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Research Article

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Study in Transient Regime by Analytical Method of Thermal Transfer Through a Two-Dimensional (2D) Kenaf-Based Insulating Material: Influence of the Thermal Exchange Coefficient on the Front Face

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Abstract In this article, we present a study on the influence of the exchange coefficient through the propagation of heat through a two-dimensional (2D) Kenaf-based insulating material. The study in Cartesian coordinates of a simple wall made up of Kenaf of characteristics: the density ρ= 80 $kg.\ m^{-3}$, the thermal conductivity λ= $0.038W.\ m^{-1}\ .\ K^{-1}$ and the thermal diffusivity
$\alpha = 5.237.10^{-7}\ m^{-2}.\ s^{-1}$. This study is based on the description of the different temperature profiles and the heat flow as a function of reduced space and time variables for different values of the exchange coefficient. The influence of the heat exchange coefficient was studied in transient dynamic regime. In transient dynamic regime, the evaluation of temperature and heat flux density profiles allowed us to describe the effects of heat dissipation as a function of time.

Keywords Exchange coefficient, Transient Dynamics, Temperature and Flux Density

Introduction

Various works have presented different methods for determining thermophysical parameters including thermal conductivity [1, 2], thermal effusivity, thermal diffusivity, or the heat exchange coefficient. Among the methods used, we have the static method [3, 4], the transient dynamic method [5] and the frequency dynamic method [6, 7, 8]. Artificial insulators pose an environmental problem unlike natural biodegradable insulators. Kenaf, a biodegradable natural product, is used as material.

The measurement of thermophysical parameters (diffusivity or thermal conductivity) [9] also makes it possible to make a choice on thermal insulators. In this study, we use the Kenaf material [10] as a thermal insulator.

We propose an analytical method for solving the heat equation by imposing variable heat exchange coefficients [11, 12] at the two-face levels to determine the temperature and heat flux density [13] of the kenaf material. We highlight the quality of the thermal insulation [14, 15].

Study Model

Study Device

The Kenaf material is assumed to be homogeneous and parallelepipedic in shape. The depth of the material is L=0,05m; the initial temperature of the material $T_i = 10^0C$ and that of the external ambient media $Ta_1 = Ta_2 = 30^0C$. The heat exchange coefficients at the front face and the back face are respectively h_1 and h_2. The average thermal diffusivity is α and the thermal conductivity is λ

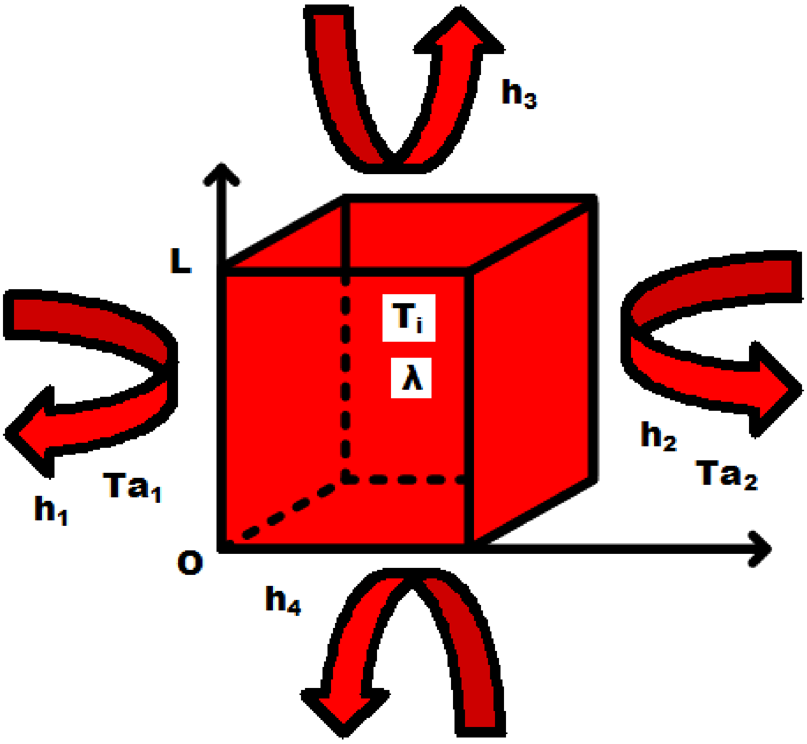

Figure 1: Study model

Theory:

The unidirectional heat transfer in the yarn-plaster thermal insulation is governed by equation (1) below:

$$\frac{\partial^2 T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x^2} + \frac{\partial^2 T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y^2} - \frac{1}{\alpha}\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial t} = 0;(1)$$

$T = T(x,y,h_1,h_2,h_3,h_4,t)$ is the temperature inside the material; x the depth and t the time. Equation (2) gives the expression of the diffusivity α.

$$\alpha = \frac{\lambda}{\rho c};(2)$$

α is the coefficient of thermal diffusivity ($m^2.s^{-1}$)

λ is the thermal conductivity ($W.m^{-2}.c^{-1}$)

ρ is the density of the material ($kg.m^{-2}$)

Boundary conditions

$$\begin{cases} \lambda \left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x}\right|_{x=0} = h_1\left[T(0,y,t) - T_a\right];(3) \\ \lambda \left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x}\right|_{x=L} = -h_2\left[T(L,y,t) - T_a\right];(4) \\ \lambda \left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y}\right|_{y=0} = h_3\left[T(x,0,t) - T_a\right];(5) \\ \lambda \left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y}\right|_{y=L} = -h_4\left[T(x,L,t) - T_a\right];(6) \\ T(x,y,h_1,h_2,h_3,h_4,t=0) = T_i;(7) \end{cases}$$

Dimensionless heat equation

$$\theta(u,v,\tau)=\frac{T(x,y,t)-T_a}{T_i-T_a};(8)$$

with $\theta(u,v,\tau)$: reduced temperature;

$u=\frac{x}{L}$; is a space reduced variable

$v=\frac{y}{L}$; is a space reduced variable

and $\tau=\frac{\alpha t}{L^2}=F_0$

F_0 : Reduced time variable or Fourier number

The heat equation (1) becomes:

$$\frac{\partial^2\theta(u,v,\tau)}{\partial u^2}+\frac{\partial^2\theta(u,v,\tau)}{\partial v^2}=\frac{\partial\theta(u,v,\tau)}{\partial\tau};(9)$$

The boundary conditions (3), (4), (5) and (6) become (10), (11), (12) and (13):

$$\begin{cases}\left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=0}=\frac{h_{1x.}L}{\lambda}\theta(0,\tau);(10)\\ \left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=1}=-\frac{h_{2x.}L}{\lambda}\theta(1,\tau);(11)\\ \left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=0}=\frac{h_{1y.}L}{\lambda}\theta(0,\tau);(12)\\ \left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=1}=\frac{h_{2y.}L}{\lambda}\theta(1,\tau);(13)\end{cases}$$

Let us find the solution of equation (9) in the form of reduced variables separable in space and time given by relation (14):

$\theta(u,v,\tau)=U(u)V(v)W(\tau);(14)$

Using the relations (9) and (14) we obtain that of (15)

$$\frac{1}{U(u)}\frac{\partial^2U(u)}{\partial u}+\frac{1}{V(v)}\frac{\partial^2V(v)}{\partial v}+\frac{1}{W(\tau)}\frac{\partial W(\tau)}{\partial\tau}=-\gamma^2;(15)$$

γ is a positive constant.

From relation (15) we obtain two differential equations:

- The differential equation in time is given by (16):

$$\frac{1}{W(\tau)}\frac{\partial W(\tau)}{\partial\tau}=-\gamma^2;(16)$$

- The differential equation in space (17) is written:

$$\frac{1}{U(u)}\frac{\partial^2 U(u)}{\partial u} = -\beta^2 ;(17)$$

The boundary conditions space:

$$\begin{cases} \left.\dfrac{\partial\theta(0,\tau)}{\partial\tau}\right|_{u=0} = B_{i1x}\theta(0,\tau);(18) \\ \left.\dfrac{\partial\theta(1,\tau)}{\partial\tau}\right|_{u=1} = -B_{i2x}\theta(1,\tau);(19) \\ \left.\dfrac{\partial\theta(0,\tau)}{\partial\tau}\right|_{u=0} = B_{i1y}\theta(0,\tau);(20) \\ \left.\dfrac{\partial\theta(1,\tau)}{\partial\tau}\right|_{u=1} = -B_{i2y}\theta(1,\tau);(21) \end{cases}$$

Avec $B_{i1x} = \frac{h_{1x}.L}{\lambda}$; $B_{i2x} = \frac{h_{2x}.L}{\lambda}$; $B_{i1y} = \frac{h_{1y}.L}{\lambda}$ et $B_{i2y} = \frac{h_{2y}.L}{\lambda}$

respectively the Biot numbers on the front face and on the back face.

The general solution of the reduced temperature is in the form

$$\theta(u,v,\tau) = \sum_n \left[\left(a_n \cos(\beta_n u) + b_n \sin(\beta_n u)\right)\right]\left[c_n \cos(\mu_n v) + d_n \sin(\mu_n v)\right]e^{-\gamma^2\tau};(22)$$

$$\begin{cases} \beta_n b_n = B_{i1x} a_n ;(23) \\ -\beta_n a_n \sin(\beta_n L) + \beta_n b_n \cos(\beta_n L) = -B_{i2x}\left(a_n \cos(\beta_n L) + b_n \sin(\beta_n L)\right);(24) \end{cases}$$

$$\sin(\beta_n L)(a_n\beta_n - B_{i2x}b_n) = \cos(\beta_n L)(b_n\beta_n + B_{i2x}a_n);(25)$$

$$\tan(\beta_n L) = \frac{b_n\beta_n + B_{i2x}a_n}{a_n\beta_n - B_{i2x}b_n};(26)$$

The following transcendent equation x:

$$\tan(\beta_n L) = \frac{\frac{h_{1x}L}{\lambda}\beta_n + \frac{h_{2x}L}{\lambda}\beta_n}{\beta_n^2 - \frac{h_{1x}h_{2x}L}{\lambda^2}};(27)$$

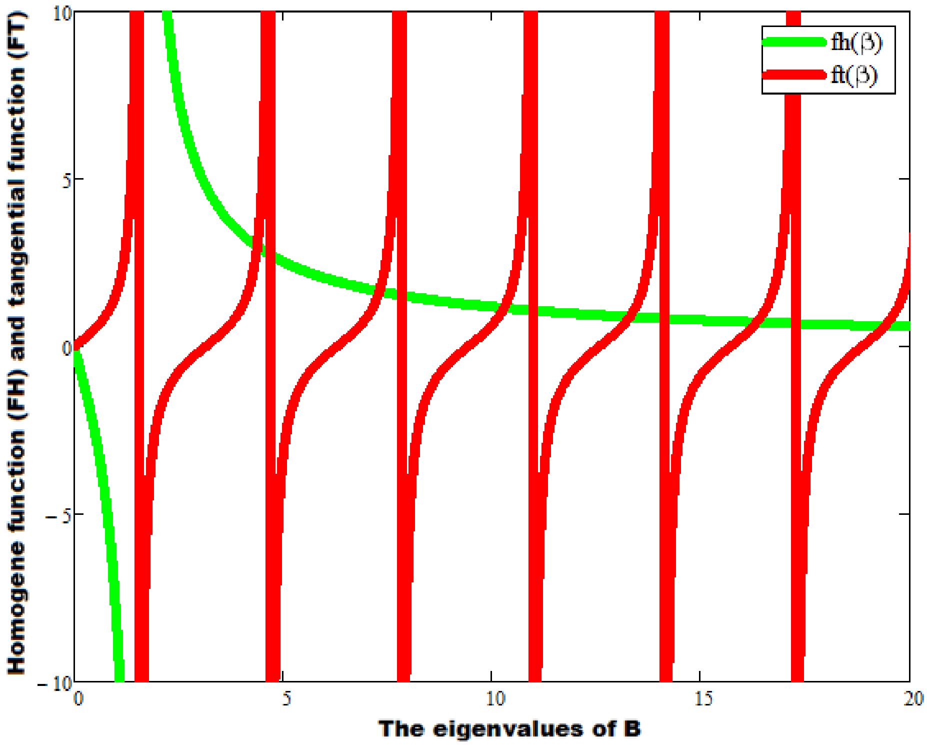

Figure 2: Curve of the following transcendent equation

The intersection of the two curves fh (β_n) and ft (β_n) corresponds to the solution.

Table 1 summarizes the eigenvalues found of β_n

Table 1: The eigenvalues β_n the equation

n	**1**	**2**	**3**	**4**	**5**
β_n	4.7	7.8	10.2	13.5	16.6

Transcendent equation following y

$$\begin{cases} \mu_n d_n = B_{i1y} c_n ;(28) \\ -\mu_n c_n \sin(\mu_n L) + \mu_n d_n \cos(\mu_n L) = -B_{i2y}\left(c_n \cos(\mu_n L) + d_n \sin(\mu_n L)\right);(29) \end{cases}$$

$$\sin(\mu_n L)(c_n \mu_n - B_{i2y} d_n) = \cos(\mu_n L)(c_n \mu_n + B_{i2y} c_n);(30)$$

$$\tan(\mu_n L) = \frac{d_n \mu_n + B_{i2y} c_n}{c_n \mu_n - B_{i2y} d_n};(31)$$

$$\tan(\mu_n L) = \frac{\frac{h_{1y} L}{\lambda}\mu_n + \frac{h_{2y} L}{\lambda}\mu_n}{\mu_n^2 - \frac{h_{1y} h_{2y} L}{\lambda^2}};(32)$$

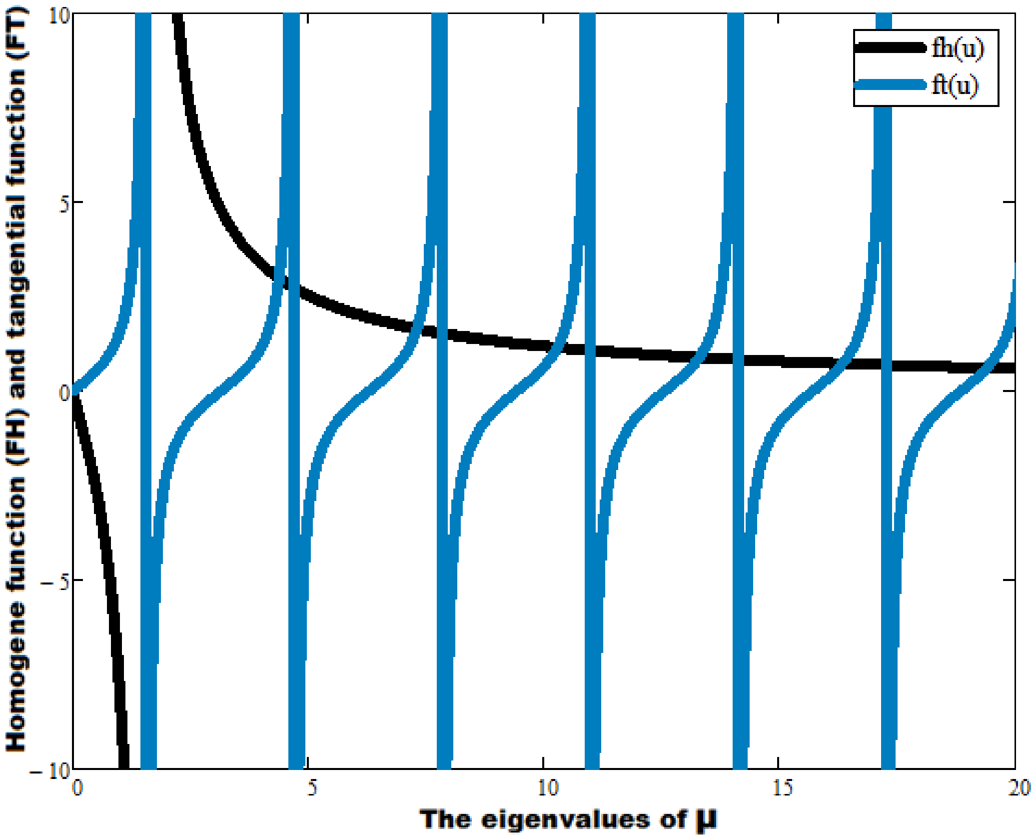

Figure 3: Curve of the following transcendent equation

The intersection of the two curves fh (μ_n) and ft (μ_n) corresponds to the solution.

Table 1 summarizes the eigenvalues found of μ_n

Table 2: The eigenvalues μ_n the equation

n	1	2	3	4	5
μ_n	4.7	7.8	10.2	13.5	16.6

$$\theta(u;v,\tau)=\frac{T(x,y,h_{1x},h_{2x},h_{1y},h_{2y},t)-T_a}{T_i-T_a};(33)$$

$$T(x,y,h_{1x},h_{2x},h_{1y},h_{2y},t)=T=T_a+(T_i-T_a)\theta(u;v,\tau);(34)$$

The general solution of temperature:

$$T=T_a+(T_i-T_a)\sum_n\left[a_n\left(\cos\left(\beta_n\frac{x}{L}\right)+\frac{\frac{h_{1x}L}{\lambda}}{\beta_n}\sin\left(\beta_n\frac{x}{L}\right)\right)c_n\left(\cos\left(\mu_n\frac{y}{L}\right)+\frac{\frac{h_{1y}L}{\lambda}}{\mu_n}\sin\left(\mu_n\frac{y}{L}\right)\right)\right]e^{-\frac{\alpha t}{L^2}\gamma^2};(35)$$

Heat flux density:

We get the expression for the density of the heat flow (or surface heat flow)

Which is the heat flux per unit area (W.m^{-2}) as follows:

$$\vec{\varphi}(x, y, h_{1x}, h_{2x}, h_{1y}, h_{2y}, t) = -\lambda \overrightarrow{grad} T(x, y, h_{1x}, h_{2x}, h_{1y}, h_{2y}, t); (36)$$

$$\vec{\varphi}(x, y, h_{1x}, h_{2x}, h_{1y}, h_{2y}, t) = \overrightarrow{\varphi_x}(x, y, h_{1x}, h_{2x}, h_{1y}, h_{2y}, t) + \overrightarrow{\varphi_y}(x, y, h_{1x}, h_{2x}, h_{1y}, h_{2y}, t); (37)$$

From these two expressions we get, the final expression of the heat flux density

$$\varphi(x,y,t) = \lambda(Ti - Ta)\left[\begin{array}{l}\left[\sum_n an\left(-\frac{\beta n}{L}\sin\left(\beta n\frac{x}{L}\right) + \frac{Bi1x}{L}\cos\left(\beta n\frac{x}{L}\right)\right)cn\left(\cos\left(\mu n\frac{y}{L}\right) + \frac{Bi1y}{L}\sin\left(\mu n\frac{y}{L}\right)\right)\right]^2 + \\ \left[\sum_n cn\left(-\frac{\mu n}{L}\sin\left(\mu n\frac{y}{L}\right) + \frac{Bi1y}{L}\cos\left(\mu n\frac{x}{L}\right)\right)an\left(\cos\left(\beta n\frac{x}{L}\right) + \frac{Bi1x}{L}\sin\left(\beta n\frac{x}{L}\right)\right)\right]^2\end{array}\right]^{\frac{1}{2}} e^{-\frac{\alpha t}{L^2}\gamma^2};(38)$$

Results and Discussion

Evolution of Temperature and Heat Flux Density as a Function of Depth for Different Values of the Heat Exchange Coefficient

Figures 4 and 5 show the evolution of temperature and heat flux density as a function of depth in the material under the influence of the heat exchange coefficient by convection. We note a decrease in temperature and heat flux density as a function of the depth of the material under the influence of the heat exchange coefficient at the front face. The temperature gradually decreases to reach the initial temperature Ti of the material and the heat flux density transmitted decreases before reaching the rear face, this means that the material has stored most of the heat to release it. This shows that the kenaf used is a good insulator.

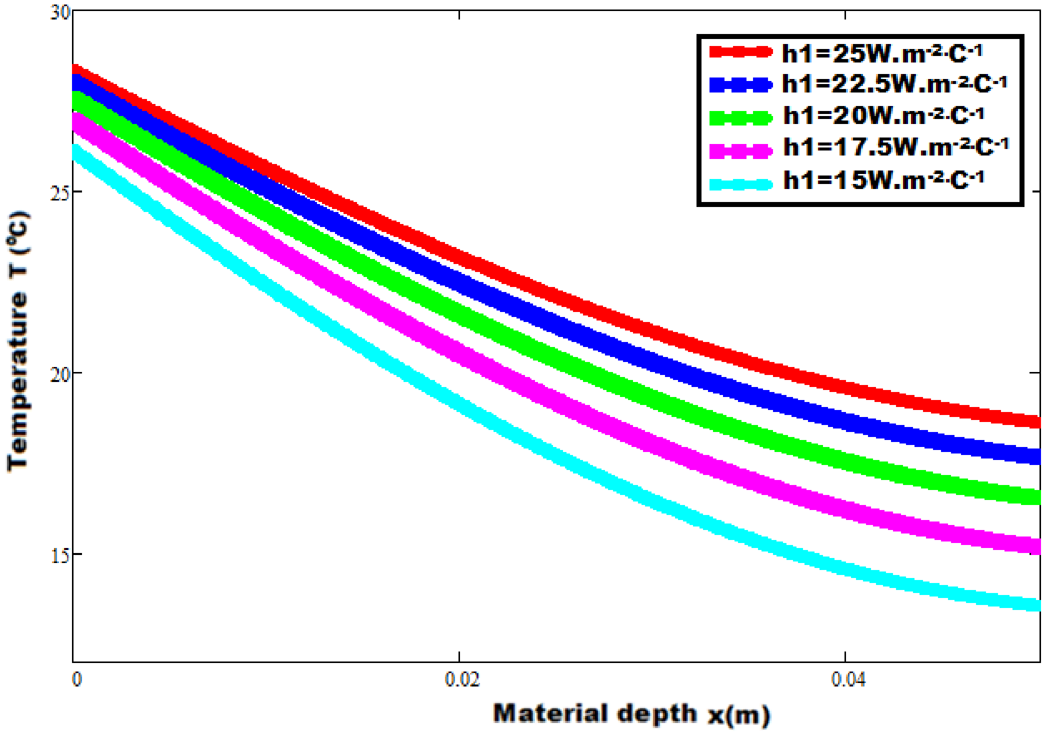

Figure 4: Temperature as a function of material depth. h_2=0.005W.m^{-2}.C^{-1} ; h_3=5W.m^{-2}.C^{-1}; h_2=0.005W.m^{-2}.C^{-1} ; t=10s.

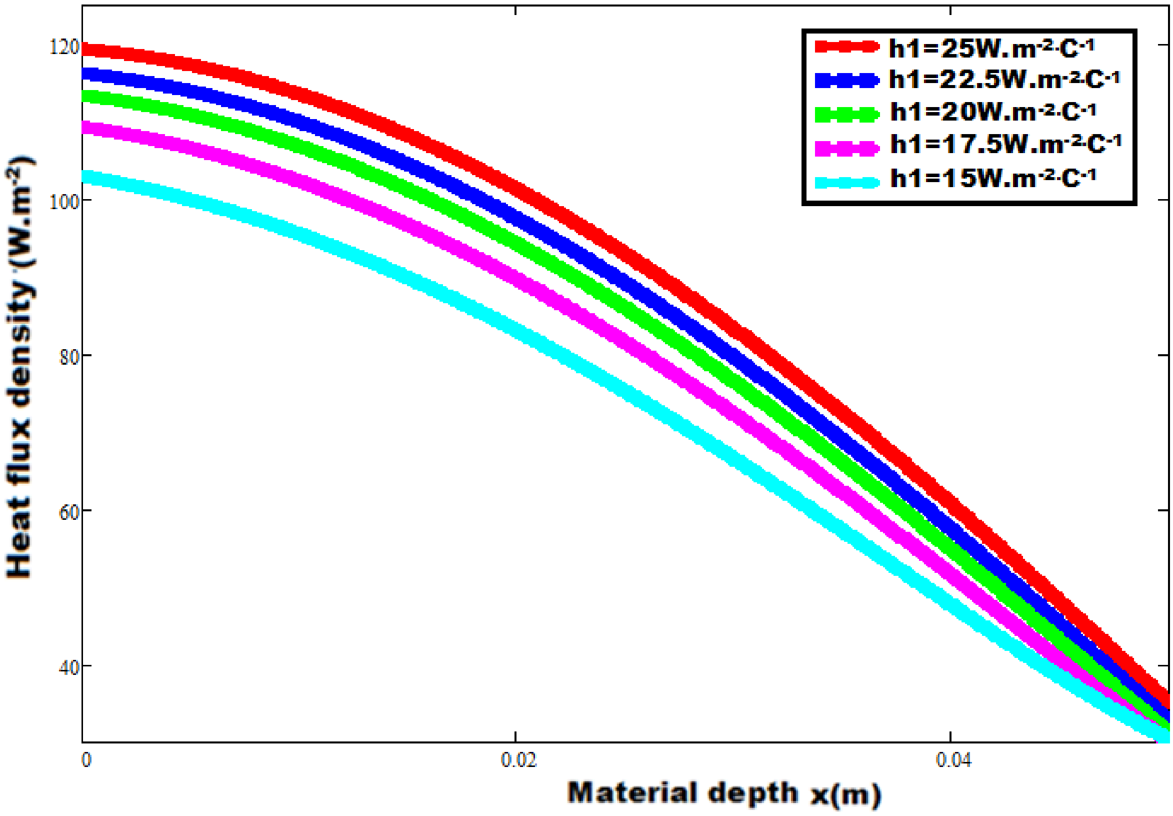

Figure 5: Heat flux density as a function of material depth. h_2=0.005W.m^{-2}.C^{-1} ; h_3=5W.m^{-2}.C^{-1}; h_2=0.005W.m^{-2}.C^{-1} ; t=10s.

Evolution of Temperature and Heat Flux Density as a Function of Time

Figures 6 and 7 show the evolution of temperature and heat flux density as a function of time under the influence of the heat transfer coefficient by convection.

We notice that the temperature increases as time increases and unlike the heat flux density which decreases with time.

The temperature is all the more important as the heat exchange coefficient on the front face is high. This means that the material stores thermal energy without releasing it. Unlike the heat flux density which decreases. This decrease is due to a loss of heat in the material. These phenomena show that the material used is a good insulator.

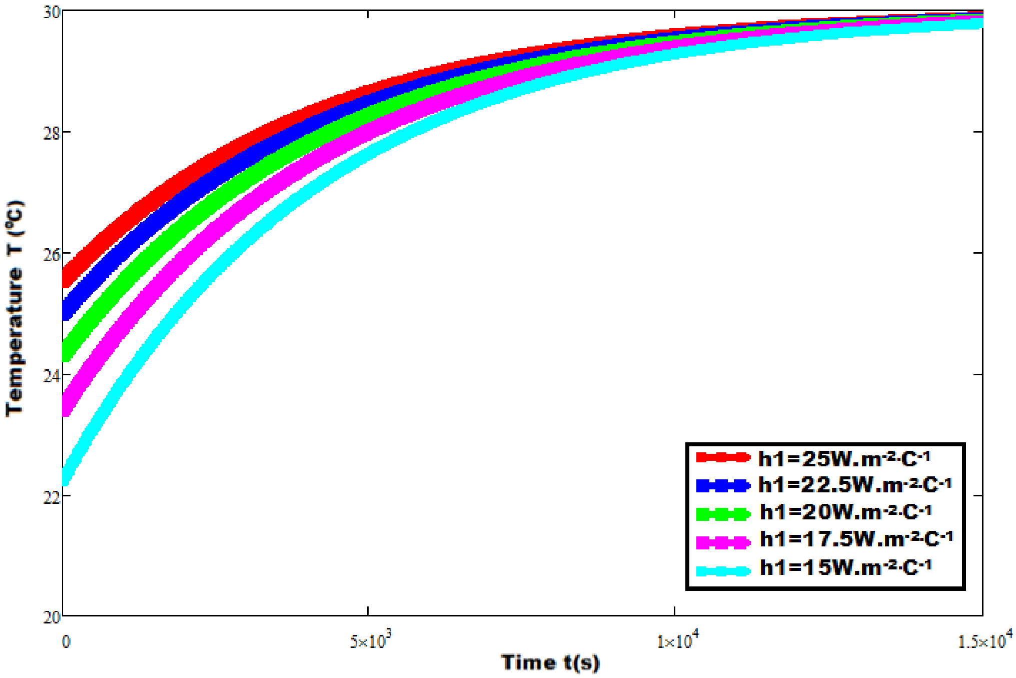

Figure 6: Evolution of the temperature as a function of material time. h_2=0.005W.m^{-2}.C^{-1} ; h_3=5W.m^{-2}.C^{-1}; h_2=0.005W.m^{-2}.C^{-1} ; t=10s.

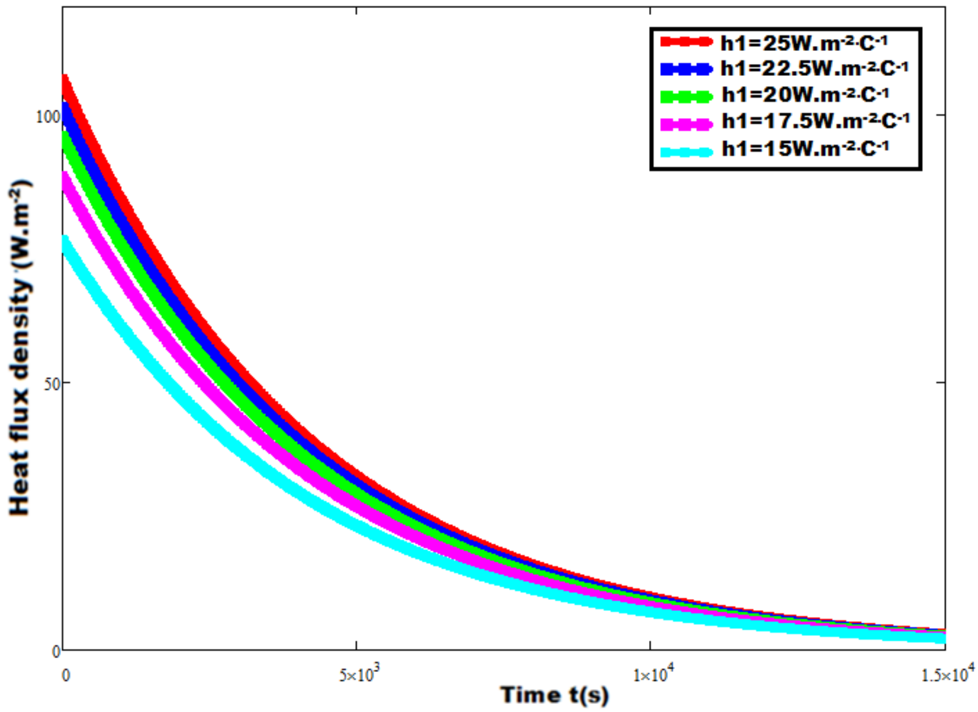

Figure 7: Evolution of the heat flux density as a function of time. h_2=0.005W.m^{-2}.C^{-1} ; h_3=5W.m^{-2}.C^{-1}; h_2=0.005W.m^{-2}.C^{-1} ; t=10s.

Conclusion

After solving the two-dimensional heat equation by the analytical method and after plotting the temperature and heat flux density. We notice when the time increases the temperature also increases Which means that the material stores thermal energy without restoring it contrary to the heat flux density which decreases. This decrease is due to a loss of heat in the material. These phenomena show that the material used is a good insulator.

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