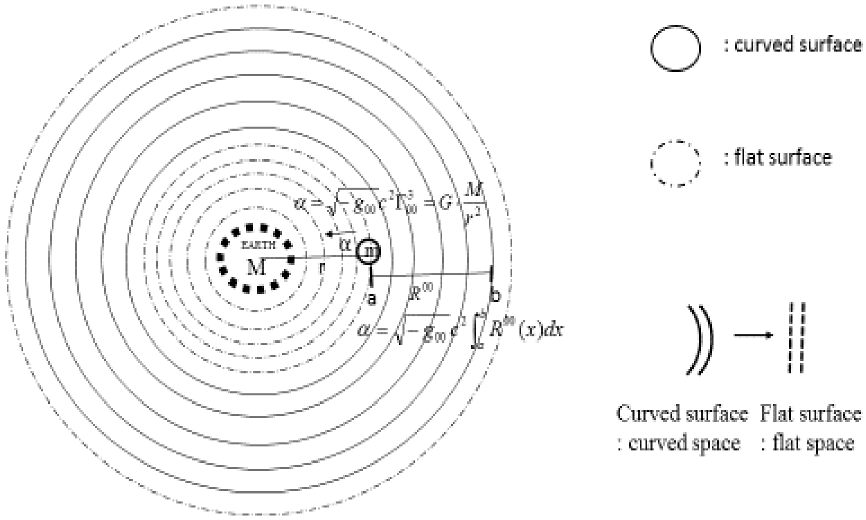


Figure 6: Propagation velocity of the change from curved space to flat space.

3.3. Related Matters: Time Dilation due to Magnetic Field

If the curvature of space is large, the progress of time will be delayed. And the curvature of space can be controlled by the magnetic field (See APPENDIX B: Curvature Control by Magnetic Field).
That is, it is possible to delay the time by magnetic field [16,17].

$$R^{00} = \frac{4\pi G}{\mu_0 c^4} \cdot B^2 = 8.2\times 10^{-38} \cdot B^2 \quad (B\ in\ Tesla) \tag{18}$$

where we let $\mu_0 = 4\pi\times 10^{-7}(H/m)$, $\varepsilon_0 = 1/(36\pi)\times 10^{-9}(F/m)$, $c = 3\times 10^{8}(m/s)$, $G = 6.672\times 10^{-11}(N\cdot m^2/kg^2)$, B is a magnetic field in Tesla and R^{00} is a major component of spatial curvature $(1/m^2)$.

From Eq.(14), $$\Delta t' = \frac{R^{00}}{R^{00}\,'}\Delta t \tag{19}$$

Although the spatial curvature at the surface of the Earth is very small value, i.e., $1.71\times 10^{-23}(1/m^2)$, it is enough value to produce 1G (9.8 m/s^2) acceleration.
Here, we estimate how much time delay will occur due to the magnetic field. The magnitude of the magnetic field that produces the curvature of space ($1.71\times 10^{-23}(1/m^2)$) on the surface of the Earth is obtained like this:

$R^{00} = 1.71\times 10^{-23} = 8.2\times 10^{-38}\cdot B^2$, then $B = 1.44\times 10^{7}\, T$.

If $B' = 1.44\times 10^{8}\, T$, $R^{00}\,' = 8.2\times 10^{-38}(1.44\times 10^{8})^2 = 1.71\times 10^{-21}$.

From Eq.(19),

$$\Delta t' = \frac{1.71\times 10^{-23}}{1.71\times 10^{-21}}\Delta t \cong 1\times 10^{-2}\,\Delta t = 0.01\Delta t$$.

Due to the application of the magnetic field $B' = 1.44\times 10^{8}$ Tesla on the ground, the passage of 1 second on the ground time becomes 1/100 second and is delayed.

By the way, a neutron star has an ultra-strong magnetic field of 800 million tesla ($8\times10^8 T$) (on its surface), the earth's magnetic field is 1 / 20,000 tesla, and the magnetic field that can be constantly created in a laboratory is currently about 100 tesla at maximum.

4. Conclusion

The time delay due to gravity is well known, but why does gravity delay time? The reason for this is mathematically explained, but the mechanism is not clear and intuitive. Gravity curves space, so this problem is synonymous with why time is delayed when space curves. This paper explains as one proposal why time is delayed when space curves based on the mechanism of gravity generation due to curved space. A clear picture and its mechanism that the time is delayed are obtained when the gravity is large, that is, the curving of space is large. It is possible to directly and pictorially derive that time is delayed when space curves. Since gravity is caused by the curving of space, the mechanism by which the curving of space delays time can provide a much more basic picture and mechanism than the explanation that gravity delays time.

After all, the radius of curvature is short, the passage of time is also short, resulting in a time delay. The mechanism of time delay due to the curved space has been clarified.

APPENDIX A: Derivation of Time Dilation Equation in the Gravitational Field

Now in general, the line element is described in:

$$ds^2 = g_{ij}dx^i dx^j = g_{00}(dx^0)^2 + g_{33}(dx^3)^2 + g_{11}(dx^1)^2 + g_{22}(dx^2)^2$$
$$= g_{00}(cdt)^2 + g_{33}(dr)^2 + g_{11}r^2(d\theta)^2 + g_{22}r^2\sin^2\theta(d\varphi)^2 \quad . \qquad \text{(A.1)}$$

We choose the spherical coordinates " $ct=x^0$, $r=x^3$, $\theta=x^1$, $\varphi=x^2$ " in space-time (see Fig. A1).

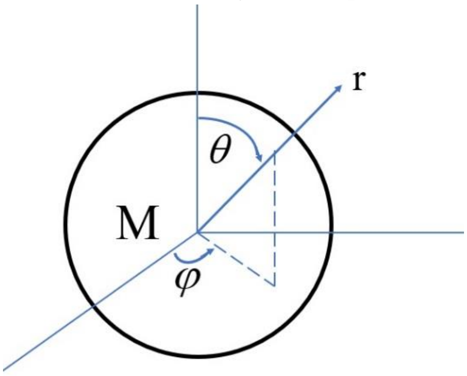


Figure A1: Spherical coordinates.

Next, let us consider External Schwarzschild Solution.

External Schwarzschild Solution is an exact solution of the gravitational field equation, which describes the gravitational field outside the spherically symmetric, static mass distribution.

The line element is obtained as follows:

$$ds^2 = -(1-\frac{r_g}{r})c^2dt^2 + \frac{1}{1-\frac{r_g}{r}}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad . \qquad \text{(A.2)}$$

The metrics are given by:

$$g_{00} = -(1-r_g/r), g_{11} = g_{22} = 1, g_{33} = 1/(1-r_g/r),$$
$$and\ other\, g_{ij} = 0\,. \qquad \text{(A.3)}$$

where r_g is the gravitational radius (i.e., $r_g = 2GM/c^2$).

The path of light satisfies the condition of $ds^2 = 0$. Now consider the light traveling in the circumferential direction instead of the central direction of the Earth. This is because consistent with the time-lag experiment due to gravity. Considering the light traveling on a curved concentric curved surface that surrounds the Earth, in the case of "$dr = d\varphi = 0$(meridian direction)", the minute distance dl that the light travels on the curved surface is $dl = rd\theta$. This is the same in the case of "$dr = d\theta = 0$", but because it's a φ direction, setting $\theta = 90°$ ($\sin\theta = 1$), then the minute distance dl that the light travels on the curved surface is $dl = rd\varphi$ (equator direction).

Assuming that the passage of time Δt on the curved surface far away ($r:\infty$) from the Earth, the passage of time $\Delta t'$ on the curved surface of the distance "r" from the Earth is as follows:

For [$dr = d\varphi = 0$]:

$$0 = -(1-\frac{r_g}{r})c^2dt^2 + r^2d\theta^2 \quad \text{(A.4)}$$

$$r^2d\theta^2 = (1-\frac{r_g}{r})c^2dt^2 \quad \text{(A.5)}$$

$$rd\theta = \pm\sqrt{1-\frac{r_g}{r}}\,cdt \quad \text{then,} \quad dl = \pm\sqrt{1-\frac{r_g}{r}}\,cdt \quad \text{(A.6)}$$

For [$dr = d\theta = 0$]:

$$0 = -(1-\frac{r_g}{r})c^2dt^2 + r^2d\varphi^2 \quad \text{(A.7)}$$

$$r^2d\varphi^2 = (1-\frac{r_g}{r})c^2dt^2 \quad \text{(A.8)}$$

$$rd\varphi = \pm\sqrt{1-\frac{r_g}{r}}\,cdt \quad \text{then,} \quad dl = \pm\sqrt{1-\frac{r_g}{r}}\,cdt \quad \text{(A.9)}$$

Namely, from Eq.(A.6) or Eq.(A9),

$$\frac{dl}{c} = \sqrt{1-\frac{r_g}{r}}\,dt \quad \text{then,} \quad \frac{dl}{c} = dt' = \sqrt{1-\frac{r_g}{r}}\,dt \quad \text{(A.10)}$$

Finally, We get the following from Eq.(A.10):

$$\Delta t' = \sqrt{1-\frac{r_g}{r}}\,\Delta t = \sqrt{1-\frac{2GM}{c^2r}}\,\Delta t \quad \text{(A.11)}$$

If r is ∞, then $\Delta t' = \Delta t$, so Δt may be the time at a distance from the Earth ($r = \infty$), and $\Delta t'$ is the time at the distance r from the Earth. This indicates a time dilation due to gravity, as the meaning is clear.

The closer it is to a celestial body, the smaller the passage of time.

To understand it, as shown in Figure A2, we can think of it as the time $\Delta t'_h$ at height "h" on the concentric sphere of the Earth is smaller than the time $\Delta t'_{h+a}$ at height "h + a" on the concentric sphere of the Earth a little away from the Earth.

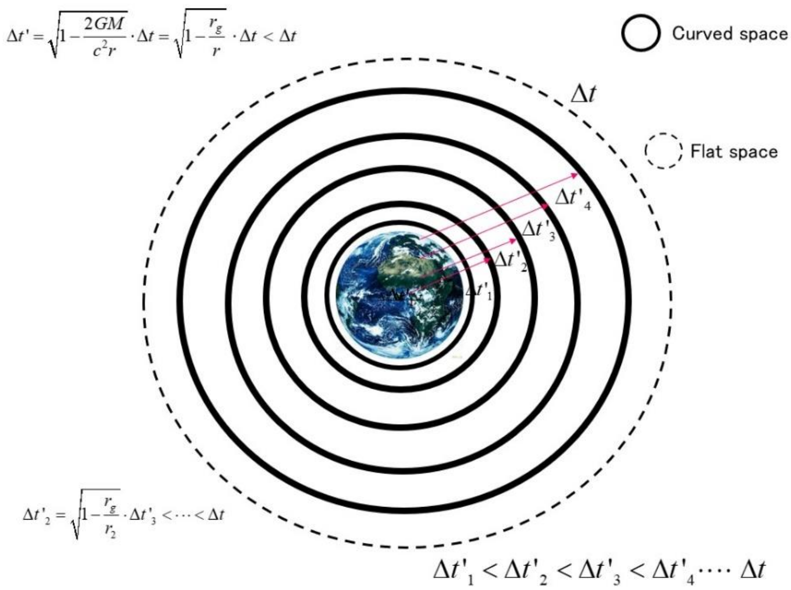


Figure A2: Time Dilation in the Gravitational Field.

As the radius of curvature increases (as the curvature decreases), the interval of time increases.

APPENDIX B: Curvature Control by Magnetic Field

Let us consider the electromagnetic energy tensor M^{ij}. In this case, the solution of metric tensor g_{ij} is found by

$$R^{ij} - \frac{1}{2}\cdot g^{ij}R = -\frac{8\pi G}{c^4}\cdot M^{ij} . \tag{B.1}$$

Eq.(B.1) determines the structure of space due to the electromagnetic energy.

Here, if we multiply both sides of Eq.(B.1) by g_{ij}, we obtain

$$g_{ij}\left(R^{ij} - \frac{1}{2}\cdot g^{ij}R\right) = g_{ij}R^{ij} - \frac{1}{2}\cdot g_{ij}g^{ij}R = R - \frac{1}{2}\cdot 4R = -R , \tag{B.2}$$

$$g_{ij}\left(\frac{-8\pi G}{c^4}\cdot M^{ij}\right) = -\frac{8\pi G}{c^4}\cdot g_{ij}M^{ij} = \frac{-8\pi G}{c^4}\cdot M_i^{\ i} = \frac{-8\pi G}{c^4}M . \tag{B.3}$$

The following equation is derived from Eqs.(B.2) and (B.3)

$$R = \frac{8\pi G}{c^4}\cdot M . \tag{B.4}$$

Substituting Eq. (B.4) into Eq.(B.1), we obtain

$$R^{ij} = -\frac{8\pi G}{c^4}\cdot M^{ij} + \frac{1}{2}\cdot g^{ij}R = -\frac{8\pi G}{c^4}\cdot\left(M^{ij} - \frac{1}{2}\cdot g^{ij}M\right) . \tag{B.5}$$

Using antisymmetric tensor f_{ij} which denotes the magnitude of electromagnetic field, the electromagnetic energy tensor M^{ij} is represented as follows;

$$M^{ij} = -\frac{1}{\mu_0}\cdot\left(f^{i\rho}f_\rho^{\ j} - \frac{1}{4}\cdot g^{ij}f^{\alpha\beta}f_{\alpha\beta}\right), \quad f^{i\rho} = g^{i\alpha}g^{\rho\beta}f_{\alpha\beta} . \tag{B.6}$$

Therefore, for M, we have

$$M = M_i^i = g_{ij} M^{ij} = -\frac{1}{\mu_0} \cdot \left(g_{ij} f^{i\rho} f_\rho^j - \frac{1}{4} \cdot g_{ij} g^{ij} f^{\alpha\beta} f_{\alpha\beta} \right)$$
$$= -\frac{1}{\mu_0} \cdot \left(f^{i\rho} f_{i\rho} - \frac{1}{4} \cdot 4 f^{\alpha\beta} f_{\alpha\beta} \right) = -\frac{1}{\mu_0} \cdot \left(f^{i\rho} f_{i\rho} - f^{i\rho} f_{i\rho} \right) = 0 \quad \text{(B.7)}$$

Accordingly, substituting $M = 0$ into Eq.(B.5), we get

$$R^{ij} = -\frac{8\pi G}{c^4} \cdot M^{ij} \quad \text{(B.8)}$$

Although Ricci tensor R^{ij} has 10 independent components, the major component is the case of $i = j = 0$, i.e., R^{00}. Therefore, Eq.(B.8) becomes

$$R^{00} = -\frac{8\pi G}{c^4} \cdot M^{00} \quad \text{(B.9)}$$

On the other hand, 6 components of antisymmetric tensor $f_{ij} = -f_{ji}$ are given by electric field E and magnetic field B from the relation to Maxwell's field equations

$$f_{10} = -f_{01} = \frac{1}{c} \cdot E_x,\ f_{20} = -f_{02} = \frac{1}{c} \cdot E_y,\ f_{30} = -f_{03} = \frac{1}{c} E_z$$
$$f_{12} = -f_{21} = B_z,\ f_{23} = -f_{32} = B_x,\ f_{31} = -f_{13} = B_y$$
$$f_{00} = f_{11} = f_{22} = f_{33} = 0 \quad \text{(B.10)}$$

Substituting Eq.(B.10) into Eq.(B.6), we have

$$M^{00} = -\left(\frac{1}{2} \cdot \varepsilon_0 E^2 + \frac{1}{2\mu_0} \cdot B^2 \right) \approx -\frac{1}{2\mu_0} \cdot B^2 \quad \text{(B.11)}$$

Finally, from Eqs.(B.9) and (B.11), we have

$$R^{00} = \frac{4\pi G}{\mu_0 c^4} \cdot B^2 = 8.2 \times 10^{-38} \cdot B^2 \quad (B \text{ in Tesla}) \quad \text{(B.12)}$$

where we let $\mu_0 = 4\pi \times 10^{-7} (H/m)$, $\varepsilon_0 = 1/(36\pi) \times 10^{-9} (F/m)$, $c = 3 \times 10^8 (m/s)$, $G = 6.672 \times 10^{-11} (N \cdot m^2/kg^2)$, B is a magnetic field in Tesla and R^{00} is a major component of spatial curvature $(1/m^2)$.

The relationship between curvature and magnetic field was derived by Minami and introduced it in 16th International Symposium on Space Technology and Science (1988) [17].

Eq.(B12) is derived from general method.

On the other hand, Levi-Civita also investigated the gravitational field produced by a homogeneous electric or magnetic field, which was expressed by Pauli (Pauli, 1981) [15]. If x^3 is taken in the direction of a magnetic field of intensity F (Gauss unit), the square of the line element is of the form;

$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + \frac{(x^1 dx^1 + x^2 dx^2)^2}{a^2 - r^2}$$
$$- \left[c_1 \exp(x^3/a) + c_2 \exp(-x^3/a) \right]^2 (dx^4)^2 \quad \text{(B.13)}$$

where r= $\sqrt{(x^1)^2+(x^2)^2}$, c_1 and c_2 are constants, $a=\dfrac{c^2}{\sqrt{kF}}$, k is Newtonian gravitational constant(G), and $x^1 \ldots x^4$ are Cartesian coordinates ($x^1 \ldots x^3$=space, $x^4=ct$) with orthographic projection.

The space is cylindrically symmetric about the direction of the field, and on each plane perpendicular to the field direction the same geometry holds as in Euclidean space on a sphere of radius a, that is, the radius of curvature a is given by

$$a=\frac{c^2}{\sqrt{kF}} \quad \text{(B.14)}$$

Since the relation of between magnetic field B in SI units and magnetic field F in CGS Gauss units are described as follows: $B\sqrt{\dfrac{4\pi}{\mu_0}} \Leftrightarrow F$, then the radius of curvature "*a*" in Eq.(B14) is expressed in SI units as the following (changing symbol, $k \to G, F \to B$):

$$a=\frac{c^2}{\sqrt{GF}}=\frac{c^2}{\sqrt{G}\cdot B\sqrt{\dfrac{4\pi}{\mu_0}}} \qquad \approx (3.484\times 10^{18}\,\frac{1}{B}\quad meters).$$

While, scalar curvature is represented by $R^{00}\approx R=\dfrac{1}{a^2}=\dfrac{GB^2\dfrac{4\pi}{\mu_0}}{c^4}=\dfrac{4\pi G}{\mu_0 c^4}B^2$,

which coincides with (B.12).

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