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## Calculation of the Coupling Matrices on the Discontinuity between Rectangular Waveguide and Substrate Integrated Waveguide

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**Abstract** In this article, we have characterized the matrices of couplings on the discontinuities between a rectangular waveguide and a substrate integrated waveguide (SIW) in order to improve the reception power by proceeding on the calculation of couplings' matrices between these waveguides. The results that we have obtained, give us an idea on the different possible couplings during the propagation of the electromagnetic field TE and TM modes at the discontinuity surface.

**Keywords** discontinuities, waveguide, SIW, TE and TM modes

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### 1. Introduction

The electronics industry tries to keep circuit manufacturing costs as low as possible to satisfy the consumer as well as to be more competitive. High-frequency circuits, which are used in all spheres of telecommunications, are sometimes heavy and imposing depending on the technologies used, which is not attractive for the integration of portable systems (cell phones, mp3 players, mp4 players, etc.). To pass the signal from one point to another, in an integrated circuit, you need a transmission medium. For microwave signals, there are many variants of media such as planar lines (microstrip, coplanar, ...), coaxial line and waveguide to name a few.

Rectangular waveguides are a good example of components with very high performance, but bulky [1-2]. An alternative technology emerged a few years ago to overcome this problem: the substrate integrated waveguide (SIW) [3-5]. Indeed, this guide accomplishes the same functions as conventional waveguides. However, they have a much better integration density and their costs are lower.

In many microwave devices such as filters, impedance transformers, adapters, resonant circuits ..., transitions play a fundamental role between metal guides [2]. They are widely used in satellites and terrestrial communication system, and with the increasing complexity of wireless satellite communication equipment.

However, several types of transitions have been designed to adapt the waveguides [1] [3]. In order to improve the received power and the reflection coefficient in order to avoid interference due to standing waves in the field of radio transmission or satellite transmission. However, it will be a question of determining the coupling matrices between rectangular waveguide and substrate integrated waveguide into their junction.

### 2. Reminder of Maxwell's equations

The general theory governing any electromagnetic phenomenon is based on Maxwell's equations [2] which constitute a set of four coupled vector partial differential equations of the first order. They were formulated in 1873 by James Clerk Maxwell, who was able to prove mathematically by these equations the propagation of the electromagnetic wave.



$$\begin{cases} \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \\ \nabla \cdot \vec{D} = \rho \\ \nabla \cdot \vec{B} = 0 \end{cases} \quad (1)$$

with:  $\vec{E}$  electric field;  $\vec{H}$  magnetic field;  $\vec{D}$  electrical induction;  $\vec{B}$  magnetic induction. For an isotropic medium, the constitution laws are written:

$$\begin{cases} \vec{D} = \epsilon \vec{E} \\ \vec{B} = \mu \vec{H} \\ \vec{j} = \sigma \vec{E} \end{cases} \quad (2)$$

with  $\epsilon$  and  $\mu$  respectively the permittivity and the permeability of the medium. Note that equations (1) and (2) apply to a given point in space, hence the term "local forms" of Maxwell's equations.

From these Maxwell equations we get the inhomogeneous Helmholtz equations:

$$\nabla^2 \vec{H} + \omega^2 \mu \epsilon \vec{H} = -\nabla \times \vec{j} \quad (3)$$

$$\nabla^2 \vec{E} + \omega^2 \mu \epsilon \vec{E} = j\omega \mu \vec{j} + \nabla \left( \frac{\rho}{\epsilon} \right) \quad (4)$$

For the study of propagation or radiation waves we place ourselves outside areas containing sources of charges and currents. The propagation equations for such a study are then written in the following forms:

$$\nabla^2 \vec{H} + k^2 \vec{H} = 0 \quad (5)$$

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0 \quad (6)$$

with  $k$  the wave number defined by:  $k = \omega \sqrt{\mu \epsilon}$ .

### 3. Rectangular Waveguides

The homogeneous rectangular waveguide is a guide structure with a single conductor in the form of a hollow tube of rectangular section, the hollow part of which is filled with dielectric. Thus, Figure 1 shows the geometry of a rectangular waveguide of width  $a$  and height  $b$  filled with a homogeneous dielectric with constant permittivity  $\epsilon$  and permeability  $\mu$ .

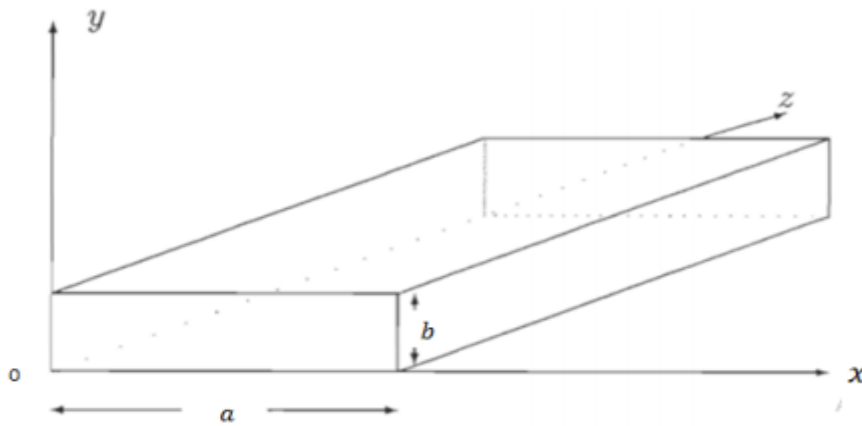


Figure 1: Right section of the rectangular waveguide

#### a) Expressions of the transverse components of the fields as a function of the longitudinal components $E_z$ and $H_z$

The tangential fields  $\vec{E}_t$  and  $\vec{H}_t$  break up in the following way in the base  $\vec{e}_x$  and  $\vec{e}_y$

$$\begin{cases} \vec{E}_t = E_x \vec{e}_x + E_y \vec{e}_y \\ \vec{H}_t = H_x \vec{e}_x + H_y \vec{e}_y \end{cases} \quad (7)$$

where  $E_x$  and  $H_x$  are following components (Ox);  $E_y$  and  $H_y$  of the following components (Oy).



$$\begin{cases} E_x = -\frac{j}{k_c^2} \left[ \omega \mu_0 \frac{\partial H_z}{\partial y} + \beta \frac{\partial E_z}{\partial x} \right] \\ E_y = \frac{j}{k_c^2} \left[ \omega \mu_0 \frac{\partial H_z}{\partial x} - \beta \frac{\partial E_z}{\partial y} \right] \\ H_x = -\frac{j}{k_c^2} \left[ \beta \frac{\partial H_z}{\partial x} - \omega \mu_0 \frac{\partial E_z}{\partial y} \right] \\ H_y = -\frac{j}{k_c^2} \left[ \beta \frac{\partial H_z}{\partial y} + \omega \mu_0 \frac{\partial E_z}{\partial x} \right] \end{cases} \quad (8)$$

Relations (3.4) show that the transverse components of the electric and magnetic fields are expressed as a function of their longitudinal components  $E_z$  and  $H_z$ . We can therefore distinguish two main types of propagation modes [6, 7]: the modes TE for which  $E_z = 0$  and  $H_z \neq 0$ ; The TM modes for which  $E_z \neq 0$  and  $H_z = 0$ .

### b) Wave equation in Cartesian coordinates

$F_z$  obeys the following Helmholtz equation

$$\nabla^2 F(x, y, z) + k^2 F(x, y, z) = 0 \quad (9)$$

Where  $F(x, y, z)$  is one of the components of the electric or magnetic field.

A solution of this equation is:

The components of the electric and magnetic fields of the modes  $TE_{mn}$

For the electric field

$$\begin{cases} E_x = \frac{j\sqrt{2}}{k_{mn}} \left( \frac{n\pi}{b} \right) \cos \left( \frac{m\pi}{a} x \right) \sin \left( \frac{n\pi}{b} y \right) e^{-j\beta z} \\ E_y = \frac{j\sqrt{2}}{k_{mn}} \left( \frac{m\pi}{a} \right) \sin \left( \frac{m\pi}{a} x \right) \cos \left( \frac{n\pi}{b} y \right) e^{-j\beta z} \\ E_z = 0 \end{cases} \quad (10)$$

For the magnetic field

$$\begin{cases} H_x = \frac{j\sqrt{2}\beta}{\omega \mu_0 k_{mn}} \left( \frac{m\pi}{a} \right) \sin \left( \frac{m\pi}{a} x \right) \cos \left( \frac{n\pi}{b} y \right) e^{-j\beta z} \\ H_y = \frac{j\sqrt{2}\beta}{\omega \mu_0 k_{mn}} \left( \frac{n\pi}{b} \right) \cos \left( \frac{m\pi}{a} x \right) \sin \left( \frac{n\pi}{b} y \right) e^{-j\beta z} \\ H_z = \frac{\sqrt{2}k_c^2}{\omega \mu_0 k_{mn}} \cos \left( \frac{m\pi}{a} x \right) \cos \left( \frac{n\pi}{b} y \right) e^{-j\beta z} \end{cases} \quad (11)$$

The components of the electric and magnetic fields of the modes  $TM_{mn}$

For the electric field

$$\begin{cases} E_x = -\frac{2j}{k_c \sqrt{ab}} \left( \frac{m\pi}{a} \right) \cos \left( \frac{m\pi}{a} x \right) \sin \left( \frac{n\pi}{b} y \right) e^{-j\beta z} \\ E_y = -\frac{2j}{k_c \sqrt{ab}} \left( \frac{n\pi}{b} \right) \sin \left( \frac{m\pi}{a} x \right) \cos \left( \frac{n\pi}{b} y \right) e^{-j\beta z} \\ E_z = \frac{2k_c}{\beta \sqrt{ab}} \sin \left( \frac{m\pi}{a} x \right) \cos \left( \frac{n\pi}{b} y \right) e^{-j\beta z} \end{cases} \quad (12)$$

For the Magnetic Field:

$$\begin{cases} H_x = -\frac{2j\omega \epsilon}{\beta k_c \sqrt{ab}} \left( \frac{n\pi}{b} \right) \sin \left( \frac{m\pi}{a} x \right) \cos \left( \frac{n\pi}{b} y \right) e^{-j\beta z} \\ H_y = -\frac{2j\omega \epsilon}{\beta k_c \sqrt{ab}} \left( \frac{m\pi}{a} \right) \cos \left( \frac{m\pi}{a} x \right) \sin \left( \frac{n\pi}{b} y \right) e^{-j\beta z} \\ H_z = 0 \end{cases} \quad (13)$$

The following figure shows that,  $\frac{|E_z|}{|E_{(zmax)}|}$  for  $n=m=1$ .



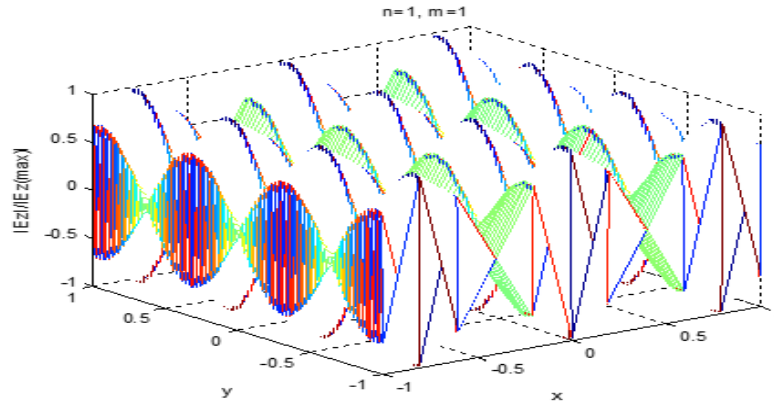


Figure 2: Representation of the longitudinal field  $E_z$  in the rectangular waveguide for  $n = m = 1$ .

**c) Expressions of the electric and magnetic fields of the substrate integrated waveguide**

To determine the expressions of the fields in a waveguide integrated into the substrate, we use the same expressions for a standard guide and replacing  $a$  by  $a_{SIW}$ . The choice of dimensions of the SIW must be judicious to obtain an efficient system. Figure 3 shows the main parameters of the SIW.

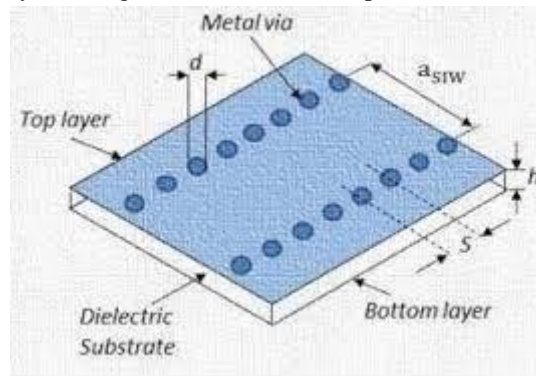


Figure 3: Substrate integrated waveguide

$$\begin{cases} d < \frac{\lambda_g}{5} \\ s < 2d \end{cases} \quad (14)$$

where  $\lambda_g$  is the guided wavelength

$$\lambda_g = \frac{2\pi}{\sqrt{(2\pi f)^2 \epsilon_r - (\frac{\pi}{a})^2}} \quad (15)$$

The determination of the width of the SIW guide is summarized in the literature by:

$$a_s = a_d + \frac{d^2}{0.95p} \quad (16)$$

With  $a_s$  and  $a_d$  are respectively the widths of the SIW and its equivalent of the classic rectangular guide,  $d$  represents the diameter of the metallized holes and  $p$  the spacing (pitch) between them.

$$a_d = \frac{a}{\sqrt{\epsilon_r}} \quad (17)$$

**4. Integral coupling between the two waveguides**

To study the interactions between propagation modes in waveguides, we are interested in the calculations of integrals of couplings between TE-TE, TE-TM, TMTE, and TM-TM modes in Figure 4.

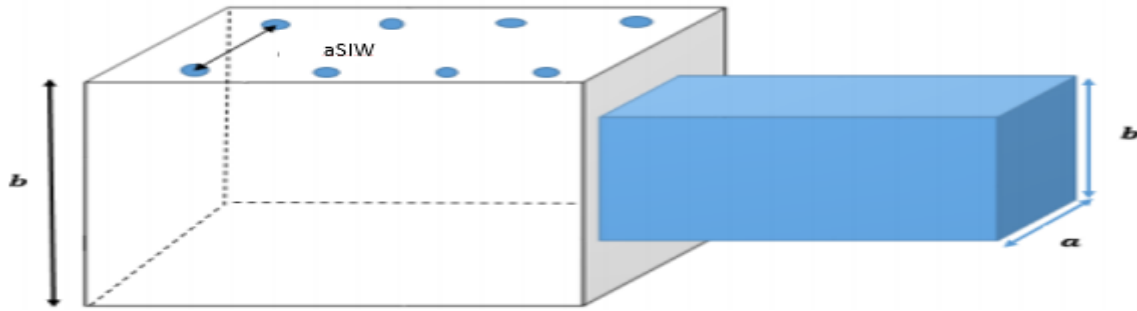


Figure 4: Discontinuity (or junction) between waveguide integrated into the substrate and rectangular guide whose surface of the lower rectangle has the surface of the SIW.

The coupling between field of modes i and j in guides (1) and (2) the integral.

$$\int_s^1 \vec{E}_i^{(1)} \times \vec{H}_j^{*(2)} ds \tag{18}$$

where s: denotes the surface of discontinuity based on a closed contour in this relation;  $\vec{E}_i^{(1)}$  designates the electric field of mode i in the rectangular waveguide (1);  $\vec{H}_j^{*(2)}$  designates the magnetic field of mode j in the waveguide integrated into the substrate (2).

**Calculation of integrals of couplings**

**Coupling between TE-TE mode**

This is a cross between the electric field of the TE modes propagating in the rectangular waveguide (1) with the magnetic field of the TE modes propagating in the GIS (2). The expression of the calculations is as follows:

$$M^{(TE-TE)} = \iint_s \vec{E}_i^{(1)} \times \vec{H}_j^{*(2)} ds = \iint_s (E_x^{(1)} H_y^{*(2)} - E_y^{(1)} H_x^{*(2)}) ds \tag{19}$$

After calculation, we find:

$$M^{(TE-TE)} = \begin{pmatrix} 0.0033 & 0.0056 & 0.0097 & 0.0140 & 0.0185 & 0.0229 \\ 0.0048 & 0.0066 & 0.0103 & 0.0144 & 0.0188 & 0.0232 \\ 0.0066 & 0.0080 & 0.0112 & 0.0151 & 0.0193 & 0.0236 \\ 0.0084 & 0.0096 & 0.0124 & 0.0160 & 0.0199 & 0.0241 \\ 0.0103 & 0.0113 & 0.0137 & 0.0170 & 0.0208 & 0.0248 \\ 0.0122 & 0.0130 & 0.0152 & 0.0182 & 0.0217 & 0.0256 \end{pmatrix}$$

This matrix shows that, during the junction between the waveguide integrated in the substrate and the rectangular waveguide, coupling between TE-TE modes is possible, the electric fields of the TE modes and the magnetic fields of the TE modes propagating at separation surface of these two waveguides. We also note that the power increases as m and n increase.

**- Coupling between TM-TM mode**

This is a cross between the electric field propagating in the guide (1) in TM mode and the magnetic field propagating in the guide (2) in TM mode.

$$M^{(TM-TM)} = \iint_s \vec{E}_i^{(1)} \times \vec{H}_j^{*(2)} ds = \iint_s (E_x^{(1)} H_y^{*(2)} - E_y^{(1)} H_x^{*(2)}) dx dy \tag{20}$$

$$M^{(TM-TM)} = \begin{pmatrix} 0 & 0.4624 & 0.1533 & 0.0724 & 0.0416 & 0.00269 \\ 0 & 0.5247 & 0.2524 & 0.1318 & 0.0788 & 0.0519 \\ 0 & 0.4183 & 0.2869 & 0.1711 & 0.1083 & 0.0735 \\ 0 & 0.3091 & 0.2781 & 0.1902 & 0.1289 & 0.0907 \\ 0 & 0.2284 & 0.2496 & 0.1935 & 0.1407 & 0.1031 \\ 0 & 0.1723 & 0.2161 & 0.1865 & 0.1452 & 0.1110 \end{pmatrix}$$

This matrix shows that, during the junction between the waveguide integrated in the substrate and the rectangular waveguide, coupling between TM-TM modes is possible, the electric fields of the TM modes and the magnetic fields of the TM modes propagating at the separation surface of these two waveguides when  $m \neq 0$  and  $n \neq 0$ .

-Coupling between TE-TM mode

This is a cross between the electric field propagating in the guide (1) in TE mode and the magnetic field propagating in the guide (2) in TM mode.

$$M^{(TE-TM)} = \begin{pmatrix} -0.0021 & 0.0008 & 0.0006 & 0.0005 & 0.0004 & 0.0003 \\ -0.0014 & 0.0001 & 0.0004 & 0.0004 & 0.0003 & 0.0003 \\ -0.0010 & -0.0002 & 0.0002 & 0.0003 & 0.0003 & 0.0003 \\ -0.0008 & -0.0004 & 0.0001 & 0.0002 & 0.0002 & 0.0002 \\ -0.0007 & -0.0004 & -0.0001 & 0.0001 & 0.0002 & 0.0002 \\ -0.0005 & -0.0004 & -0.0001 & 0.0000 & 0.0001 & 0.0001 \end{pmatrix}$$

We find that, the value of (TM-T E)mn decreases when m and n is large. For this case, the mode (TM-T E)<sub>10</sub> is greater and its value is 0.1035 regardless of m and n. This matrix justifies the propagation of the electromagnetic wave at the junction of two waveguides.

5. Impedance of the discontinuity between waveguides

One designates by discontinuity any modification intervening in a guiding structure, such as the change of geometry, direction of propagation or physical parameters (permittivity, permeability) which affect the symmetry of translation of the guiding structure and lead to energy reflections, hence the appearance of impedance.

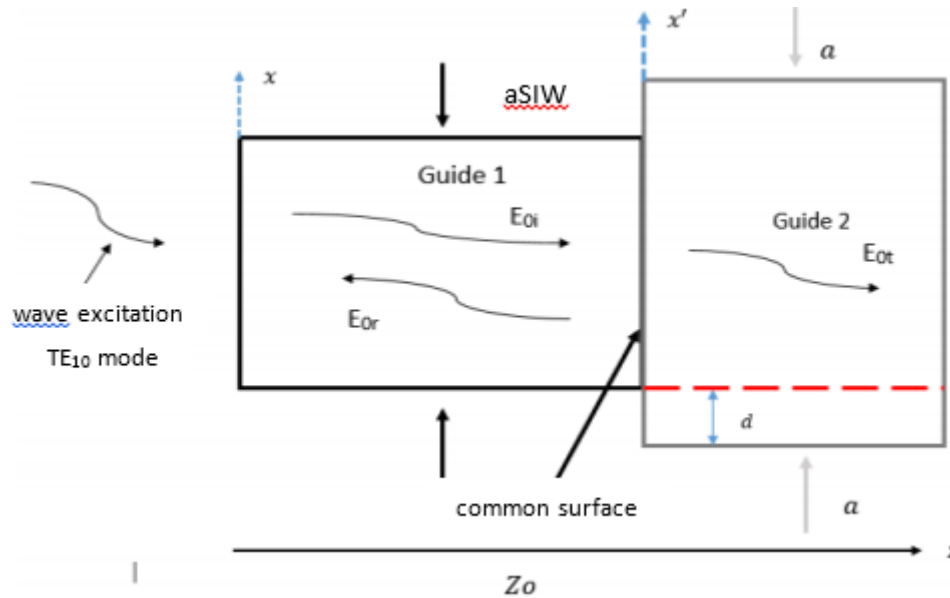


Figure 5: Discontinuity between waveguides (Incidence, transmission and reflection of a wave)

$$\vec{E} = \begin{cases} E_x = 0 \\ E_y = C_{m0} \sin\left(\frac{m\pi}{a_s} x\right) \\ 0 \end{cases} \quad (21)$$

$$\vec{H} = \begin{cases} H_x = D_{m0} Y_{acm} \sin\left(\frac{m\pi}{a_s} x\right) \\ H_y = 0 \\ H_z = A_{mn} \cos\left(\frac{m\pi}{a_s} x\right) \end{cases} \quad (22)$$

In the absence of surface currents at the interface between the two waveguides, the continuity of the transverse components of the electric and magnetic fields leads to the following relationships.

$$\begin{cases} E_{oi} + E_{or} = E_{ot} \\ H_{oi} + H_{or} = H_{ot} \end{cases} \quad (23)$$

- For the Substrate Integrated Waveguide

$$\begin{cases} E_y^{(1)} = \sum_{m=1}^{\infty} \sin\left(\frac{m\pi}{a_s}\right) (C_{m0} e^{-j\beta_m z} + C'_{m0} e^{j\beta_m z}) \\ H_x^{(1)} = \sum_{m=1}^{\infty} Y_{aem} \sin\left(\frac{m\pi}{a_s}\right) (C_{m0} e^{-j\beta_m z} - C'_{m0} e^{j\beta_m z}) \end{cases} \quad (24)$$

- For the Waveguide

$$\begin{cases} E_y^{(2)} = \sum_{q=1}^{\infty} \sin\left(\frac{q\pi}{a}(x+d)\right) (D_{q0}e^{-j\beta_{qz}} + D'_{q0}e^{j\beta_{qz}}) \\ H_x^{(2)} = \sum_{q=1}^{\infty} Y_{aem} \sin\left(\frac{q\pi}{a}(x+d)\right) (D_{q0}e^{-j\beta_{qz}} - C'_{q0}e^{j\beta_{qz}}) \end{cases} \quad (25)$$

The coefficients  $C_{m0}$  and  $C'_{m0}$  represent the amplitudes of the incident and reflected waves of mode  $m$  in the first guide.

By applying the conditions of continuity of the tangential components of the fields, at the interface  $Z = 0$  of the two guides.  $E_y^{(1)} = 0 \Leftrightarrow E_y^{(1)} = E_y^{(2)}$  the same for  $H_x^{(1)} = H_x^{(2)}$ .

$$\begin{aligned} \sum_{m=1}^{\infty} [C_{m0} + C'_{m0}] \sin\left(\frac{m\pi}{a_s}x\right) &= \sum_{m=1}^{\infty} [D_{q0} + D'_{q0}] \sin\left(\frac{q\pi}{a}(x+d)\right) \\ \sum_{m=1}^{\infty} [C_{m0} - C'_{m0}] Y_{acm} \sin\left(\frac{m\pi}{a_s}x\right) &= \sum_{m=1}^{\infty} [D_{q0} - D'_{q0}] Y_{am} \sin\left(\frac{q\pi}{a}(x+d)\right) \end{aligned} \quad (26)$$

By projecting relations (3.13) and (3.14) on the modes and taking into account the properties of orthogonality between modes, we have:

$$Z_e = \frac{1+[S_{11}]}{1-[S_{11}]} \quad (27)$$

This notion of input impedance can prove useful in the analysis of propagation problems, and constitutes a basis for the formal analysis of guided wave theory. Indeed, this impedance is transformed like the reduced impedance.

## 6. Conclusion

We have presented in this manuscript the study of the transition between rectangular waveguide and waveguide integrated in the substrate. The results obtained in our work are very satisfactory and allow us to predict the propagation phenomena of electromagnetic waves.

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