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**Research Article**

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# Design of maximum output power of wave energy

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**Abstract** In this paper, differential equations are used to describe the heave motion of a wave energy device consisting of a float vibrator system in the ocean, and then an optimization model for the maximum output power of wave energy is established.

**Keywords** Differential equation; Optimization model; Newton's second law

## 1. Introduction

Wave energy device is a system composed of floater, vibrator and other accessories. Under the excitation of wave, the float moves and drives the vibrator to heave and pitch together. When the float and vibrator generate relative motion, the linear damping and rotary damping at the connection will lose the system energy and output work. In this paper, we need to establish a mathematical model to describe the motion law of the float oscillator system, and then optimize the design of the connection damping, so as to maximize the average output power of the float oscillator system under given external conditions.

## 2. Model assumptions and symbol description

### 2.1 Model assumptions

(1) Ignore all kinds of friction between axle, base, compartment and air.
Reason: The vibrator, central shaft and PTO are all sealed inside the float as a capacity conversion device. The conversion efficiency should be improved as much as possible to reduce unnecessary friction.
(2) Ignore the mass of other small components of the float and vibrator system.
Reason: The quality of other small components is relatively small, and secondary factors are ignored to simplify the model.

### 2.2 Symbol Description

Only the main symbols are given here, and other symbols are described where they appear in the text. The International System of Units is adopted by default for each physical quantity and its unit symbol is omitted.

**Table 1:** Symbol Description

| Symbol | Symbol Description | Unit |
|---|---|---|
| $m_f$ | Mass of float | $kg$ |
| $m_v$ | Mass of vibrator | $kg$ |
| $v_f(t)$ | Instantaneous velocity of float | $m/s$ |

| | | |
|---|---|---|
| $v_v(t)$ | Instantaneous velocity of vibrator | $m/s$ |
| $x_f(t)$ | Float displacement | $m$ |
| $x_v(t)$ | Displacement of vibrator | $m$ |

## 3 Establishment of model

### 3.1 Hydrostatic resilience $F_{hr}$

Here, we take the equilibrium position of the system as the displacement reference point. When the float produces displacement $x$, the float also changes. The change of buoyancy is the gravity of a cylindrical water body with height of and a bottom radius of $r_c$ By Archimedes buoyancy law

$$\boldsymbol{F_{hr}(x) = \rho g V = -\rho g \pi r_c^2 x}$$

Where $\rho = 1000, r_c = 1$.

### 3.2 Additional inertial mass $m_h$

When seawater pushes the float to move, it also pushes the flow around the float to move with it, which increases the mass relative to the motion system. Therefore, it is necessary to consider the additional inertial force, and a virtual mass, that is, an additional mass $m_h = 1165.99$ , can be added to the float mass.

### 3.3 Wave-making resistance

When the float swings in seawater, the rising waves will hinder the float's movement, and the wave-making resistance $F_w$ is proportional to the swaying speed and proportional coefficient is $K_{wF} = 167.8395$.

### 3.4 Spring elasticity

A spring is connected between the float and the oscillator, which can play a buffering role: when the relative motion of the float and the oscillator is intense, part of the kinetic energy of the float-oscillator system is stored in the spring in the form of elastic potential energy; When the relative motion of float and oscillator is relatively gentle, the spring recovers its original length and releases elastic potential energy, which is converted into kinetic energy of float-oscillator system. Spring elasticity $F_T$ is a conservative force, which only transforms the mechanical energy of float-oscillator system without consuming its mechanical energy.

According to Hooke's Law, the elastic force of a spring is proportional to its deformation. In the initial state, the vibrator compresses the spring to produce a compression amount $\frac{m_v g}{k_T}$, the displacement $x_f(t)$ of the float causes the spring to produce a corresponding compression amount $x_f(t)$, and the displacement $x_v(t)$ of the vibrator causes the spring to produce a corresponding stretch amount. Taking the initial equilibrium position of the vibrator as a reference point, it can be obtained that the elongation amount of the spring at $t$ time is $x_v(t) - x_f(t)$, and the corresponding contraction elastic force generated by the spring is $F_T = k_T(x_v(t) - x_f(t))$. $k_T = 80000$ is the spring stiffness. According to Newton's third law, the spring force on the float is vertically upward, while the spring force on the oscillator is vertically downward.

### 3.5 Damping force

The damping connecting float and oscillator plays a role in consuming the mechanical energy of float-oscillator system. When the float and oscillator move relatively, the damper acts to disperse the mechanical energy consumption of the float-oscillator system to the outside of the system, that is, to output work to the outside.

It is assumed that the damping force of the linear damper is proportional to the relative velocity of the oscillator and the float. If the damping coefficient is $k_d$, the damping force is $f_d = k_d(x_v(t) - x_f(t))$, where $v_f(t)$ is the instantaneous velocity of the float and $v_v(t)$ is velocity of the oscillator.

### 3.6 Description of the motion law of float-oscillator system--differential equations

The force of float and oscillator is analyzed, and Newton's Second Law of Motion-Force and Acceleration is listed

$$f\cos\omega t - K_{wF}\times v_f(t) - \rho g\pi r_c^2 x - k_T\left(v_v(t)-x_f(t)\right) - k_d\left(v_v(t)-x_f(t)\right) = (m_f+m_h)a_f$$

$$k_T\left(v_v(t)-x_f(t)\right) + k_d\left(v_v(t)-x_f(t)\right) = m_v a_v$$

Among them, float acceleration is $a_f = \frac{d^2x_f}{dt^2}$, float velocity $v_f = \frac{dx_f}{dt}$, oscillator acceleration $a_v = \frac{d^2x_v(t)}{dt^2}$ and oscillator velocity $v_v = \frac{dx_v}{dt}$, and $x_v(t)$ is the displacements of the oscillator relative to its translational position on the still sea surface.

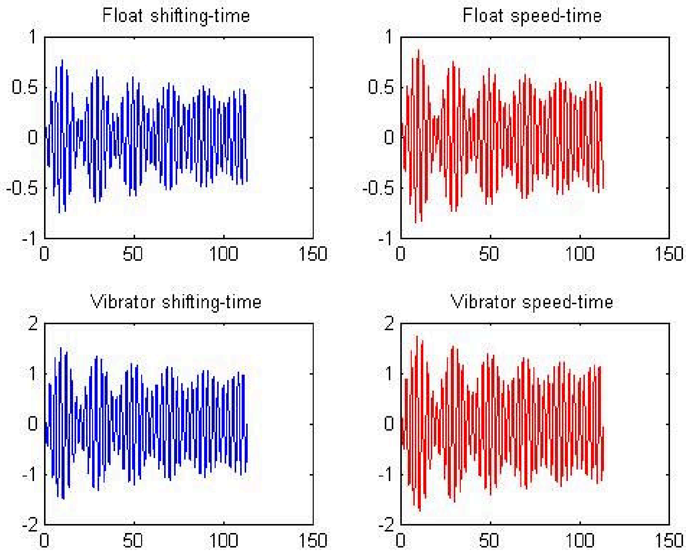
At the initial moment, both the oscillator and float are stationary and have initial value conditions:

$x_f(0) = x_v(0) = 0, v_f(0) = v_v(0) = 0.$

The heave motion law of float-oscillator can be characterized by the following differential equations:

$$\begin{cases} f\cos\omega t - K_{wF}\times v_f(t) - \rho g\pi r_c^2 x - k_T\left(v_v(t)-x_f(t)\right) - k_d\left(v_v(t)-x_f(t)\right) \\ \qquad = (m_f+m_h)a_f \\ k_T\left(v_v(t)-x_f(t)\right) + k_d\left(v_v(t)-x_f(t)\right) = m_v a_v \\ x_f(0) = x_v(0) = 0 \\ v_f(0) = v_v(0) = 0 \end{cases}$$

By solving this differential equation system, $x_f(t)$, $x_v(t)$, $v_f(t)$, and $v_v(t)$ can be obtained. According to the specific values of each physical quantity, the differential equations of motion of float-oscillator system can be solved discretely. After solving, the numerical solution is shown as follows.




### 3.7 Output work and output power of wave energy device under heave motion mode

Connecting the damping of float and oscillator to do work, the mechanical energy consumption of float-oscillator system is scattered outside the system, that is, the external output work $W$. Take a very short time interval $[t, t+dt]$, during which the velocity $v_f(t)$and $v_v(t)$ and displacement $x_f(t)$ and $x_v(t)$ of float and oscillator change so little that they can be regarded as approximately constant, so the damping force can also be regarded as approximately constant, and its magnitude is constant as $f_d = k_d\left[v_f(t) - v_v(t)\right]$. In analogy with the relative sliding process of high physical connection, the mechanical energy loss is the sliding friction force multiplied by the relative displacement. It can be inferred that in the process of relative displacement of float-oscillator system, the damping force does positive work for one of them and negative work for the other, and the mechanical energy loss of the system is related to the relative displacement. The relative displacement is $\Delta x = \left|v_f(t) - v_v(t)\right|dt$, so the infinitesimal work done by damping force is

$$dW = f_d . \Delta x = k_d\left[v_f(t) - v_v(t)\right]^2 dt.$$

Take a suitable observation period $[0, T]$, such as 4000 wave periods, and integrate to obtain the output work $W$ of damping force:

$$W = \int_0^T k_d\left[v_f(t) - v_v(t)\right]^2 dt.$$

Then the output power of wave energy is obtained:

$$P = \frac{W}{T} = \frac{1}{T}\int_0^T k_d\left[v_f(t) - v_v(t)\right]^2 dt.$$

## 4. Model Solution

### 4.1 Discrete algorithm for solving optimization model

Although the heave and pitch motion of the float-oscillator system can be completely described by the differential equations in the model, it is difficult to find the formula solution of the differential equations. The discretization method can be used for numerical solution. The specific algorithm steps are as follows.

**Step 1** Initialize the preparation. Determine the initial state of the float-oscillator system, including the initial heave displacement $x_f(0) = 0$ of the float, the initial heave velocity $v_f(0) = 0$ of the float, the initial heave displacement $x_v(0)$ of the oscillator and the initial heave velocity $v_v(0)$ of the oscillator. Choose the appropriate time step $dt$ (0.2 seconds) and a long enough observation period $[0, T]$ ($T$ takes 4000 wave cycles).

**Step 2** Cyclic variation of decision variables $k_d \in [0,10000]$. After being determined $k_d$, the second derivative $a_f(t), a_v(t)$ can be determined from the second order differential equation system. For these two second derivatives at the current moment $t$, using the definitions of second derivatives $a_f(t), a_v(t)$ respectively, we can iterate one step to get the first derivative value at the next moment. For example $a_f(t)$, $a_f(t) = \frac{d^2x_f(t)}{dt^2} = \lim\limits_{dt\to 0}\frac{\frac{dx_f(t+dt)}{dt} - \frac{dx_f(t)}{dt}}{dt}$.

The approximate recurrence relation $\frac{dx_f(t+dt)}{dt} = \frac{dx_f(t)}{dt} + a_f(t). dt$ is obtained.

This recursive relation iterates one step to get $\frac{dx_f(t+dt)}{dt}$, that is, to get the linear velocity $v_f(t + dt)$ at the next moment $t + dt$.

In the same way, get the physical quantity $v_v(t + dt), \omega_f(t + dt), \omega_v(t + dt)$ at the next moment $t + dt$.

$$v_f(t) = \frac{dx_f(t)}{dt} = \lim_{dt\to 0}\frac{x_f(t + dt) - x_f(t)}{dt}.$$

For the four first-order derivatives $v_f(t), v_v(t), \omega_f(t), \omega_v(t)$ at the current moment $t$, using the definitions of the first-order derivatives respectively, we can iterate one step to get the original function value at the next moment. For example $v_f(t)$, $v_f(t) = \frac{dx_f(t)}{dt} = \lim\limits_{dt\to 0}\frac{x_f(t+dt) - x_f(t)}{dt}$.

The heave displacement $x_f(t + dt)$ of the float at the next moment $t + dt$ can be obtained by iterating this recursive relation in one step.

Get the physical quantity $x_v(t + dt), \theta_f(t + dt), \theta_v(t + dt)$ at the next moment $t + dt$.

Based on each physical quantity at time $t + dt$, the state parameters of float-oscillator system at each discrete time are obtained by continuous iteration:

$\{x_f(n.dt)\}_{n=0}^{n=+\infty}, \{x_v(n.dt)\}_{n=0}^{n=+\infty}, \{v_f(n.dt)\}_{n=0}^{n=+\infty}, \{v_v(n.dt)\}_{n=0}^{n=+\infty}$.

Calculate the discrete objective function

$$P(k_d, k_{dt}) = \frac{1}{T}\sum_{n=0}^{n=N=T/dt} k_d\left(v_f(n.dt)\cos\big(\theta_v(0+n.dt)\big) - v_v(n.dt)\right)^2 dt + \frac{1}{T}\sum_{n=0}^{n=N=T/dt} k_{dt}\left(\omega_f(n.dt) - \omega_v(n.dt)\right)^2 dt$$

Therein $k_{dt}$ is damping torque.

**Step 3** By changing $k_d \in [0,100000]$, the maximum output power $P^*$ of wave energy is obtained, and the corresponding $k_d^*$ is the best design parameter of wave energy device, that is,

$$\underset{k_d\in[0,100000]}{\arg max}\ P(k_d).$$

### 4.2 Numerical solution of maximum output power model based on search algorithm

According to the specific values of each physical quantity, the optimization model is solved under the condition that the damping coefficient of linear damper is constant

$$\max_{k_d\in[0,100000]} P(k_d) = \frac{1}{T}\int_0^T k_d\left[v_f(t)\cos\left(\theta_v(t)\right) - v_v(t)\right]^2 dt.$$

The optimal damping coefficient is $k_d = 3500$, and the maximum average output power is $P = 246$. The search results are shown in Figure 4-1.

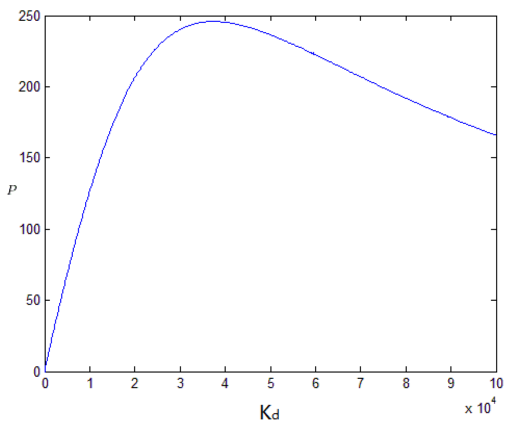



*Figure 4-1: Relationship between damping coefficient and output power*

## 5. Conclusion

In the process of modeling, this paper uses the combination of basic physical knowledge and mathematical differential knowledge to establish a model, which makes the physical meaning of this model clear. The established differential equation is easy to be solved discretely and programmed.

However, this model only considers heave, and does not consider the other dimension of wave energy device in 3D space. If 3D motion can be considered comprehensively, the model will be closer to the actual production. If the convergence rate and cumulative error of the algorithm can be estimated theoretically, the model can be further improved. The motion law of wave energy device is solved only from the point of view of differential equations, without considering other factors, such as the related connecting members may pull the float so that it will not sink too deep into the seabed or fly away from the sea surface, which also makes the solution results of this model deviate from the actual situation.

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