



Bit-to-Error Rate Performance Analysis of Massive Multiple-Antenna Downlink System

Muoghalu, C. N.*¹, Ekengwu, B. O.², Asiegbu, N. C.², Eke-Nnodi, C. O.²

*¹Department of Electrical and Electronic Engineering, Chukwuemeka Odumegwu Ojukwu University, Uli, Nigeria

²Department of Electronic Engineering, University of Nigeria, Nsukka, Nigeria

Abstract This paper has presented bit-to-error rate (BER) performance analysis of multiuser (MU) massive multiple-antenna downlink system. Two precoding algorithms were used namely, zero-forcing (ZF) and maximum ratio transmission (MRT). Simulations were carried out in MATLAB environment to examine the performance of massive multiple-antenna system in terms of BER against the transmit signal to noise (SNR) with respect to increasing base station antennas M at a fixed user equipment, K . The BER for ZF is compared with MRT to examine the effectiveness of the precoding algorithms on the massive antenna configuration in terms of BER versus SNR. The results obtained indicated that as the SNR increases, the BER ratio decreases. Generally, BER reduces significantly with large antenna using MRT precoding to 2.08×10^{-6} . The use of MRT precoding technique as part of massive multiple antennas provides better BER versus SNR ratio performance.

Keywords BER, Multiple-antenna, MRT, SNR, ZF

Introduction

The demand for high speed and widespread network access in mobile communications increases everyday as the number of users increase and applications are constantly developed with greater demand for network resources. As a result of this trend, mobile communications have experienced significant developments within the last two decades resulting to tremendous studies that have been carried out. Multi-User multiple input multiple output (MU-MIMO) system is the technology sort for in recent wireless communication standards such as LTE for improved 4G and in extension to 5G, where massive multiple antenna arrays have been deployed.

Over the years, starting from single input single output (SISO) systems, single user (SU) and MU-MIMO, the rate of data transfer in wireless networks has successively improved. While significant benefits have been offered by MU-MIMO over previous technology, massive MIMO is aimed at enhancing these advantages. For instance, massive MIMO provides wireless network speed of 10 Gbps and more using hundreds of antennas taking advantage of advances in parallel digital signal processing and high-speed electronics [1].

As a multiple antenna technique, massive MIMO aims to enhance the data transmission using hundreds of transmit antennas. This means that the more the BS or transmit antennas employed, the more the data streams that can be transmitted to serve more user equipments (UEs) or terminals and thereby minimizing the radiated power, even as the data rate is boosted. Also, the reliability of the link is improved through spatial diversity and, offers additional degree of freedom (DOF), and enhances the performance regardless of the measurement noisiness. As a result of the wide range of states of freedom and greater selectivity in data stream transmission and reception possessed by massive MIMO, cancellation of interference is enhanced [1]. In massive MIMO, transmission into undesired directions to reduce negative interference that causes low latency can be fairly



averted by BS. Furthermore, proper use of beamforming schemes by massive MIMO minimizes fading drops, which additionally improves signal to noise ratio (SNR), bit rate and reduces latency [2].

In the analysis of the performance of wireless network, many parameters are considered such as spectral efficiency, transmission capacity of the channel, throughput, bit-to-error rate (BER), and others. This paper considers BER performance of massive MIMO.

Downlink MU Massive MIMO System

System Model

A general downlink large scale Multi-User (MU)- multiple antenna system for the proposed solution is shown in Figure 1 having a base station (e-Node B) equipped with large number of M transmit antennas and with K number of users such that each user is equipped with one single antenna (with $M \gg K$). The K users are simultaneously serviced by the base station. In order to reduce model complexity, the following assumptions have been made in this work:

- The same time and frequency resources are shared among the K user equipment.
- There is no pilot contamination.
- The base station is considered to have ideal (or perfect) channel state information (CSI). That is transmitter is assumed to know the channel parameters.
- The channel is estimated at the base station via reverse link (uplink pilots) suppose channel reciprocity.
- Each of the user equipment (UE) has single antenna.

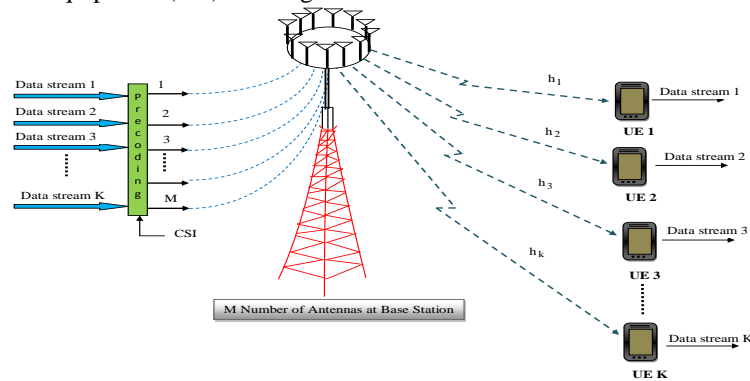


Figure 1: A downlink massive multiuser multiple antenna system model

During the training sequence at the base station, the channel information is acquired. The kind of training techniques relies on the time-division duplex (TDD) and frequency-division duplex (FDD) algorithms. When using FDD at the base station, it is difficult to determine CSI as the number of downlink resources needed for training pilots happens to be proportional to number of base station antennas.

Conversely, when using TDD operation, the CSI can be obtained by the base station from the uplink training phase as a result of the use of channel reciprocity. Thus, operating cost as a result of pilots being proportional to the users makes TDD operation preferable to FDD in most of the scenarios involving massive multiuser multiple antenna system [3-6]. Thus a TDD multiuser massive multiple antennas technique for downlink transmission is considered in this paper.

Mathematical Description of Data Transmission

This subsection considers the mathematical definition of downlink data transmission. Now, consider the simplified down link massive multiple antenna system in a single cell shown in Figure 2.



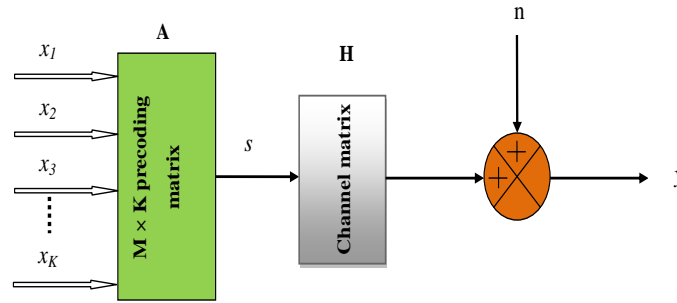


Figure 2: A model of downlink massive multiple antenna system in a single cell

Assuming a perfect channel given by $K \times M$ channel matrix $H = [h_1^T \ \dots \ h_K^T]^T$, where h_k represents the $1 \times M$ channel vector among K^{th} user equipment (UE) and base station. With the propagation channel usually designed by assuming large-scale and small-scale fading [3], small-scale fading has been considered in this work assuming the elements of H defined as the channel matrix among base station antennas and users' equipment given by Eq. (1) are separately and identically distributed Gaussian distribution with unit variance and zero mean.

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} & \dots & h_{1M} \\ h_{21} & h_{22} & h_{23} & \dots & h_{2M} \\ h_{31} & h_{32} & h_{33} & \dots & h_{3M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{K1} & h_{K2} & h_{K3} & \dots & h_{KM} \end{bmatrix} \tag{1}$$

Since the K number of users will receive their respective information signal from the vector $K \times 1$, so the received signal is defined by the input-output relationship of the massive multiuser multiple antenna channel considering Figure 3.2 and is given by:

$$y = Hs + n \tag{2}$$

where s denotes the $M \times 1$ vector of precoded transmitted symbols (or signals), y is the $K \times 1$ vector of signals received by UEs, n is $K \times 1$ user equipment additive white Gaussian noise (AGWN) vector having independently and identically distributed Gaussian distribution with zero-mean and with variance N_0 .

Assuming linear precoding at the base station for both effectiveness and analytical simplicity, s in Eq. 2 can be defined looking at Fig. 2 without considering the power of the transmitted signal by:

$$s = Ax \tag{3}$$

where A represents a $M \times K$ precoding matrix, and x denotes input vector such that $x = [x_1 \ x_2 \ x_3 \ \dots \ x_K]^T$ with x_k the data symbol of k^{th} user which is assumed independently and identically distributed Gaussian distribution with zero-mean with unit variance.

Considering the precoding matrix having $\|A\|=1$ [3] and P_s representing the transmitted power from the base station (that is the total average energy available at the base station in one symbol period). Hence, the transmitted signal from the base station is defined such that Eq. (4) becomes:

$$s = \sqrt{P_s} Ax \tag{4}$$

Substituting Eq. (4) into Eq. (2) gives:

$$y = \sqrt{P_s} HAx + n \tag{5}$$

Using the notation $A = [a_1 \ a_2 \ a_3 \ \dots \ a_k]$ the transmitted signal on the i^{th} antenna can be expressed by:



$$s_i = \sqrt{P_s} \sum_{j=1}^k a_{ij} x_j \quad (6)$$

The k^{th} user equipment received signal can be defined by:

$$y_k = \sqrt{P_s} \sum_{j=1}^K \sum_{m=1}^M h_{k,m} a_{m,j} x_j + n_k \quad (7)$$

Looking at Eq. (7), the desired signal part when $j=k$ and the interference part can be expressed by:

$$y_k = \underbrace{\sqrt{P_s} \sum_{m=1}^M h_{k,m} a_{m,k} x_k}_{\text{desired signal}} + \underbrace{\sqrt{P_s} \sum_{j \neq k}^K \sum_{m=1}^M h_{k,m} a_{m,j} x_j}_{\text{interference}} + \underbrace{n_k}_{\text{noise}} \quad (8)$$

It can be seen from Eq. (7) that received signal of each user (that is k^{th} user equipment) consists of desired signal, interference known as multiuser interference (MUI), and noise (which is an AWGN) having zero-mean with variance $N_0 = 1$, such that $n_k \in CN(0,1)$ for all users.

Therefore, the received signal-to-interference-plus-noise ratio (SINR) of the k^{th} user can be written as given by:

$$\text{SINR}_k = \frac{\sqrt{P_s} \sum_{m=1}^M h_{k,m} a_{m,k} x_k}{\sqrt{P_s} \sum_{j \neq k}^K \sum_{m=1}^M h_{k,m} a_{m,j} x_j + n_k} \quad (9)$$

Equation (9) is a function of transmit precoding matrix \mathbf{a}_k and x_k , which is transmit signal vector for k^{th} user. The same holds for all K users connected in downlink multiuser multiple antenna system.

Downlink Precoding Techniques

At the base station, precoding or pre-filtering scheme takes place during downlink operation. This technique is used to direct the transmitted signal towards the intended users and it is designed to cancel inter-user interference signal arising from multiple users accessing the link at the simultaneously. Since Eq. (8), which is the received signal at user's equipment (UE), contains desired signal, signal interference and noise, mitigating the interference can be achieved using linear precoding schemes. This section is set to define mathematical modelling of two precoding techniques namely; zero-forcing (ZF) precoding and maximum ratio transmission (MRT) precoding that will later be used to develop a MATLAB algorithm to evaluate the performance of the proposed solution in terms of spectral efficiency while mitigating multiuser interference (MUI) under increasing base station antenna and number of users.

Zero-Forcing Precoding Technique

This scheme is a linear pre-filtering process that allows a multiuser transmission without producing MUI. It is a type of beamforming that is practicable when the number of transmit antennas M at the base station is greater than or equal to K number of users (that is $M \geq K$) with user equipment having single antenna. The use of ZF precoding can guarantee interference cancellation at each user. This precoding scheme is assumed to implement a pseudo-inverse of the channel matrix. Therefore, ZF precoding used by base station considering the $M \times K$ linear precoding matrix \mathbf{A} and $K \times M$ channel matrix \mathbf{H} is given by:

$$\mathbf{A}_{\text{ZF}} = \mathbf{H}^* (\mathbf{H} \times \mathbf{H}^*)^{-1} \quad (10)$$



where the expression $H^*(H \times H^*)^{-1}$ is known as pseudo inverse of complex channel matrix H .

MRT Precoding Technique

The maximum ratio transmission (MRT) precoding scheme is one of the common method that optimizes the SNR of desired received signal at each k^{th} user terminal implementing the hermitian of channel matrix that is conjugate transpose of channel matrix. The precoding scheme for MRT is given by:

$$A_{\text{MRT}} = H^* \quad (11)$$

It should be noted that A_{ZF} and A_{MRT} are the precoding matrices A of the precoding schemes respectively and consisting of each column vector a_k that is $A = (a_1 \ a_2 \ a_3 \ \dots \ a_k)$.

BER and SNR

Bit-to-error rate (BER) is essential parameter in evaluating the performance of data channels. During data transmission from one point to another over a wireless link, one key parameter to know is number of errors that will appear in the data at the receive end. The BER as wireless system assessment parameter can assess the full end to end performance of a system including the transmitter, the channel and the receiver. Thus, BER is defined as the rate at which errors occur in a transmission system. Mathematically, BER is given by:

$$\text{BER} = \frac{N_E}{\sum N_B} \quad (12)$$

where N_E is the number of errors, and N_B is the number of bits transmitted.

Signal to noise ratio (SNR) is defined as the ratio of signal power to noise power. With the proposed system, is assumed that the intended signal vector x and the vector noise are independently and identically distributed complex Gaussian random variables with zero mean and unit variance, and is given by:

$$\text{SNR} = \frac{P_{\text{signal}}}{P_{\text{noise}}} = 10 \log_{10} \left(\frac{P_{\text{signal}}}{P_{\text{noise}}} \right) \text{ in (dB)} \quad (13)$$

where P_{signal} signal power, and P_{noise} is the noise power.

Simulation Result and Discussion

This section examines the performance of MU massive MIMO downlink system in terms of BER against SNR by means of simulations in MATLAB environment. Modulation scheme employing quadrature phase shift keying (QPSK) was used considering data transmission over an assumed uncorrelated spatially flat Rayleigh fading channel, and the noise distributed complex Gaussian variables with zero mean and unit channel power standard deviation. The base station (BS) antennas (M) was varied from (300 to 600), while keeping the number of receive antennas at 50. The number of active users K on the network was taken as 50 such that the number of antennas per user is equal to one.

The simulation results of the massive multiple antenna system performance in terms of bit error rate (BER) versus signal to noise ratio (SNR) for number of base antennas, $M = 1:300, 400, 500, \text{ and } 600$, when the number of users, $K = 50$ as shown in Figures 3 to 6. Table 1 is the summary of the performance of the system for different BS antenna configuration scenarios considered.



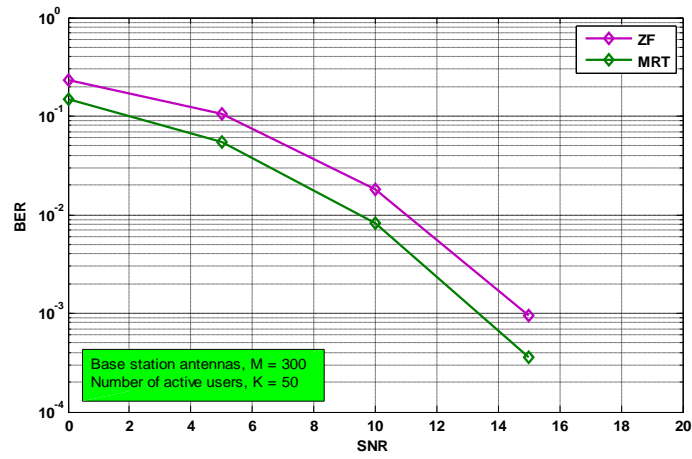


Figure 3: BER with respect to ZF and MRT for $M = 300$, $K = 50$

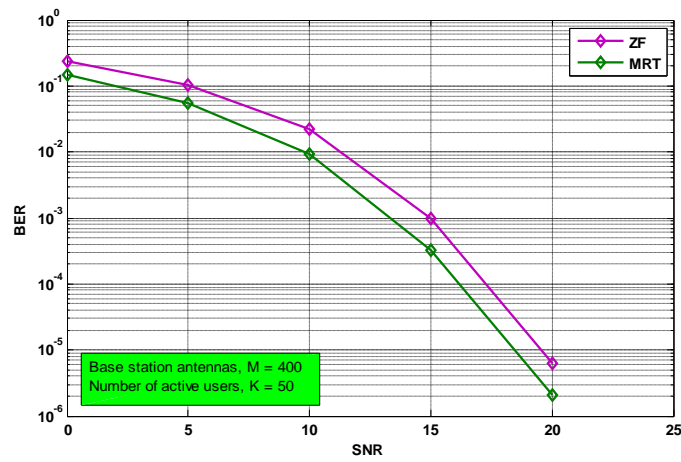


Figure 4: BER with respect to ZF and MRT for $M = 400$, $K = 50$

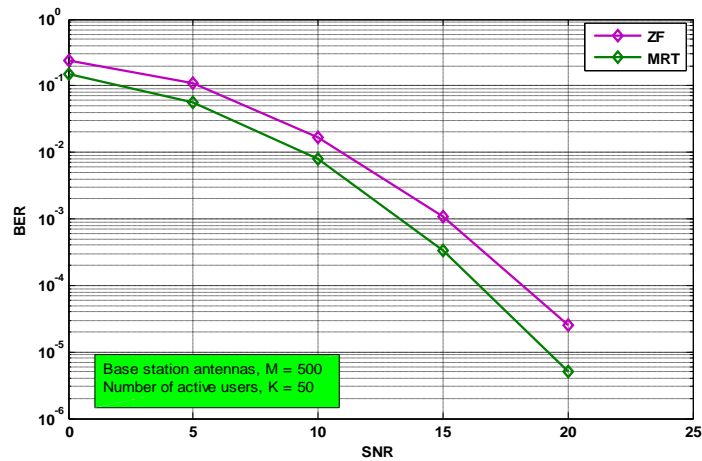


Figure 5: BER with respect to ZF and MRT for $M = 500$, $K = 50$

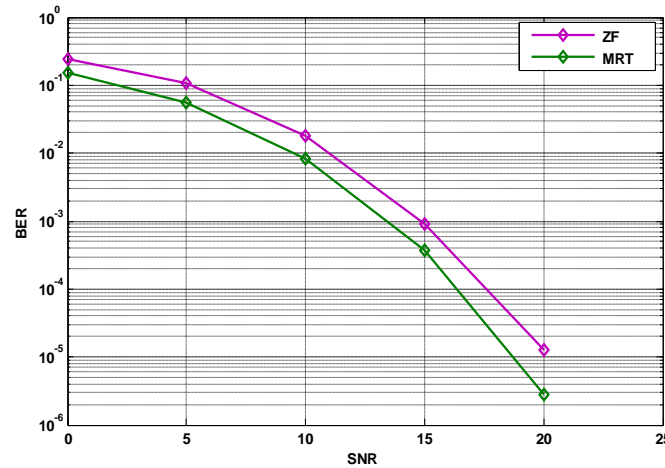


Figure 6: BER with respect to ZF and MRT for $M = 600, K = 50$

Table 1: BER performance analysis (at $K = 50$)

SNR (dB)	Number of base station antennas (M) (BER for ZF precoding)				Number of base station antennas (M) (BER for MRT precoding)			
	300	400	500	600	300	400	500	600
0	0.2384	0.2414	0.2368	0.1497	0.1491	0.1498	0.1508	0.238
5	0.1053	0.1047	0.1085	0.1092	0.05519	0.0557	0.05619	0.05617
10	0.02001	0.02191	0.01702	0.001814	0.008703	0.009229	0.007905	0.008311
15	0.0006972	0.0009729	0.0008096	0.0008958	0.0003028	0.0003292	0.000285	3.64e-04
20	-	6.25e-06	2.5e-05	1.25e-05	-	2.083e-06	5e-06	2.77e-06

Without loss of generality, but for simplicity purpose, the number of received antennas for each user is the same. That is, the simulation is conducted assuming that each user is assigned a single receive antenna such that for K number of users equipment = 50, there are N number of received antennas = 50.

Subsequently, the BER performance of the system for the different configuration is examined and the analysis of the results obtained is presented in Table 1.

The BER performances for each stream when a ZF and an MRT precoding schemes were used for the downlink operation (transmission from base station to the users' equipment) are presented Figures 3 to 6. The BER curves are plotted against the transmit SNR with base station configuration of $M = 300, 400, 500,$ and 600 for $N = 50$ and $K = 50$. The BER for ZF and MRT precoding algorithms are compared with the same antenna arrangement. As can be seen in Table 1, MRT gives better BER than ZF. It is observed that as the SNR increases, the BER ratio decreases. Generally, BER reduces significantly with large antenna using MRT precoding. This observation is validated by the simulation analysis of BER against SNR for downlink MU-MIMO systems carried out by Zhang et al. [7], where it was reported that MRT precoding scheme provided better BER than ZF precoding technique.

Conclusion

The simulation analysis of BER against SNR in MU massive multiple antennas down link system has been studied. The plot of BER against the transmit SNR was presented with respect to increasing base station antennas M at a fixed user equipment, K . The BER for ZF is compared with MRT to examine the performance of the proposed massive antenna configuration in terms of BER versus SNR. The results obtained indicated that as the SNR increases, the BER ratio decreases. Generally, BER reduces significantly with large antenna using MRT precoding.



References

- [1]. Hassan, N., and Fernando, X. (2017). Massive MIMO wireless networks: An overview. *MDPI Journal, Electronics*, 6(63), 1-29.
- [2]. Gupta, A., and Jha, R.K. (2015). A survey of 5G Network: Architect and emerging technologies. *IEEE Access*, 3, 1206-1232.
- [3]. Singh, J., and Kedia, D. (2019). Performance of ZF and CB precoding techniques for large-scale MU-MIMO system. *International Journal of Recent Technology and Engineering*, 8(2), 5529-5536.
- [4]. Hoydis, J., Ten Brink, S., and Debbah, M. (2013). Massive MIMO in the UL/DL of cellular networks: How many antennas do we need? *IEEE Journal on Selected Areas in Communication*, 31(2), 160-171.
- [5]. Marzetta, T.L. (2010). Noncooperative cellular wireless with unlimited numbers of base station antennas. *IEEE Transactions on Wireless Communications*, 9(11), 3590-3600.
- [6]. Jose, J., Ashikhmin, A., Marzetta, T.L., and Vishwanath, S. (2011). Pilot Contamination and Precoding in multi-cell TDD system. *IEEE Transactions on Wireless Communications*, 10(8), 2640-2651.
- [7]. Zhang, Y., Gao, J., and Liu, Y. (2016). MRT precoding in downlink multi-user MIMO systems. *EURASIP Journal on Wireless Communications and Networking*, 2016(241), 1-7.

