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Relative Motion Analysis of a Floating Production Storage and Offloading Vessel in the Gulf of Guinea

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Abstract In this study, the analysis of the relative motion at the bow between an FPSO and a head sea wave in the gulf of guinea was done. Based on the strip theory and regular wave model, the Response Amplitude Operators (RAOs) and the excitation forces were determined. Moreover, spectral analysis was done on the result obtained from the regular wave analysis using the one hundred year return period storm metocean data of the area of operation of the vessel to obtain its most probable maximum responses in a real sea condition which is highly irregular. Hence, the relative motion spectrums and responses, namely relative displacement and relative velocity, of the vessel were obtained from the analysis of the relative motion RAOs. Furthermore, the study analysed the probability of green water and slamming occurrence and predicted the number of green water per hour, and number of slamming per hour in order to determine the operability of the vessel on the basis of the operability limit criteria. The result indicates that the probabilities of green water and slamming occurrence is very small, and hence the number of green water and slamming per hour. What is more, the study predicted that the vessel is operable in the gulf of guinea.

Keywords Relative motion, Relative velocity, Slamming, Green water, Operability limit criteria

Nomenclature

A	Area of cross section
B	Beam of vessel
D	Draught of vessel
L	length of vessel
k	Wave number
g	Acceleration due to gravity
t	Any time instant
Φ	Velocity potential
ω	Circular wave frequency
$\hat{\xi}$	Amplitude of wave elevation
$\hat{\eta}$	Response amplitude
$\dot{\eta}_3, \dot{\eta}_5$	Velocity terms of heave and pitch response
η	Vessel response
ρ	Density of sea water
RAO_3	Heave motion response amplitude operator
RAO_5	Pitch motion response amplitude operator

1.0 Introduction

When the sea keeping qualities of a ship is assessed, the green water and slamming occurrences are important factors taken into consideration. Interestingly, these phenomena are directly governed by the relative motion of the vessel. The relative motion is a measure of the vertical motion of a ship with respect to the wave profile. This can be analysed either numerically or theoretically in the frequency domain, or in the time domain

The relative motion between the wave and the vessel at the bow was treated in the frequency domain in [1]. It was recommended that the formation of the transfer function in the frequency domain be done for the relative vertical motion between the ship and wave profile for any heading and at any chosen station along the longitudinal (x) axis of the vessel (ship and ship-shaped offshore structures), where the axial axis y =0. Furthermore, he reported that "green water will appear on deck if the relative motion is larger than the freeboard" and then gave a statistical description of the probability of occurrence of green water on deck, and bottom emergence having obtained the relative vertical motion of the vessel.

The vertical motion modes which are introduced by hydrostatic restoring forces significantly affect the response of a ship or floating platforms subjected to wave loads in deep waters [2]. These modes of ship motion are also responsible for the relative motion of the vessel. In [3], it was recommended that the result got from uncoupled motion analysis for a moored FPSO in harsh environments are valid for use in the early design stage of the vessel and its mooring system. Hence, this paper adopts the uncoupled approach for the analysis done here.

[4] presented an orientation into the various aspects that contributes considerably to the green water loading on FPSO vessels on the basis of model tests. Hence, he investigated the phenomena leading to green water loading, the impact of metocean conditions such as wave height, wave period, currents speed among others, and also how the shape of the bow, position of deck equipment and shape of breakwaters influence green water loading.

From the findings of [5] and [6] in connection with the study of FPSOs, their recommendation has been one of case studies. Moreover, because of the cost associated with carrying out seakeeping experiments, nowadays numeric software are deployed in ship motion research. Where this software are not available or accessible, researchers have resorted to developing their own computer programmes for such analyses. This is the case in this research that modeled the vessel as a rectangular box.

[7] used a wave spectrum to evaluate the relative vertical motion between the wave and the vessel at the bow, and then the freeboard exceedance having obtained the maximum heave and pitch responses. The results obtained enabled him to develop the computer programme for the analysis and prediction of green water on a ship-shaped offshore structure [8].

In this paper, the vertical motions of the Bonga FPSO were analysed in the frequency domain using the regular wave theory and considering that the vessel is acted upon by a head sea. Once the amplitudes of the heave and pitch motions were obtained, they were used for the relative motion analysis to get the amplitudes of the relative displacement and relative velocity between the wave and the vessel at bow. Furthermore, the Response Amplitude Operators (RAO) of the relative displacement and relative velocity which was derived from the regular wave theory, were subjected to spectral analysis in order to account for the irregular nature of the sea.

2.0 Regular Wave Analysis

2.1 Wave Parameter

Based on the regular wave theory and a coordinate system fixed with respect to the mean position of the vessel, with x in the direction of forward motion, z vertically upward through the centre of gravity of the vessel, and the origin in the plane of the undisturbed still water surface. The wave profile thus derived from the velocity potential for head sea is:

$$\xi = \hat{\xi}\, sin(kx + \omega t) \tag{1}$$

Also, the vertical velocity of the fluid particle got from the velocity profile is

$$V_z = \frac{\partial \Phi}{\partial z} = \hat{\xi}\omega e^{kz}\, cos(kx + \omega t) \tag{2}$$

The vertical particle acceleration is the time derivative of the vertical velocity. So,

$$\dot{V}_z = \frac{\partial V_z}{\partial t} = -\hat{\xi}\omega^2 e^{kz}\, sin(kx + \omega t) \tag{3}$$

The dynamic pressure of the wave is a function of the time derivative of the velocity profile and is given as

$$P = -\rho\frac{\partial\Phi}{\partial t} = \rho g\hat{\xi}e^{kz}\,sin(kx+\omega t) \tag{4}$$

Since the vessel operates in deep water, the dispersion relationship derived from the velocity profile and then applying the Cauchy-Poisson condition gives

$$k = \frac{\omega^2}{g} = \frac{2\pi}{\lambda} \tag{5}$$

2.2 Relative Displacement

At any point x, along the length of the vessel, the relative displacement between the wave and the vessel at the bow can be given as:

$$\eta_r = \eta_3 - x\eta_5 - \xi \tag{6}$$

At the bow where $x = +\frac{L}{2}$ from amidships, then

$$\eta_r = \eta_3 - \frac{L}{2}\eta_5 - \hat{\xi}\,sin\left(\omega t + \frac{kL}{2}\right) \tag{7}$$

$$\eta_r = \hat{\eta}_3\,sin(\omega t) - \frac{L}{2}\hat{\eta}_5\,cos(\omega t) - \hat{\xi}\,sin\left(\omega t + \frac{kL}{2}\right) \tag{8}$$

Expanding $sin\left(\omega t + \frac{kL}{2}\right)$ and then substituting into the above equation gave us:

$$\eta_r = \hat{\eta}_3\,sin(\omega t) - \frac{L}{2}\hat{\eta}_5\,cos(\omega t) - \hat{\xi}\left\{sin(\omega t)\,cos\left(\frac{kL}{2}\right) + cos(\omega t)\,sin\left(\frac{kL}{2}\right)\right\} \tag{9}$$

$$\eta_r = \left[\hat{\eta}_3 - \hat{\xi}\,cos\left(\frac{\pi L}{\lambda}\right)\right]sin(\omega t) - \left[\frac{L}{2}\hat{\eta}_5 + \hat{\xi}\,sin\left(\frac{\pi L}{\lambda}\right)\right]cos(\omega t) \tag{10}$$

The magnitude of the amplitude of the relative motion between the wave motion and the heave motion of the vessel at the bow will thus be:

$$\hat{\eta}_r = \begin{Bmatrix} \left[\hat{\eta}_3 - \hat{\xi}\,cos\left(\frac{\pi L}{\lambda}\right)\right]^2 \\ + \left[\frac{L}{2}\hat{\eta}_5 + \hat{\xi}\,sin\left(\frac{\pi L}{\lambda}\right)\right]^2 \end{Bmatrix}^{0.5} \tag{11}$$

Dividing by the amplitude of the wave profile yields the Response Amplitude Operator for the relative motion which is:

$$RAO_r = \frac{\hat{\eta}_r}{\hat{\xi}} = \begin{Bmatrix} \left[\frac{\hat{\eta}_3}{\hat{\xi}} - cos\left(\frac{\pi L}{\lambda}\right)\right]^2 \\ + \left[\frac{L\hat{\eta}_5}{2\hat{\xi}} + sin\left(\frac{\pi L}{\lambda}\right)\right]^2 \end{Bmatrix}^{0.5} \tag{12a}$$

In terms of RAO_3 and RAO_5, the above equation is expressed as:

$$RAO_r = \begin{Bmatrix} \left[RAO_3 - cos\left(\frac{\pi L}{\lambda}\right)\right]^2 \\ + \left[\frac{L}{2}RAO_5 + sin\left(\frac{\pi L}{\lambda}\right)\right]^2 \end{Bmatrix}^{0.5} \tag{12b}$$

2.3 Relative Velocity

The relative velocity between the wave and the vessel at any station x along the longitudinal direction of the vessel where slamming is to be analyzed, is defined as given by the relationship:

$$\dot{\eta}_r = \dot{\eta}_3 - x\dot{\eta}_5 - \dot{\xi} \tag{13}$$

Thus, the relative velocity of the ship at the bow ($x = +\frac{L}{2}$) would be:

$$V_r = \dot{\eta}_r = \omega\hat{\eta}_3\cos\omega t + \frac{L}{2}\omega\hat{\eta}_5\sin\omega t - \omega\hat{\xi}\cos\left(\omega t + \frac{kL}{2}\right) \tag{14}$$

$$V_r = \omega\hat{\eta}_3\cos\omega t + \frac{L}{2}\omega\hat{\eta}_5\sin\omega t - \omega\hat{\xi}\cos\frac{kL}{2}\cos\omega t + \omega\hat{\xi}\sin\frac{kL}{2}\sin\omega t \tag{15}$$

$$V_r = \omega \begin{bmatrix} \left(\hat{\eta}_3 - \hat{\xi}\cos\frac{\pi L}{\lambda}\right)\cos\omega t \\ + \left(\frac{L}{2}\omega\hat{\eta}_5 + \omega\hat{\xi}\sin\frac{\pi L}{\lambda}\right)\sin\omega t \end{bmatrix} \tag{16}$$

The amplitude of the relative velocity will therefore be:

$$\hat{V}_r = \omega \begin{bmatrix} \left(\hat{\eta}_3 - \hat{\xi}\cos\frac{\pi L}{\lambda}\right)^2 \\ + \left(\frac{L}{2}\omega\hat{\eta}_5 + \omega\hat{\xi}\sin\frac{\pi L}{\lambda}\right)^2 \end{bmatrix}^{0.5} \tag{17}$$

The response amplitude operator would then be:

$$RAO_{Vr} = \omega . RAO_r \tag{18}$$

3.0 Spectral Analysis

3.1 Wave Data and FPSO Particulars

Using the Pierson-Moskowitz wave spectrum which has been expressed in terms of the zero up-crossing period (Tz) and significant wave height (Hs), and the 100 year return period storm data of the gulf of guinea (in West Africa) which describes the wave environment, the analysis was done. The wave data of the gulf of guinea is H_S = 2.7m, and T_Z =7.6s. The modified Pierson-Moskowitz wave spectrum is expressed thus as:

$$S_w(\omega) = \frac{124}{T_z^4} H_s^2 \omega^{-5} e^{\left(-\frac{496}{T_z^4}\omega^{-4}\right)} \tag{19}$$

The particulars of the FPSO used for this analysis are: L (305m), B (58m), D (32m), T (23.4m).

3.2 Relative Displacement

The response spectrum of the relative motion displacement as a function of wave frequency is given as:

$$S_r(\omega) = (RAO_r)^2 . S_w(\omega) d\omega \tag*{20)}$$

The zeroth moment with which the significant and most probable maximum response amplitude is obtained is the area under the response spectrum curve, and is expressed as:

$$M_{0r} = \int_0^\infty S_r(\omega) . d\omega = \int_0^\infty \left|\frac{\hat{\eta}_r}{\hat{\xi}}\right|^2 . S_w(\omega)\, d\omega \tag{21}$$

In terms of the significant response amplitude of the relative motion:

$$M_{0r} = \left(\frac{\hat{\eta}_{rs}}{2}\right)^2 \tag{22}$$

Thus, the most probable maximum response amplitude of the relative motion is:

$$\hat{\eta}_{rmx} = 1.86\hat{\eta}_{rs} \tag{23}$$

3.3 Exceedance of the Freeboard

Having determined the most probable maximum amplitude of the relative displacement, it is common practice to extend the analysis to determine the freeboard exceedance of FPSO units in order to prevent green water on deck. The exceedance of the freeboard is defined by the relation:

$$f_e = \hat{\eta}_{rmx} - D + T \tag{24}$$

The difference between the depth (D), and the mean draught (T), of the vessel gives the freeboard (f_b) which when exceeded during the relative motion, leads to water on deck.

The probability density function of the maximum and minimum values for a spectrum whose frequency range is not too wide, is given by the Rayleigh distribution:

$$f(\hat{\eta}_{rmx}) = \frac{4\hat{\eta}_{rmx}}{\hat{\eta}_{rs}{}^2} . exp\left\{-2\left(\frac{\hat{\eta}_{rmx}}{\hat{\eta}_{rs}}\right)^2\right\} \tag{25}$$

Thus the probability that the amplitude $\hat{\eta}_{rmx}$ would exceed a threshold value $(D - T)$ so that green water will occur is:

$$P(gw) = P(f_e) = exp\left\{-2\left(\frac{f_b}{\hat{\eta}_{rs}}\right)^2\right\} \tag{26}$$

Furthermore, the average number of times per hour that the above probability and hence green water occur is:

$$Tn_{gw} = \frac{3600}{T_z}.P(f_e) \tag{27}$$

Based on the general operability limiting criteria for merchant vessels (Journée and Massie 2001), the probability of green water occurrence is:

$$P(gw) = 0.05 \tag{28}$$

3.4 Relative Velocity

The response spectrum of the relative velocity as a function of wave frequency is:

$$S_{Vr}(\omega) = (RAO_{Vr})^2.S_w(\omega)d\omega \tag{29}$$

The variance of the relative velocity will thus be:

$$M_{0Vr} = \int_0^\infty (RAO_{Vr})^2.S_w(\omega)\,d\omega \tag{30}$$

Also, in terms of the significant response amplitude of the relative velocity,

$$M_{0Vr} = \left(\frac{\hat{V}_{rs}}{2}\right)^2 \tag{31}$$

Hence, the most probable maximum response amplitude of the relative velocity is:

$$\hat{V}_{rmx} = 1.86\hat{V}_{rs} \tag{32}$$

3.5 Bow Emergence

Slamming phenomenon occurs when two statistically independent conditions are met. According to [9], the ship's bow must emerge at the instance the amplitude of the vertical relative displacement at 90 percent of the length of the ship is larger than the draught at that location. For the FPSO, the mean draught is given as T. The probability of bow emergence is approximately:

$$P(\hat{\eta}_{rmx} > T) = exp\left\{-2\left(\frac{T}{\hat{\eta}_{rs}}\right)^2\right\} \tag{33}$$

Also, a threshold vertical relative velocity (V_{rt}) that has no forward speed effect, between the wave surface and the bow of the ship at the point of impact must be exceeded. This is defined as:

$$V_{rt} = 0.093(gL)^{\frac{1}{2}} \tag{34}$$

The probability of exceedance of this threshold value by the amplitude of the relative velocity is therefore:

$$P(\hat{V}_{rmx} > V_{rt}) = exp\left\{-2\left(\frac{V_{rt}}{\hat{V}_{rs}}\right)^2\right\} \tag{35}$$

Since the above two conditions are statistically independent, then the probability of slamming occurring is thus estimated as:

$$P(sl) = exp\left[-2\left(\left(\frac{V_{rt}}{\hat{V}_{rs}}\right)^2 + \left(\frac{T}{\hat{\eta}_{rs}}\right)^2\right)\right] \tag{37}$$

The number of times slamming occurs per hour can thus be estimated as:

$$Tn_{sl} = \frac{3600}{T_z}.exp\left[-2\left(\begin{array}{c}\left(\frac{V_{rt}}{\hat{V}_{rs}}\right)^2 \\ +\left(\frac{T}{\hat{\eta}_{rs}}\right)^2\end{array}\right)\right] \tag{38}$$

The probability of slamming according to the general operability limiting criteria for merchant vessels with length greater than 300m has been given as:

$$P(slamming) = 0.01 \tag{39}$$

4.0 Results and Discussions

Figure 1 shows a steady rise in the RAO of the relative motion at lower frequencies up to 0.39rad/s. At this frequency, the vessel attained its maximum RAO which also began to decline as the frequency increased to about 0.49rad/s where the RAO hovered around 1.0exp-3 to the frequency of 1.0 rad/s. Beyond that the RAO became constant at 1.0exp-3 irrespective of the increase in wave frequency.

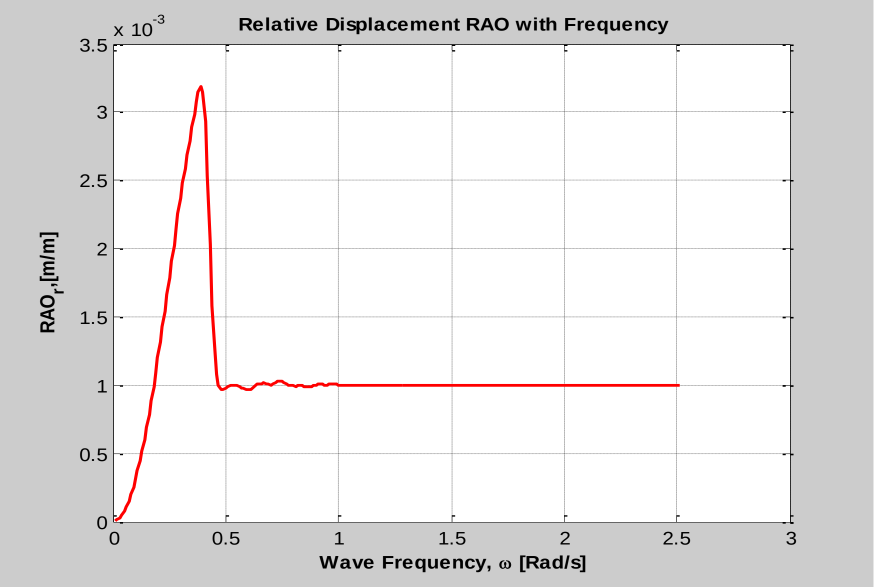

Figure 1: Relative Displacement RAO with Frequency

In terms of wavelength, Figure 2 indicates that at higher wavelengths –which corresponds to lower values of $\frac{L}{\lambda}$ and are wavelengths greater than the length of the vessel- the RAO consistently increased to a peak of 3.19x10^{-3} m/m, at a wavelength of about 407m ($\frac{L}{\lambda}$ value of 0.75).

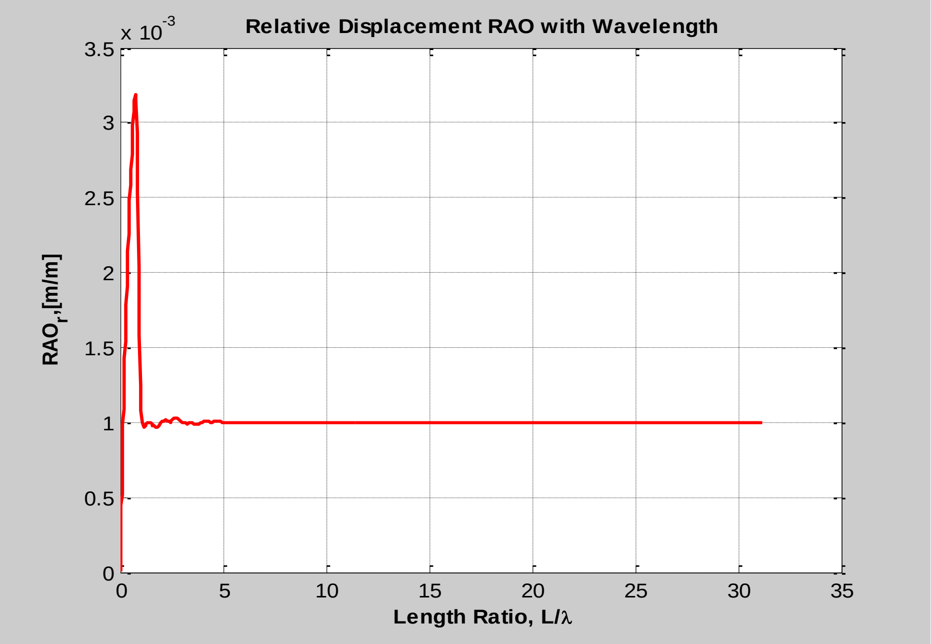

Figure 2: Variation of Relative Displacement with Wavelength

Beyond this it consistently decreased to $\frac{L}{\lambda}$ value of 1.88 (wavelength of 162m). At wavelengths smaller than $\frac{1}{5}$ of the length of the vessel, the RAO of the relative displacement is noticed to be constant at 1x10^{-3} m/m.

As indicated in Figure 3, at very low frequencies the RAO of the relative velocity steadily increased with an increase in frequency up to 0.4rad/s and then decreased steadily to a frequency of 0.48rad/s. Beyond that frequency, a linear variation was noticed between the RAO and the wave frequency.

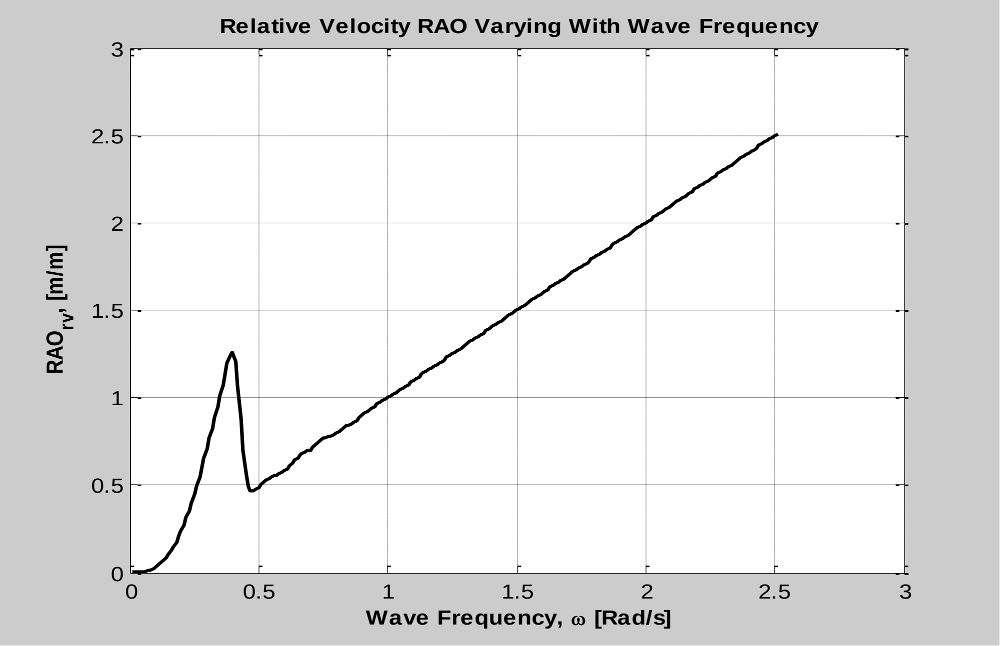

Figure 3: RAO of Relative Velocity in Variation with Frequency

Furthermore, Figure 4 shows an increase in RAO of the relative velocity at wavelengths longer than the length of the vessel. Moreover, as the $\frac{L}{\lambda}$ value of 0.792 (wavelength of 385m) decreased to a wavelength of 267.5m, there was a resultant decrease in RAO. Below this wavelength, the RAO steadily increased.

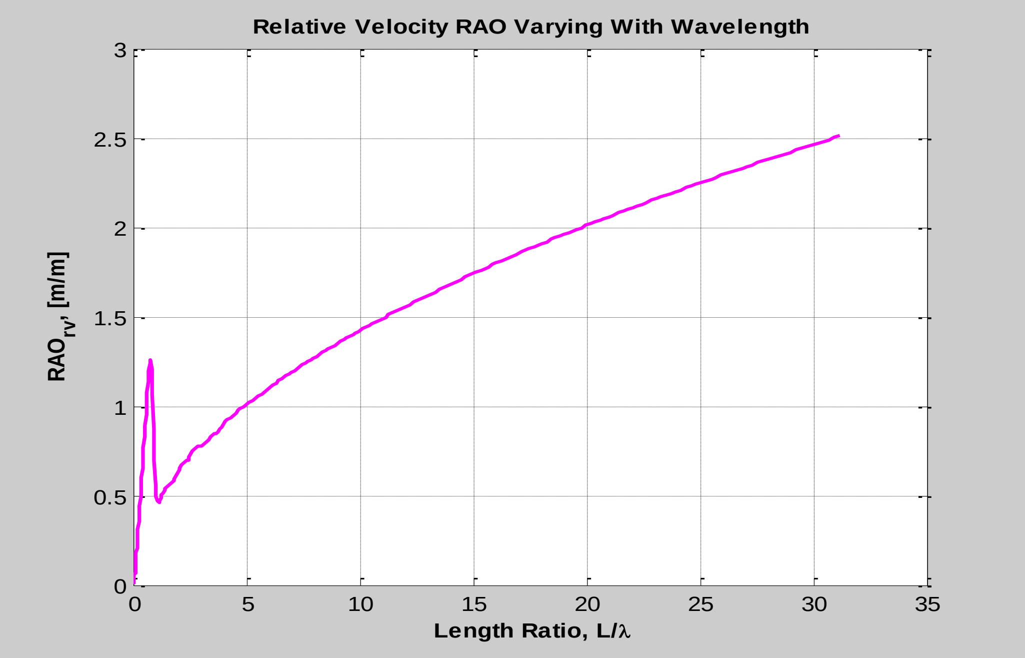

Figure 4: Variation of Relative Velocity RAO with Wavelength

From Figure 5, which is the graph of the relative motion response used to determine the above probability from the most probable maximum response amplitude. It can be seen that at very low wave frequencies-corresponding to longer wave lengths, the vessel does not experience any significant response. This is also true at wave frequencies greater than or equal to 2rad/s as the graph indicates, which is also when the wave is shorter. However, from wave frequencies 0.4rad/s to about 2rad/s, there is significant response by the vessel. It was also noticed that there is a sudden spike in the response of the vessel at approximately 4rad/s, giving it its maximum response.

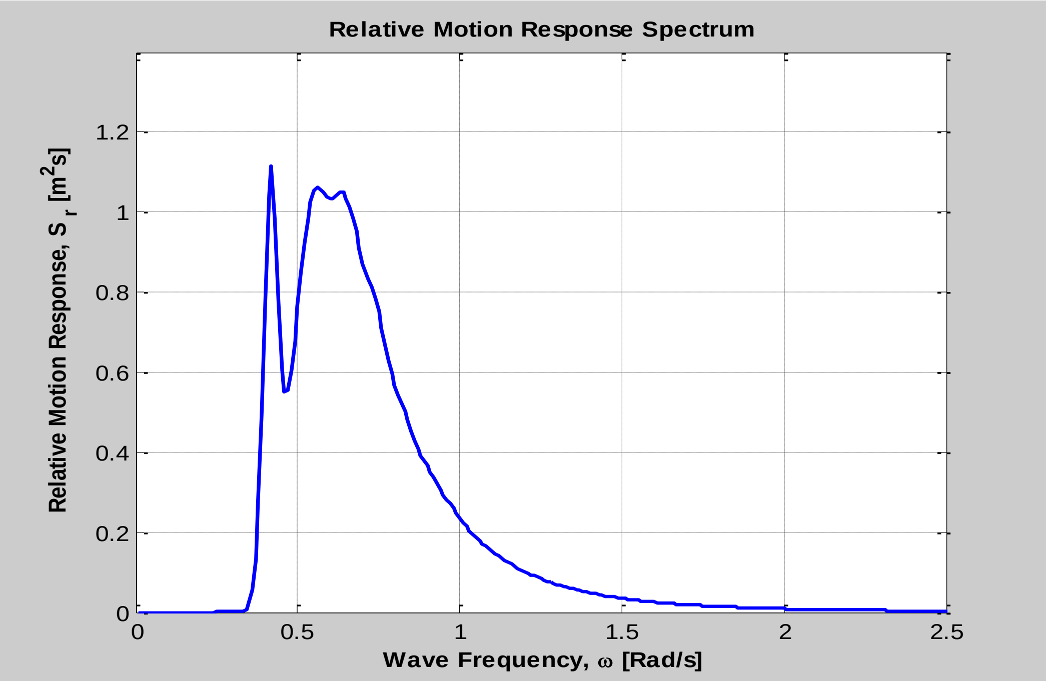

Figure 3: Relative Motion Response Varying with Wave Frequency

The most probable maximum amplitude of relative motion, $\eta_{\mathrm{rmx}} = 2.6314\mathrm{m}$

Probability of green water, P (green water) =$8.0006\mathrm{e}^{-033}$

Number of green water per hour, $Tn_{gw} = 3.7898e^{-030}$

Figure 6 is the relative velocity response spectrum from which we got the most probable maximum response amplitude of the relative velocity. The graph indicates that at lower frequencies below 0.3rad/s, the vessel experiences an insignificant response due to the relative velocity between the wave and the bow. Above that frequency, the response increases and attains a peak at approximately 0.7rad/s. beyond this point, the vessel steadily declines in its response due to the relative velocity even at high frequencies and short wave length.

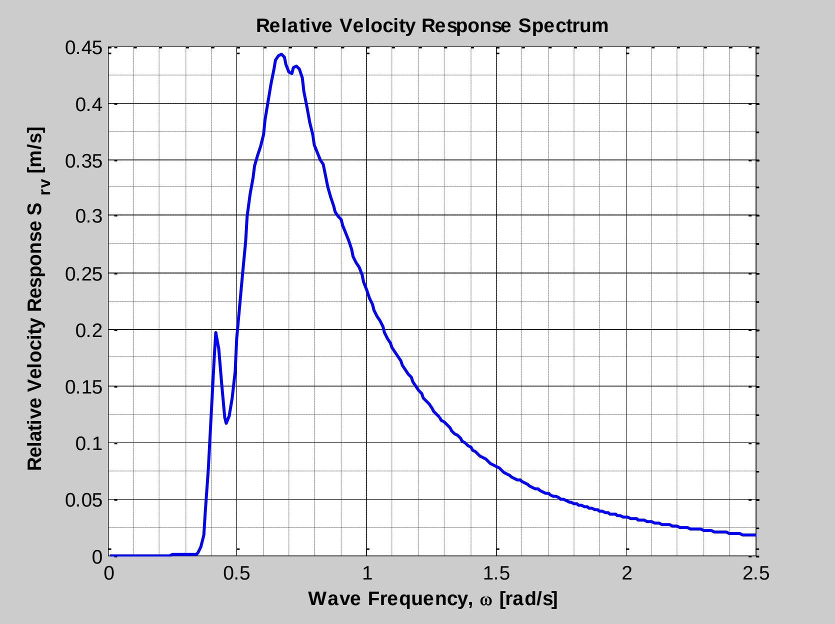

Figure 3: Relative Velocity in Variation with Wave Frequency

From the result, we have that the most probable maximum amplitude of the relative velocity $\mathrm{V}_{\mathrm{rmx}} = 2.03\mathrm{m/s}$

Probability of Slamming occurring, P (slamming) = $3.173\mathrm{e}^{-257}$ which is far less than the operability limiting criteria.
Number of slamming incidence per hour, $Tn_{sl} = 1.503e^{-254}$.

4.0 Conclusion

For low wave frequencies (long waves) with respect to the relative displacement, there is a translation of the waves into corresponding ship motions with varying amplitude and phase. But for high frequencies (short waves), the resulting ship motions transferred by the waves has same amplitude and phase. With respect to the relative displacement, the motion characteristics of the vessel is such that at low frequencies the motion varies in amplitude and phase, but varies only in amplitude while the phase remains the same at higher frequencies.

The relative displacement due to the relative motion between the wave and the vessel at the bow does not have a significant effect on the response of the vessel at very low wave frequencies and high wave frequencies. The response of the vessel to the relative velocity is insignificant only at very low wave frequencies. On the whole, the effect of the responses of the vessel due to relative motion is minimal at best as the value of the responses are quite small

Both the probability of slamming occurring and the number of slamming at the bow per hour is very small yes, insignificant. The result also shows an infinitesimal occurrence of green water on deck. Thus, deck-wetness will seldom occur as the susceptibility of the vessel to green water is slim.

Hence, on the basis of the operability limiting criteria, the vessel is thus adjudged to be operable in the gulf of guinea.

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