



Study in Transient Regime by Analytical Method of Heat Transfer through a Kapok-plaster Insulating Material has one Dimension: Influence of the Heat Exchange Coefficient

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Abstract The study in Cartesian coordinates of a form material of a simple wall consisting of Kapok-plaster with the following characteristics ($\lambda = 0,1W.m^{-1}.^{\circ}C^{-1}$ and $\alpha = 4,73.10^{-7} m^2.s^{-1}$).

A study of the thermal behavior in transient regime of Kapok associated with plaster as a binder is carried out. Analysis of the results from representations of temperature and heat flux density allowed characterization of the Kapok-plaster thermal insulation from the one-dimensional transient dynamic regime study. The resolution of the one-dimensional heat diffusion equation by the analytical method made it possible to determine the evolution of the temperature and the flux density according to the depth and the thickness under the influence of the heat exchange coefficient at the front face. The study showed that the Kapok-plaster material is a good thermal insulator.

Keywords Thermal behavior-Kapok plaster-Regime Dynamics Transient

Introduction

Artificial insulation poses an environmental problem unlike natural biodegradable insulation [1]. Kapok [2] a natural biodegradable product, is used as a thermal insulator [3-5] in association with plaster as a binder. The study is part of improving the use of natural local products on thermal insulation. The heat transfer through the kapok-plaster material [6] is studied considering a material of parallelepiped shape. The proposed study is done in one dimension in a Cartesian coordinate system. The imposed heat exchanges take place on the front and rear faces. Researchers have worked on several local materials of plant [7], animal [8, 9] or synthetic [10-12] origin for thermal comfort [13-15].

In this work, we study the transient heat transfer [16-18] in a material consisting of Kapok-plaster under the influence of the heat exchange coefficient [19, 20].

Depending on the depth and the heat exchange coefficient we present the profiles of the temperature and density curves of heat flow.

After using the different types of results we highlight the quality of the Kapok-plaster thermal insulation.

Materials and Methods

Figure 1 represents the study model, it is a simple wall in kapok-plaster of thickness L.

The thermal exchanges between the material and the two faces (exterior and interior) are assumed to be convective. They are quantified by heat transfer coefficients on the front and back sides.

- T_{a1} and T_{a2} : temperature in dynamic transient mode of indoor and outdoor environment respectively;



- $T(x, h_1, h_2, t = 0) = T_i = 10^0 C$: initial temperature of the kapok-plaster insulating material;
- $L = 0,05m$: material length along the axis x ;
- h_1 and h_2 ($W.m^{-2}.^0 C^{-1}$) : heat transfer coefficient from single wall to front and back side respectively.

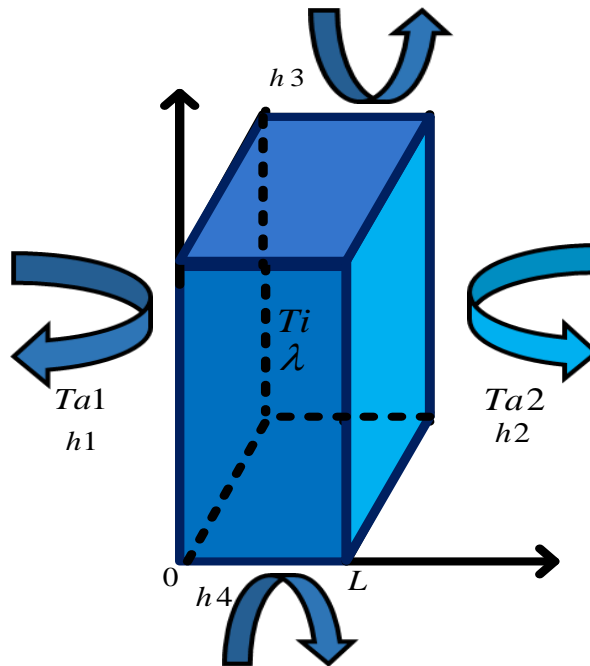


Figure 1: Sample of Kapok-plaster

where:

- $\rho(kg.m^{-3})$: density of material;
- $C(J.kg^{-1}.^0 C^{-1})$: mass thermal capacity;
- $\lambda(W.m^{-1}.^0 C^{-1})$: thermal conductivity of material;
- $P(W.m^{-3})$: internal heat supply (heat sink) of material;
- $x(m)$: depth position.

Simplified form of this equation, in absence of internal heat sinks and for constant thermal conductivity (assumed isotropic material) is given by:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho C_p} \Delta T; (1)$$

Theory

Unidirectional transfer of heat in thermal insulating tow-plaster is regulated by equation (2) below:

$$\frac{\partial^2 T(x, h_1, h_2, t)}{\partial x^2} - \frac{1}{\alpha} \frac{\partial T(x, h_1, h_2, t)}{\partial t} = 0; (2)$$

$T = T(x, h_1, h_2, t)$ is temperature inside material; x depth and t time. Equation (3) gives expression of the diffusivity α .

$$\alpha = \frac{\lambda}{\rho.c}; (3)$$

α thermal diffusivity ($m^2.s^{-1}$)

λ thermal conductivity ($W.m^{-2}.^0 C^{-1}$)



ρ density of the material (kg.m^{-2})

Boundary conditions:

$$\left\{ \begin{aligned} \lambda \frac{\partial T(x, h_1, h_2, t)}{\partial x} \Big|_{x=0} &= h_1 [T(0, h_1, h_2, t) - T_a] ; (4) \\ \lambda \frac{\partial T(x, h_1, h_2, t)}{\partial x} \Big|_{x=L} &= -h_2 [T(L, h_1, h_2, t) - T_a] ; (5) \\ T(x, h_1, h_2, t = 0) &= T_i ; (6) \end{aligned} \right.$$

To solve the equation (1) we make dimensionless by asking:

$$\theta(u, \tau) = \frac{T(x, h_1, h_2, t) - T_a}{T(x, h_1, h_2, t = 0) - T_a} = \frac{T(x, h_1, h_2, t) - T_a}{T_i - T_a} ; (7)$$

With

$\theta(u, \tau)$: reduced temperature;

$$u = \frac{x}{L} \quad ; \text{ reduced space variable}$$

and

$$\tau = \frac{\alpha.t}{L^2} ; \tau = F_0$$

F_0 : reduced variable of time or number of Fourier

Heat equation (1) becomes:

$$\frac{\partial^2 \theta(u, \tau)}{\partial u^2} = \frac{\partial \theta(u, \tau)}{\partial \tau} ; (8)$$

Boundary conditions (4) and (5) become (9) and (10):

$$\left\{ \begin{aligned} \frac{\partial \theta(u, \tau)}{\partial \tau} \Big|_{u=0} &= \frac{h_1.L}{\lambda} \theta(0, \tau); (9) \\ \frac{\partial \theta(u, \tau)}{\partial \tau} \Big|_{u=1} &= -\frac{h_2.L}{\lambda} \theta(1, \tau); (10) \end{aligned} \right.$$

Seek the solution of equation (7) in the form of equation (10):

$$\theta(u, \tau) = U(u).H(\tau); (11)$$

Using equations (7) and (10) we obtain that (11)

$$\frac{1}{U(u)} \cdot \frac{\partial^2 U(u)}{\partial u^2} = \frac{1}{H(\tau)} \cdot \frac{\partial H(\tau)}{\partial \tau} = -\beta^2 ; (12)$$

β positive constant.

From equation (11) we obtain two differential equations:

i) Differential equation in time is given by (13):

$$\frac{1}{H(\tau)} \cdot \frac{\partial H(\tau)}{\partial \tau} = -\beta^2 ; (13)$$

ii) Differential equation in space (14) is written:

$$\frac{1}{U(u)} \cdot \frac{\partial^2 U(u)}{\partial u^2} = -\beta^2 ; (14)$$

Boundary conditions of space:



$$\left\{ \begin{aligned} \left. \frac{\partial^2 U(u)}{\partial u} \right|_{u=0} &= B_{i1} U(0); (15) \\ \left. \frac{\partial^2 U(u)}{\partial u} \right|_{u=L} &= -B_{i2} U(1); (16) \end{aligned} \right.$$

With $B_{i1} = \frac{h_1 \cdot L}{\lambda}$ and $B_{i2} = \frac{h_2 \cdot L}{\lambda}$

Biot numbers respectively to the front side and the back side.

Temporal equation (14) has the solution:

$$H(\tau) = H(0) \cdot e^{-\beta^2 \tau} = H(0) \cdot e^{-\frac{\tau}{\tau_d}}; (17)$$

$$\tau_d = \frac{1}{\beta^2};$$

τ_d : reduced time constant and $\beta_n \neq 0$ Differential equation (14) has the solution:

$$U(u) = a_n \cos(\beta_n u) + b_n \sin(\beta_n u); (18)$$

with, $n \in \mathbb{N}$

a_n and b_n coefficients determined from the boundary conditions.

$$\left\{ \begin{aligned} \beta_n b_n &= B_{i1} a; (19) \\ -\beta_n a_n \sin(\beta_n) + \beta_n b_n \cos(\beta_n) &= -B_{i2} [a_n \cos(\beta_n u) + b_n \sin(\beta_n u)]; (20) \end{aligned} \right.$$

Expression (20) allows us to find transcendental equation (21):

$$\tan(\beta_n) = \frac{\frac{h_2 L}{\lambda} \beta_n + \beta_n \frac{h_1 L}{\lambda}}{\beta_n^2 - \frac{h_1 \cdot h_2 \cdot L^2}{\lambda^2}}; (21)$$

Transcendental equation is divided into two functions:

i) Trigonometric function denoted $ft(\beta_n)$

$$ft(\beta_n) = \tan(\beta_n); (22)$$

ii) Homogeneous function denoted $fh(\beta_n)$

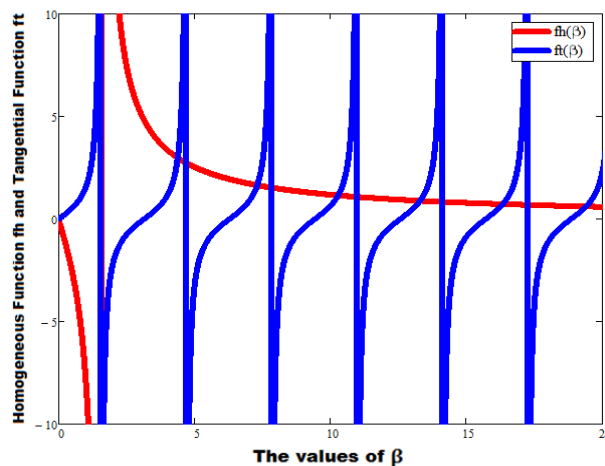


Figure 2: Graphical determination of the eigenvalues (β_n) of the transcendent equation



In this figure we present the evolution of $ft(\beta_n)$ and $fh(\beta_n)$ as a function of the eigenvalues of the transcendent equation for different values of the coefficients.

The intersection of the two curves $fh(\beta_n)$ and $ft(\beta_n)$ corresponds to the solution.

Table 1 summarizes the eigenvalues found of β_n

Table 1: The eigenvalues of β_n of the equation

| n | 1 | 2 | 3 | 4 | 5 |
|-----------|-----|-----|------|------|------|
| β_n | 4.6 | 7.5 | 10.5 | 13.4 | 16.6 |

2.1. Temperature expression

The Euler-Fourier integrals allow us to obtain the constants $H_{1n}(0)$ and $H_{2n}(0)$

$$H_{1n}(0) = \frac{\sqrt{\gamma}}{(\beta_n^2 + \gamma) \left[\sqrt{\gamma} \cosh(\sqrt{\gamma}) + \frac{h_2 L}{\lambda} \sinh(\sqrt{\gamma}) \right]} \left\{ \left[\sqrt{\gamma} \sinh(\sqrt{\gamma}) + \beta_n \sin(\beta_n) \right] + \frac{h_2 L}{\lambda} \left[\cosh(\sqrt{\gamma}) - \cos(\beta_n) \right] \right\}; \quad (23)$$

$$H_{2n}(0) = \frac{h_1 L}{\lambda(\beta_n^2 + \gamma) \left[\sqrt{\gamma} \cosh(\sqrt{\gamma}) + \frac{h_2 L}{\lambda} \sinh(\sqrt{\gamma}) \right]} \left\{ \left[\sqrt{\gamma} \cosh(\sqrt{\gamma}) - \sin(\beta_n) \right] + \frac{h_2 L}{\lambda} \left[\sinh(\sqrt{\gamma}) - \frac{\sqrt{\gamma}}{\beta_n} \cos(\beta_n) \right] \right\}; \quad (24)$$

We get the expression for the final temperature:

$$T(x, h_1, h_2, t) = T_a + (T_i - T_a) \cdot \delta\theta_0 \cdot \sum_n a_n^2 \left[\cos\left(\frac{x}{L} \cdot \beta_n\right) + \frac{h_1 L}{\lambda \beta_n} \sin\left(\frac{x}{L} \cdot \beta_n\right) \right] \cdot [H_{1n}(0) + H_{2n}(0)] \cdot e^{-\frac{\alpha \cdot t}{L^2} [\beta_n^2]}; \quad (25)$$

2.2. Expression of heat flux density

By generalizing the relation to a one-dimensional configuration:

$$\Phi(x, h_1, h_2, t) = -\lambda \cdot \vec{\nabla} T(x, h_1, h_2, t); \quad (26)$$

$$\vec{\nabla} = \frac{\partial}{\partial x}$$

With, the operator $(\frac{\partial}{\partial x})$ designates the gradient vector

We derive the temperature, $T(x, h_1, h_2, t)$ as a function of x.

$$\Phi(x, h_1, h_2, t) = -\lambda \frac{\partial T(x, h_1, h_2, t)}{\partial x}; \quad (27)$$

By applying the product of the derivative of the temperature with respect to the following component x and the thermal conductivity λ . We get the expression for the heat flux density:

$$\Phi(x, h_1, h_2, t) = \lambda \cdot (T_i - T_a) \cdot \delta\theta_0 \cdot \sum_n a_n^2 \left[\frac{\beta_n}{L} \sin\left(\frac{x}{L} \cdot \beta_n\right) - \frac{h_1}{\lambda} \cos\left(\frac{x}{L} \cdot \beta_n\right) \right] \cdot [H_{1n}(0) + H_{2n}(0)] \cdot e^{-\frac{\alpha \cdot t}{L^2} [\beta_n^2]}; \quad (28)$$

From expression (28) we plot the heat flux density according to the different parameters.

3. Results and Discussions

3.1 Evolution of the temperature and the density of the heat flow as a function of the depth for different values of the exchange coefficient

This wall is a material is made of Kapok-plaster. In this study we have a single wall with a thickness of 5cm.

Heat flux is from the front side assuming that from the back side is assumed to be low. The initial temperature of the material is fixed at 10 ° C. The outdoor environment has a temperature of 30 ° C.

For figures 3 and 4 we have plotted the temperature and the heat flux density as a function of the depth under the influence of the heat exchange coefficient at the front face h1.

We note a decrease in temperature as well as in heat flux density depending on the depth of the wall made of Kapok-plaster material. This thermal drop in depth shows that this material is a good insulator.



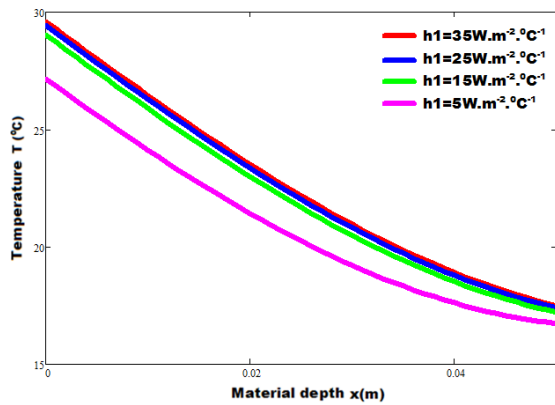


Figure 3: Temperature as a function of material depth. $h_2=0.005W.m^{-2}.C^{-1}$; $t=10s$

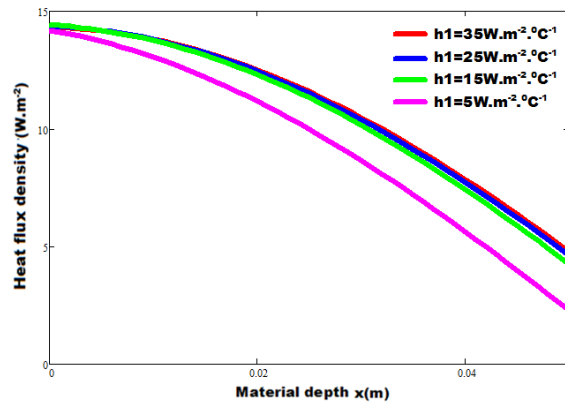


Figure 4: Heat flux density as a function of material depth. $h_2=0.005W.m^{-2}.C^{-1}$; $t=10s$

3.2. Evolution of the temperature and the density of heat flow as a function of the exchange coefficient

For low values of the exchange coefficient on the front face, the temperature and the heat flux density evolve exponentially before stabilizing at temperature values depending on the depth. We note that as one penetrates the material layer, the effect of the exchange coefficient at the front face in the evolution of heat flow becomes less and less important.

For figure 5 we see for each point of the material, the temperature increases exponentially as a function of h_1 and then reaches a level which corresponds to the maximum temperature that this point can take where the stored energy is maximum at this point.

For figure 6 the heat flux density increases exponentially as a function of h_1 then reaches a plateau and it hardly increases any more. The material seems to store thermal energy, hence a situation of saturation of the energy stored in large values of the exchange coefficient.

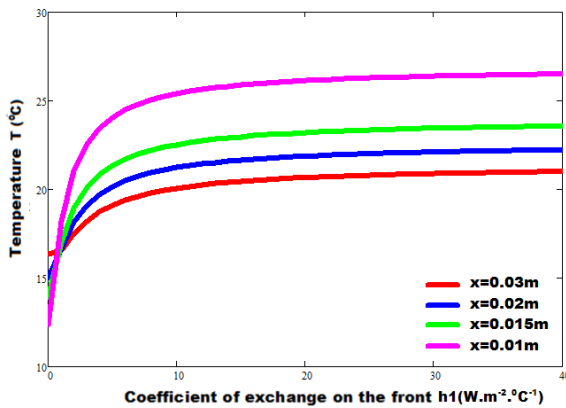


Figure 5: Temperature as a function to the heat exchange coefficient on the front face h_1 of the material. $h_2=0.005W.m^{-2}.C^{-1}$; $t=10s$

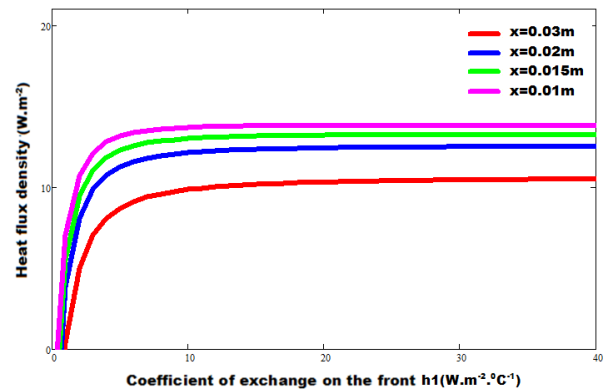


Figure 6: Heat flux density as a function of the exchange coefficient at the front face h_1 material. $h_2=0.005W.m^{-2}.C^{-1}$; $t=10s$.

3.3. Evolution of temperature and heat flow density as a function of time

Figures 7 and 8 give the temperature and heat flux density of the samples as a function of time under the influence of the heat exchange coefficient at the front face.

For figure 7 the temperature increases as a function of time. This shows the material heats up over time. This translates into a storage of thermal energy.



For Figure 8 the heat flux density decreases as a function of time inside the material. This decrease is due to a loss of heat in the kapok-plaster material.

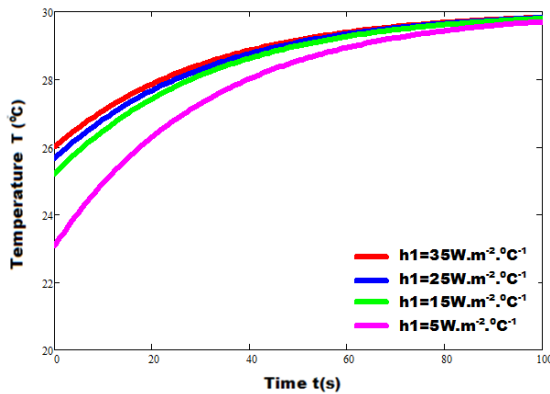


Figure 7 : Evolution de la température en fonction du temps matériau ; $x=0.01m$; $h_2=0.005W.m^{-2}.C$

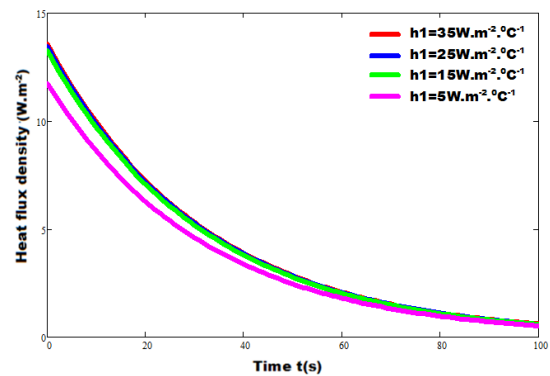


Figure 8 : Evolution de la densité de flux de chaleur en fonction du temps ; $x=0.01m$; $h_2=0.005W.m^{-2}.C^{-1}$

4. Conclusion

The study of the thermal behavior of the Kapok-plaster insulating material through a model of the temperature and the heat flux density has made it possible to highlight the quality of the material.

The influence of the exchange coefficient at the front face on the temperature and heat flux density of the Kapok-plaster material, under transient conditions, is noted. This shows that the Kapok-plaster used is a good insulator.

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