Available online www.jsaer.com

***Journal of Scientific and Engineering Research*, 2021, 8(9):176-198**

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

On the Influence of Viscosity in an Unsteady MHD Flow in a Vertical Channel

Areo A. O.[1, *], Akinpelu, F. O.[1], Adetunde, I. A.[2]

Ladoke Akintola University of Technology, Department of Pure and Applied Mathematics, Ogbomoso. Oyo State. Nigeria
Prof I. A. Adetunde Consults, 8 Westmount Close, Collyhurst Street, Manchester. UK

Abstract This article is on the influence of combination of viscosity in an unsteady MHD flow in a vertical channel. The governing equations of motion were employed and solved analytically. Transform is carried out on the governing equations to form sets of nonlinear coupled equation by suitable transformation using stream functions and non-dimensionalize the non-linear partial differential governing equations to ordinary differential equations using the non-dimensional quantities. Solving the resulting sets of ordinary differential equations by using perturbation technique to obtain expressions for the velocity, temperature and concentration profiles. We analyse the effects of velocity, temperature and concentration of the fluid at the various physical parameters; Womersley parameter (AI), Eckert number (Ec), Reynolds number (Re), Prandlt number (Pr), Grashof number (Gr), Schmidt number (Sc), Chemical reaction parameter (K_1), and the Mass Grashof number (Gc). Graphical results are presented and discussed.

Keywords MHD flow, Viscocity, Concentration, Physical Parameters, Vertical Channel

Introduction

The unsteady and steady flow of Newtonian and non-Newtonian fluids in a porous media in which the main driving force is gravitational force has attracted the attention of many scientists in the recent times. This is due to its large area of applications and significant roles in Engineering and sciences particularly in applied Geophysics, Geology, Ground water flow, Food Technology, Filtration process, enhanced oil recovery, oil reservoir engineering and oil recovery processes. Gravity flow of non-Newtonian fluid through porous medium is involved in some important engineering applications: enhanced oil recovery by thermal methods, polymer solutions, geothermal power, geothermal reservoirs, food stuff processing and emulsions of oil and foam solutions acting as display fluids in certain oil fields. The term "non-Newtonian" implies that the viscosity is not only dependent upon temperature and pressure, but also on the rate of shear that is applied to the fluid. However, Newtonian fluid will have essentially the same viscosity no matter the rate of shear applied. The non-Newtonian behavior of many fluids has been recognized for a long time. However, the science of rheology is still in its infancy in many respects and as such, new phenomena are being discovered on a constant basis with new theories propounded. In the current study, attempts have been made to obtain relationships, mathematical and empirical descriptions of the flow of non-Newtonian fluids (power-law) through porous media.

Hartman and Lazarus [1] in 1937 studied the influence of a transverse uniform magnetic field on the flow of a viscous incompressible electrically conducting fluid between two infinite parallel stationary and insulating plates. Since then this pioneering work in MHD flow has received much attention and has been extended in numerous ways. Many studies are on MHD flows, notably among them are : Mostafa and Mahmoud [2] studied the variable viscosity and chemical reaction effects on mixed convection heat and mass transfer along a

semi infinite vertical plate. Ganesan and Plani [5] worked on numerical solution of transient free convection MHD flow of an incompressible viscous fluid flow past a semi infinite inclined plate with variable surface heat and mass flux. Likewise, Sparrow and Cress [8] worked on the effect of a magnetic field on free convection heat transfer. Recently Sarmal and Hazarika [7] presented their research work on effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of a magnetic field.
This work is focused on investigating the influence of the effect of viscosity in an unsteady MHD flow with viscous dissipation and concentration in a porous vertical channel and to examine the effects of some parameters on velocity, temperature and concentration profiles.

Mathematical Formulation of the Problem

Consider a steady free convection flow of a viscous and incompressible electrically conducting fluid along a porous vertical channel. The flow is as shown in Figure 1 below. The effect of a uniform transverse magnetic field B on unsteady two-dimensional electric conducting fluid flows are considered and its velocities are given as

$$q = u(x, y, t)i + v(x, y, t)j \tag{a}$$

through a symmetric vertical channel ($D: -\infty \angle x \angle +\infty, -b(x) \angle y \angle b(x)$) where (x, y) are Cartesian co-ordinates such that ox is the axis of symmetry of the channel and $y = \pm b(x)$ are the rigid and impermeable walls of the channel. The walls of the channel are kept at a constant temperature T_w . The fluid is incompressible with uniform properties i.e. density ρ , kinematic viscosity ν and electrical conductivity σ . A volume flux with oscillating frequency δ and pulse m is prescribed as in figure 1 below:

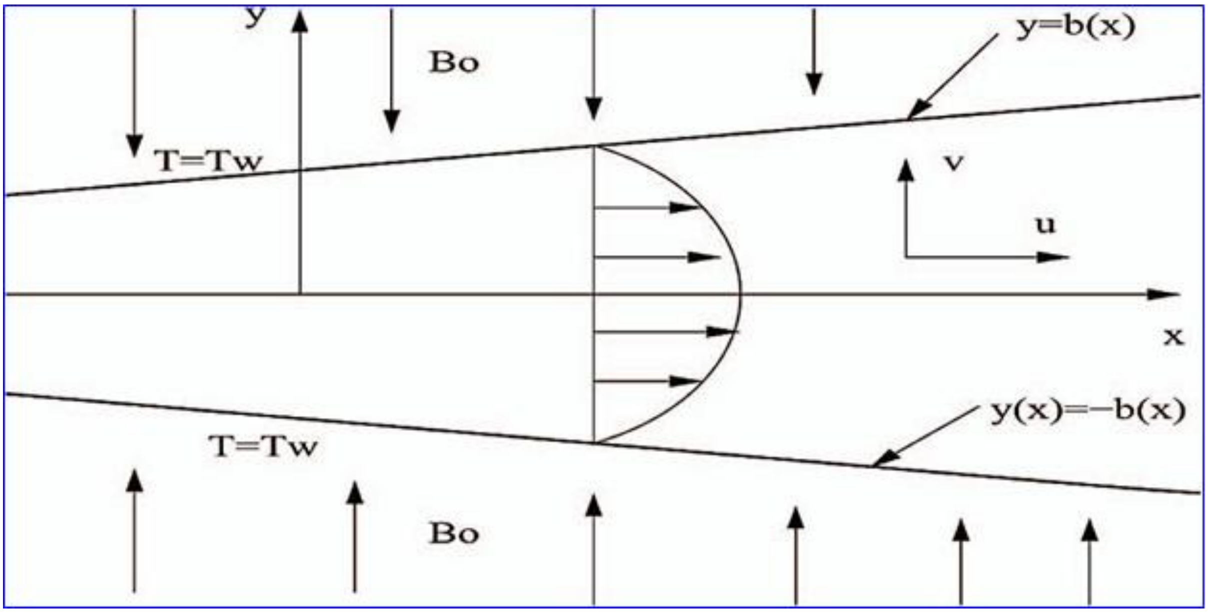

Figure 1: Problem Geometry (Source Mhone and Makinde, 2006)

$$\int_0^{b(x)} u dy = Q(1 + me^{i\delta t}) \tag{b}$$

A uniform magnetic force is applied in the $y-\text{direction}$. A very small magnetic Reynolds number is assumed and therefore the induced magnetic field is neglected. Two key physical effects occur when the fluid moves into the magnetic field; the first one is that an electric field E is induced in the flow. There is no excess charge density and then $\nabla.E = 0$. Neglecting the induced magnetic field implies that $\nabla \times B = 0$ and therefore the induced electric field is negligible. The second key effect is dynamical i.e. a Lorentz force ($J \times B$), where J is the current density acts on the fluid and modifies its motion. Therefore, there is a transfer of energy ($J.E$) from the electromagnetic field to the fluid. In this study\, relativistic effects are neglected, and J is given by Ohm's law:

$$J = \sigma(q \times B) \tag{c}$$

Within the framework of these assumptions the magneto-hydrodynamic flow relevant to the problem is governed by the set of equations

Mathematical Analysis

The governing equations for motion under the auspices of the Continuity equation, Momentum equation, Energy equation and Concentration equation are considered.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\nabla^2 u - \frac{\sigma B_0^2 u}{\rho} - \frac{\mu u}{\rho k} + g\beta(T - T_0) + g\beta(C - C_0) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + v\nabla^2 v \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} =$$

$$\frac{k}{\rho Cp}\nabla^2 T + 2\frac{v}{Cp}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2\right] + \frac{\sigma B_0^2 u^2}{\rho Cp} + g\beta(T - T_0) + g\beta(C - C_0) \tag{4}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + D_1\frac{\partial^2 T}{\partial y^2} + K_L(C - C_0) \tag{5}$$

With conditions:

Symmetry: $\frac{\partial u}{\partial y} = 0$, v = 0, $\frac{\partial T}{\partial y} = 0$, C = 0 on y = 0 (6)

Non-slip: $u + v\frac{db}{dx} = 0$, $T = T_w$, $C = C_w$ on y = b(x) (7)

It is convenient to introduce the stream function ψ defined by

$$u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x} \tag{8}$$

So that

$$\frac{\partial u}{\partial x} = \frac{\partial^2 \psi}{\partial x \partial y} \qquad \frac{\partial v}{\partial x} = -\frac{\partial^2 \psi}{\partial x^2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial^2 \psi}{\partial y^2} \qquad \frac{\partial v}{\partial y} = -\frac{\partial^2 \psi}{\partial x \partial y} \tag{9}$$

Substituting equation (9) into (1) gives

$$\left[\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial^2\psi}{\partial x\partial y}\right]=0$$ It satisfied continuity equation. (10)

But $\left[\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right]=\omega$ which can also be written as

$$\nabla^2\psi=\omega \tag{11}$$

To eliminate the pressure term from (2) and (3), differentiate (2) with respect to y and (3) with respect to x gives

$$\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right]=-\frac{1}{\rho}\frac{\partial}{\partial y}\left[\frac{\partial p}{\partial x}\right]+v\nabla^2\frac{\partial u}{\partial y}-\frac{\sigma B_0^2}{\rho}\frac{\partial u}{\partial y}-\frac{\mu}{\rho k}\frac{\partial u}{\partial y}+\frac{\partial}{\partial y}\left[g\beta(T-T_0)+g\beta(C-C_0)\right] \tag{12a}$$

$$\frac{\partial}{\partial x}\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right]=-\frac{1}{\rho}\frac{\partial}{\partial x}\left[\frac{\partial P}{\partial x}\right]+v\nabla^2\frac{\partial v}{\partial x} \tag{12b}$$

Subtract equation (12b) from (12a) to give

$$\frac{\partial}{\partial t}\left[\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right]+\frac{\partial}{\partial y}\left[v\frac{\partial u}{\partial y}-v\frac{\partial v}{\partial x}\right]+\frac{\partial}{\partial x}\left[u\frac{\partial u}{\partial y}-u\frac{\partial v}{\partial x}\right]=v\nabla^2\left[\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right]$$

$$-\frac{\sigma B_0^2}{\rho}\frac{\partial u}{\partial y}-\frac{\mu}{\rho k}\frac{\partial u}{\partial y}+\frac{\partial}{\partial y}\left[g\beta(T-T_0)+g\beta(C-C_0)\right] \tag{13}$$

Substituting (9) into (13) yields

$$\frac{\partial}{\partial t}\left[\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial x^2}\right]+\frac{\partial}{\partial y}\left[-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x^2}\right]+\frac{\partial}{\partial x}\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y^2}-\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2}\right]=$$

$$v\nabla^2\left[\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial x^2}\right]-\frac{\sigma B_0^2}{\rho}\frac{\partial^2\psi}{\partial y^2}-\frac{\mu}{\rho k}\frac{\partial^2\omega}{\partial y^2}+\frac{\partial}{\partial y}\left[g\beta(T-T_0)+g\beta(C-C_0)\right] \tag{14}$$

Simplifying (14) further to obtain

$$\frac{\partial\omega}{\partial t}+\frac{\partial(\omega,\psi)}{\partial(y,x)}=v\nabla^2\omega-\frac{\sigma B_0^2}{\rho}\frac{\partial^2\psi}{\partial y^2}-\frac{\mu}{\rho k}\frac{\partial^2\omega}{\partial y^2}+\frac{\partial}{\partial y}\left[g\beta(T-T_0)+g\beta(C-C_0)\right] \tag{15}$$

where $\frac{\partial}{\partial y}\left[-\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right]+\frac{\partial}{\partial x}\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y\partial x}\right]=\frac{\partial(\omega,\psi)}{\partial(y,x)}$ (16)

Substituting (9) into (4) leads to

$$\frac{\partial T}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{k}{\rho Cp}\nabla^2T+2\frac{v}{Cp}\left[\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2+\left(-\frac{\partial^2\psi}{\partial x\partial y}\right)^2+\frac{1}{2}\left(\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}\right)^2\right]$$

$$+\frac{\sigma B_0^2\left(\frac{\partial\psi}{\partial y}\right)^2}{\rho Cp}+g\beta(T-T_0)+g\beta(C-C_0) \tag{17}$$

Further simplifying of (17) resulted into

$$\frac{\partial T}{\partial t}+\frac{\partial}{\partial}\frac{(T,\psi)}{(y,x)}=\frac{k}{\rho Cp}\nabla^2 T+\frac{v}{Cp}\left(\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}\right)^2+\frac{4v}{Cp}\left(\frac{\partial^2\psi}{\partial x\partial y}\right)^2+\frac{\sigma B_0^2}{\rho Cp}\left(\frac{\partial\psi}{\partial y}\right)^2$$

$$+g\beta(T_\omega-T_0)+g\beta(C_\omega-C_0) \quad (18)$$

Similarly substituting (9) into (5) using the same approach as in above leads to

$$\frac{\partial C}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+D_1\frac{\partial^2 T}{\partial y^2}+K_L(C-C_0) \quad (19)$$

Simplifying equation (19) further to obtain

$$\frac{\partial C}{\partial t}+\frac{\partial}{\partial}\left(\frac{\psi,C}{y,x}\right)=D\frac{\partial^2 C}{\partial y^2}+D_1\frac{\partial^2 T}{\partial y^2}+K_L(C-C_0) \quad (20)$$

The corresponding boundary conditions are:

$$\frac{\partial^2\psi}{\partial y^2}=0,\quad \psi=0,\quad \frac{\partial T}{\partial y}=0 \quad \text{On } y=0 \quad (21)$$

$$\frac{d\psi}{dy}=\frac{\partial\psi}{\partial x}\frac{db}{dx}=0,\quad T=T_\omega,\quad \psi=Q(1+me^{i\sigma t})\ on\ y=b(x). \quad (22)$$

The function b(x) is assumed to depend upon a small parameter ε such that

$$b(x,\varepsilon)=a_0 S\left(\frac{\varepsilon x}{a_o}\right)\left(0<\varepsilon=\frac{a_0}{L}<<1\right) \quad (23)$$

where a_0 the characteristics constant half width of the channel, L is the characteristics constant length of the channel and S is the function describing the channel wall divergence geometry. This assumption helps us to simplify the problem by writing the equations in non-dimensional form. To achieve this, T_0 is defined as the reference temperature and the following non-dimensional qualities were introduced.

$$\omega'=\frac{a_0^2\omega}{Q},\quad x'=\frac{\varepsilon x}{a_0},\quad y'=\frac{y}{a_0},\quad \psi'=\frac{\psi}{Q},\quad t'=\delta t,$$

$$\theta=\frac{T-T_0}{T_\omega-T_0},\quad p'=\frac{\varepsilon a_0^2}{\rho v Q}P,\quad \phi=\frac{C-C_0}{C_\omega-C_0} \quad (24)$$

Differentiating (24) with respect to x, y and t the non-dimensional quantities and Substituting into (15, 17, 18, 19) and after neglecting terms of order ε^2 and higher order as well as the primes for charity to obtain

$$\frac{\partial^2\omega}{\partial y^2}-\alpha\frac{\partial\omega}{\partial t}=\mathrm{Re}\left[\frac{\partial}{\partial}\frac{(\omega,\psi)}{(y,x)}+\Omega\frac{\partial^2\psi}{\partial y^2}+Fs\frac{\partial^2\psi_{0s}}{\partial y^2}-Gr\frac{\partial\theta}{\partial y}-Gc\frac{\partial\phi}{\partial y}\right] \quad (25)$$

$$\frac{\partial^2\theta}{\partial y^2}-\alpha\Pr\frac{\partial\theta}{\partial t}=\mathrm{Re}\Pr\left[\frac{\partial}{\partial}\left(\frac{\theta,\omega}{y,x}\right)-\Omega E_c\left(\frac{\partial^2\psi}{\partial y^2}\right)^2-Gr\theta-Gc\phi\right]-\Pr E_c\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 \quad (26)$$

$$\frac{\partial\phi}{\partial t}+\mathrm{Re}\frac{1}{\alpha}\frac{\partial}{\partial}\left(\frac{\psi,\phi}{y,x}\right)=Sc\frac{\partial^2\phi}{\partial y^2}+Sc_1\frac{\partial^2\theta}{\partial y^2}+K_1\phi \quad (27)$$

Where $Gr = g\beta \frac{a_0 L}{Q}(T_w - T_0)$ thermal Grashof number, $Gc = g\beta \frac{a_0 L}{Q}(C_w - C_0)$ mass Grashof number, $\Omega = \frac{\sigma B_0^2 a_0 L}{\rho Q}$ is the magnetic field intensity parameter $\alpha = \frac{\delta a_0^2}{\nu}$ the womersley number, $\mathrm{Re} = \frac{Q\varepsilon}{\nu}$ the effective Reynolds number $E_c = \frac{Q^2}{a_0^2 (T_w - T_0) C_p}$ is the Eckert number $Sc = Da_0\delta$ Schmitt number and $k_1 = k_L \delta$ chemical reaction number and $\Pr = \rho C_p \frac{\nu}{k}$ the prandlt number.

The boundary conditions are:

$$\frac{\partial^2 \psi}{\partial y^2} = 0, \quad \psi = 0, \frac{\partial \psi}{\partial y} = 0, \quad \frac{\partial \theta}{\partial y} = 0 \qquad \text{On } y = 0 \tag{28}$$

$$\frac{\partial \psi}{\partial y} = 0, \quad \phi = 1, \quad \theta = 1, \quad \psi = 1 + me^{it} \qquad \text{On } y = S(x) \tag{29}$$

Due to the nonlinear nature of the equations, it is convenient to adopt a power series expansion with the effective flow Reynolds number (Re) as follows:

$$\psi = \sum_{j=o}^{\infty} \mathrm{Re}^j (\psi_{js} + me^{it}\psi_j), \qquad \omega = \sum_{j=o}^{\infty} \mathrm{Re}^j (\omega_{js} + me^{it}\omega_j)$$

$$\theta = \sum_{j=o}^{\infty} \mathrm{Re}^j (\theta_{js} + me^{it}\theta_j), \qquad \phi = \sum_{j=o}^{\infty} \mathrm{Re}^j (\phi_{js} + me^{it}\phi_j) \tag{30}$$

where $\psi_{js}, \psi_j, \omega_{js}, \omega_j, \theta_{js}, \theta_j, \phi_{js}$ and ϕ_j are functions of $S(x)$ and y. it is important to note that the real part of the equation (30) forms the solution of the problem which is physically meaningful. Substituting equation (30) into equations (25-27) and collecting terms of like order of Re and me^{it}, gives zero order:

$$\sum_{j=0}^{\infty} \mathrm{Re}^j \left(\frac{\partial^2 \omega_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \omega_j}{\partial y^2}\right) - \alpha\left(\sum_{j=0}^{\infty} \mathrm{Re}^j \left(\frac{\partial \omega_{js}}{\partial t} + mie^{it}\omega_j\right)\right) =$$
$$\mathrm{Re}\begin{pmatrix} 0 + \Omega\sum_{j=0}^{\infty} \mathrm{Re}^j \left(\frac{\partial^2 \psi_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \psi_j}{\partial y^2}\right) - Fs\left(\frac{\partial^2 \psi_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \psi_j}{\partial y^2}\right) \\ + Gr\left(\frac{\partial \theta_{js}}{\partial y} + me^{it}\frac{\partial \theta_j}{\partial y}\right) - Gc\left(\frac{\partial \phi_{js}}{\partial y} + me^{it}\frac{\partial \phi_j}{\partial y}\right) \end{pmatrix} \tag{31}$$

$$\sum_{j=0}^{\infty} \mathrm{Re}^j \left(\frac{\partial^2 \theta_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \theta_j}{\partial y^2}\right) - \alpha \Pr\left(\sum_{j=0}^{\infty} \mathrm{Re}^j \left(\frac{\partial \theta_{js}}{\partial t} + mie^{it}\theta_j\right)\right) =$$
$$\mathrm{Re}\Pr\left[0 - \Omega E_c\left(\sum_{j=0}^{\infty} \mathrm{Re}^j \left(\frac{\partial^2 \psi_{js}}{\partial y^2} + me^{it}\frac{\partial^2 \psi_j}{\partial y^2}\right)\right) - Gr\sum_{j=o}^{\infty} \mathrm{Re}^j (\theta_{js} + me^{it}\theta_j)\right]$$

$$-Gc\sum_{j=o}^{\infty}\mathrm{Re}^{j}(\phi_{js}+me^{it}\phi_{j})-E_{c}\,\mathrm{Pr}\left(\sum_{j=0}^{\infty}\mathrm{Re}^{j}(\frac{\partial^{2}\psi_{js}}{\partial y^{2}}+me^{it}\frac{\partial^{2}\psi_{j}}{\partial y^{2}})\right)^{2} \tag{32}$$

$$\sum_{j=0}^{\infty}\mathrm{Re}^{j}(\phi_{js}+mie^{it}\phi_{j})+\mathrm{Re}\frac{1}{\alpha}(0)=Sc\left(\sum_{j=0}^{\infty}\mathrm{Re}^{j}(\frac{\partial^{2}\phi_{js}}{\partial y^{2}}+me^{it}\frac{\partial^{2}\phi_{j}}{\partial y^{2}})\right)$$
$$+Sc_{1}\left(\sum_{j=0}^{\infty}\mathrm{Re}^{j}(\frac{\partial^{2}\theta_{js}}{\partial y^{2}}+me^{it}\frac{\partial^{2}\theta_{j}}{\partial y^{2}})\right)+K_{1}\left(\sum_{j=o}^{\infty}\mathrm{Re}^{j}(\phi_{js}+me^{it}\phi_{j})\right) \tag{33}$$

when j = 0, and collecting terms of like order of me^{it}, lead to Zero order (31), (32) and (33) yields

$$\frac{\partial^{2}\omega_{0}}{\partial y^{2}}=\lambda_{1}^{2}\omega_{0}, \tag{34}$$

$$\frac{\partial^{2}\psi_{0}}{\partial y^{2}}=-\omega_{0} \tag{35}$$

$$\frac{\partial^{2}\theta_{0}}{\partial y^{2}}-\lambda_{2}^{2}\theta_{0}=-2E_{c}\,\mathrm{Pr}\frac{\partial^{2}\psi_{0}}{\partial y^{2}}\frac{\partial^{2}\psi_{0s}}{\partial y^{2}} \tag{36}$$

$$\frac{\partial^{2}\phi_{0}}{\partial y^{2}}+\frac{1}{Sc}\left[K_{1}-i\right]\phi_{0}=-\frac{Sc_{1}}{Sc}\frac{\partial^{2}\theta_{0}}{\partial y^{2}} \tag{37}$$

Considering the term js in (31), (32) and (33) the following set of equations were obtained.

$$\frac{\partial^{2}\theta_{0s}}{\partial y^{2}}=E_{c}\,\mathrm{Pr}\left(\frac{\partial\psi_{0s}}{\partial y}\right)^{2} \tag{38}$$

$$\frac{\partial^{2}\omega_{0s}}{\partial y^{2}}=0 \tag{39}$$

$$\frac{\partial^{2}\psi_{0s}}{\partial y^{2}}=-\omega_{0s} \tag{40}$$

The boundary conditions are

$$\frac{\partial^{2}\psi_{0}}{\partial y^{2}}=\frac{\partial^{2}\psi_{0s}}{\partial y^{2}}=0,\quad \psi_{0}=\psi_{0s}=0,\quad \frac{\partial\theta_{0}}{\partial y}=\frac{\partial\theta_{0s}}{\partial y}=0,\quad \frac{\partial\phi_{0}}{\partial y}=\frac{\partial\phi_{0s}}{\partial y}=0,\quad on\quad y=0 \tag{41}$$

$$\frac{\partial\psi_{0}}{\partial y}=\frac{\partial\psi_{0s}}{\partial y}=0,\quad \psi_{0}=\psi_{0s}=1\quad \theta_{0}=0\quad \theta_{0s}=1,\ \phi_{0}=0\quad \phi_{0s}=1,\quad on\quad y=S(x) \tag{42}$$

$$\frac{\partial^{2}\omega_{0}}{\partial y^{2}}=\lambda_{1}^{2}\omega_{0}, \tag{43}$$

$$\frac{\partial^{2}\psi_{0}}{\partial y^{2}}=-\omega_{0} \tag{44}$$

$$\frac{\partial^{2}\theta_{0}}{\partial y^{2}}-\lambda_{2}^{2}\theta_{0}=-2E_{c}\,\mathrm{Pr}\frac{\partial^{2}\psi_{0}}{\partial y^{2}}\frac{\partial^{2}\psi_{0s}}{\partial y^{2}} \tag{45}$$

$$\frac{\partial^2 \phi_0}{\partial y^2} + \frac{1}{Sc}[K_1 - i]\phi_0 = -\frac{Sc_1}{Sc}\frac{\partial^2 \theta_0}{\partial y^2} \tag{46}$$

Considering the term js in (31), (32) and (33) the following set of equations were obtained.

$$\frac{\partial^2 \theta_{0s}}{\partial y^2} = E_c \Pr\left(\frac{\partial \psi_{0s}}{\partial y}\right)^2 \tag{47}$$

$$\frac{\partial^2 \omega_{0s}}{\partial y^2} = 0 \tag{48}$$

$$\frac{\partial^2 \psi_{0s}}{\partial y^2} = -\omega_{0s} \tag{49}$$

$$\frac{\partial^2 \phi_{os}}{\partial y^2} - \frac{k_1}{Sc}\phi_{0s} = \frac{Sc_1}{Sc}\frac{\partial^2 \theta_{0s}}{\partial y^2} \tag{50}$$

The boundary conditions are

$$\frac{\partial^2 \psi_0}{\partial y^2} = \frac{\partial^2 \psi_{0s}}{\partial y^2} = 0, \quad \psi_0 = \psi_{0s} = 0, \quad \frac{\partial \theta_0}{\partial y} = \frac{\partial \theta_{0s}}{\partial y} = 0, \quad \frac{\partial \phi_0}{\partial y} = \frac{\partial \phi_{0s}}{\partial y} = 0, \quad on \quad y = 0 \tag{51}$$

$$\frac{\partial \psi_0}{\partial y} = \frac{\partial \psi_{0s}}{\partial y} = 0, \quad \psi_0 = \psi_{0s} = 1 \quad \theta_0 = 0 \quad \theta_{0s} = 1, \ \phi_0 = 0 \quad \phi_{0s} = 1, \quad on \quad y = S(x) \tag{52}$$

When j = 1 order 1 gives:

$$\frac{\partial^2 \psi_1}{\partial y^2} = -\omega_1 \tag{53}$$

$$\frac{\partial^2 \omega_1}{\partial y^2} - \lambda_1^2 \omega_1 = \frac{\partial}{\partial}\frac{(\omega_0, \psi_{0s})}{(y,x)} + \frac{\partial}{\partial}\frac{(\omega_{0s}, \psi_0)}{(y,x)} + \Omega\frac{\partial^2 \psi_0}{\partial y^2} + Fs\frac{\partial^2 \psi_0}{\partial y^2} - Gr\frac{\partial \theta_0}{\partial y} - Gc\frac{\partial \phi_0}{\partial y} \tag{54}$$

$$\begin{aligned}\frac{\partial^2 \theta_1}{\partial y^2} - \alpha \Pr \theta_1 = \Pr\left(\frac{\partial}{\partial}\frac{(\psi_0, \theta_{0s})}{(y,x)} + \frac{\partial}{\partial}\frac{(\psi_{0s}, \omega_0)}{(y,x)} + 2\Omega E_c \frac{\partial^2 \psi_0}{\partial y^2}\frac{\partial^2 \psi_{0s}}{\partial y^2}\right) \\ - Gr \Pr \theta_0 - Gc \Pr \phi_0 - 2Ec \Pr\left(\frac{\partial^2 \psi_0}{\partial y^2}\frac{\partial^2 \psi_{1s}}{\partial y^2} + \frac{\partial^2 \psi_1}{\partial y^2}\frac{\partial^2 \psi_{0s}}{\partial y^2}\right)\end{aligned} \tag{56}$$

$$\frac{\partial^2 \phi_1}{\partial y^2} - \frac{[1 + K_1]}{Sc}\phi_1 = \frac{Sc_1}{Sc}\frac{\partial^2 \theta_1}{\partial y^2} \tag{57}$$

Considering the term js the following set of equations were obtained.

$$\frac{\partial^2 \psi_{1s}}{\partial y^2} = -\omega_{1s} \tag{58}$$

$$\frac{\partial^2 \omega_{1s}}{\partial y^2} = \frac{\partial}{\partial}\frac{(\psi_{0s}, \omega_{0s})}{(y,x)} - \Omega\frac{\partial^2 \psi_{0s}}{\partial y^2} + Fs\frac{\partial^2 \psi_{0s}}{\partial y^2} - Gr\frac{\partial \theta_{0s}}{\partial y} - Gc\frac{\partial \phi_{0s}}{\partial y} \tag{59}$$

$$\frac{\partial^2 \theta_{1s}}{\partial y^2} = \Pr\frac{\partial}{\partial}\frac{(\psi_{0s}, \theta_{0s})}{(y,x)} - \Omega E_c\left(\frac{\partial \psi_{0s}}{\partial y}\right)^2 - Gr\theta_{os} - Gc\phi_{os} - 2E_c \Pr\frac{\partial^2 \psi_{0s}}{\partial y^2}\frac{\partial^2 \psi_{1s}}{\partial y^2} \tag{60}$$

$$\frac{\partial^2 \phi_{1s}}{\partial y^2} + \frac{k_1}{Sc}\phi_{1s} = -\frac{Sc_1}{Sc}\frac{\partial^2 \theta_{1s}}{\partial y^2} \tag{61}$$

We now solving the following equations: (43), (44), (45), (46), (47), (48), (49), (50), (53), (54), (55), (56) (57), (58), (59), (60) and (61) and the stream function ψ, vorticity ω, temperature distribution θ and volume fraction ϕ were obtained thus

The solution gives:

$$\omega_0 = \frac{1+me^{it}}{\sinh \lambda_1 s(x)}\cosh \lambda_1 y \tag{62}$$

$$\omega_{0s} = \frac{1}{s(x)}y \tag{63}$$

$$\psi_0 = \frac{(1+me^{it})}{\lambda_1^2}\left[\frac{(\lambda_1^2+1)y}{S(x)} - \frac{\sinh \lambda_1 y}{\sinh \lambda_1 (x)}\right] \tag{64}$$

$$\psi_{os} = \frac{S(x)y}{2} - \frac{y^3}{6S(x)} \tag{65}$$

$$\theta_o = \frac{2E_c \Pr(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh \lambda_1 S(x)} \left[\frac{2\lambda_1 \cosh \lambda_1 S(x) + S(x)\sinh \lambda_1 S(x)}{\cosh \lambda_2 S(x)}\cosh \lambda_2 y - y\sinh \lambda_1 y\right] \tag{66}$$

$$\theta_{os} = 1 + \frac{E_c \Pr(y^4 - S(x)^4)}{12S(x)^2} \tag{67}$$

$$\phi_o = \left(\frac{2Sc_1 E_c \Pr(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)}\right)\left[\frac{\left[\begin{array}{l}\frac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\lambda_2^2\cosh\lambda_2 S(x)\\ -y\lambda_1^2\sinh\lambda_1 S(x) - 2\lambda_1\cosh\lambda_1 S(x)\end{array}\right]\cos d_1 y}{\cosh d_1 S(x)\sinh\lambda_1 S(x)} + \left(\frac{2\lambda_1 Sc}{\sinh\lambda_1 S(x)\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)} - \frac{2\lambda_1^3}{\sinh\lambda_1 S(x)\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)^2}\right)\cosh\lambda_1 y - \frac{\lambda_2^2\cosh\lambda_2 y}{\cosh\lambda_2 S(x)\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)} + \frac{\lambda_1^2 y\sinh\lambda_1 y}{\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)\sinh\lambda_1 S(x)}\right] \tag{68}$$

$$\phi_{os} = -\frac{Sc_1 \Pr Ec}{\cosh\left(\pm\sqrt{\frac{k_1}{Sc}}\right)S(x)(Sc-K_1)}\cosh\left(\pm\sqrt{\frac{k_1}{Sc}}\right)y - \frac{Ec\Pr Sc_1}{S^2(x)(2Sc-K_1)}y^2 \tag{69}$$

$$\omega_1 = a_{28}\sinh\lambda_1 y + a_{29}\cosh\lambda_2 y + a_{30} + a_{31}\sinh\lambda_1 y + a_{32}\cosh\lambda_1 y + a_{33}\sinh\lambda_2 y + a_{34}\cosh\lambda_2 y + a_{35} y\sinh\lambda_1 y + a_{36} y\cosh\lambda_1 y + a_{37} y^2 + a_{38}\sinh d_1 y \quad (70)$$

$$\psi = \frac{(1+me^{it})}{\lambda_1^2}\left(\frac{(\lambda_1^2+1)y}{S(x)} - \frac{\sinh\lambda_1 y}{\sinh\lambda_1(x)}\right) + me^{it}\left(\frac{(1+me^{it})}{\lambda_1^2}\left(\frac{(\lambda_1^2+1)y}{S(x)} - \frac{\sinh\lambda_1 y}{\sinh\lambda_1(x)}\right)\right)$$
$$+\mathrm{Re}\left(\begin{array}{l}\frac{S'(x)y^4}{48} + \frac{y^6}{720} + \Omega\left(\frac{S(x)y^2}{12} - \frac{y^5}{120S(x)}\right) + Gr\left(\frac{y^2}{2} + \frac{Ec\Pr y^7}{3240S(x)^2}\right)\\ + Gc\left(\frac{c_{24}d_{12}\sinh d_{12}y}{d_{12}^2} + \frac{c_{26}y^5}{60}\right) - \frac{c_{39}y^3}{6} - \frac{c_{40}y^2}{2} - c_{41}y - c_{42}\end{array}\right)$$

$$+\mathrm{Re}\, me^{it}\left(\begin{array}{l}(c_{34}+2c_{36}+c_{38}d_{11})\frac{\sinh\lambda_1 y}{\lambda_1^2} + c_{29}\frac{\cosh\lambda_1 y}{\lambda_1^2} + c_{30}\frac{y^2}{2} + cd_{31}\frac{\sinh\lambda_1 y}{\lambda_1^2}\\ +c_{32}\frac{\cosh\lambda_1 y}{\lambda_1^2} + c_{33}\frac{\sinh\lambda_2 y}{\lambda_2^2} + c_{34}\frac{\cosh\lambda_2 y}{\lambda_2^2} + c_{35}\left(\frac{y\cosh\lambda_1 y}{\lambda_1^2} - \frac{2\sinh\lambda_1 y}{\lambda_1^3}\right)\\ +c_{36}\left(\frac{y\sinh\lambda_1 y}{\lambda_1^2} - \frac{2\cosh\lambda_1 y}{\lambda_1^3}\right) + c_{37}\frac{y^4}{12} + c_{38}\frac{\sinh d_{11}y}{d_{11}^2} + c_{39}y + c_{40}\ (3.417)\end{array}\right) \quad (71)$$

$$\omega = \frac{1}{s(x)}y + me^{it}\left(\frac{1+me^{it}}{\sinh\lambda_1 s(x)}\sinh\lambda_1 y\right) +$$

$$\mathrm{Re}\left(\begin{array}{l}\frac{S'(x)y^2}{4} - \frac{y^4}{24} - \Omega\left(\frac{S(x)y}{2} - \frac{y^3}{6S(x)}\right) - Gr\left(1 + \frac{Ec\Pr\left(\frac{y^5}{5} - S(x)^4\right)}{12S(x)^2}\right)\\ -Gc\left(c_{24}d_{12}\sinh d_{12}y + \frac{c_{26}y^3}{3}\right) + c_{39}y + c_{40}\end{array}\right) \quad (72)$$
$$+\mathrm{Re}\, me^{it}\left(\begin{array}{l}c_{28}\sinh\lambda_1 y + c_{29}\cosh\lambda_2 y + c_{30} + c_{31}\sinh\lambda_1 y +\\ c_{32}\cosh\lambda_1 y + c_{33}\sinh\lambda_2 y + c_{34}\cosh\lambda_2 y + c_{35}y\sinh\lambda_1 y\\ +c_{36}y\cosh\lambda_1 y + c_{37}y^2 + c_{38}\sinh d_{11}y\end{array}\right)$$

$$\theta = 1 + \frac{E_c \Pr(y^4 - S(x)^4)}{12S(x)^2}$$

$$+me^{it}\left(\frac{2E_c \Pr(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}\left[\begin{array}{l}\frac{2\lambda_1\cosh\lambda_1 S(x)+S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\\ \cosh\lambda_2 y - y\sinh\lambda_1 y\end{array}\right]\right)$$

$$+\operatorname{Re}\left(\begin{array}{l}\Pr\frac{1}{24}\left(\frac{6S^2(x)S'(x)y^2+y^4S'(x)}{S^2(x)}\right)+\frac{E_c \Pr y^5 S'(x)}{30S(x)^3}-\\ \frac{\Omega}{8}\left(S(x)^2S'y^2-\frac{y^4}{3}+\frac{y^6}{15S(x)^2}\right)-Gr\left(\frac{y^2}{2}+\frac{E_c\Pr(\frac{y^6}{15}-S(x)^4y^2)}{24S^2(x)}\right)\\ -Gc\left(c_{24}\frac{\cosh d_{12}y}{d_{12}^2}\cosh d_{12}y+c_{26}\frac{y^4}{12}\right)-2E_c\Pr\left(\frac{\left(1+me^{it}\right)\sinh\lambda_1 y}{\lambda_1^2\sinh\lambda_1 S(x)}\right)\\ \left(\left(\frac{y^4}{24}-\Omega\frac{y^3}{6S(x)}+Gr\left(\frac{Ec\Pr 4y^5}{240S(x)^2}\right)+Gc\left(c_{24}\sinh d_{12}y+\frac{c_{26}y^3}{3}\right)\right)+c_{43}y+c_{44}\right)\end{array}\right) \quad (73)$$

$$+\operatorname{Re}me^{it}\left(\begin{array}{l}c_{60}\sinh d_{11}y+c_{61}\cosh d_{14}y+c_{62}\sinh d_{13}y+c_{63}\cosh d_{13}y+\\ c_{64}\sinh\lambda_1 y+c_{65}\cosh\lambda_1 y+c_{66}\sinh\lambda_2 y+c_{67}\cosh\lambda_2 y+\\ c_{68}\cosh d_{11}y+c_{69}y\sinh\lambda_1 y+c_{70}y^5+c_{71}y^4+c_{72}y^3+c_{73}y^2+c_{74}y+c_{75}\end{array}\right)$$

$$\phi = c_{24}\cosh d_{12}y + c_{26}y^2$$
$$+me^{itc}\left(\left(\frac{2Sc_1E_c\Pr(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)}\right)\left[\frac{\left[\begin{array}{l}\frac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\lambda_2^2\cosh\lambda_2 S(x)\\ -y\lambda_1^2\sinh\lambda_1 S(x)-2\lambda_1\cosh\lambda_1 S(x)\end{array}\right]\cos d_{11}y}{\cosh d_{11}S(x)\sinh\lambda_1 S(x)}+\left(\frac{2\lambda_1 Sc}{\sinh\lambda_1 S(x)\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)}-\frac{2\lambda_1^3}{\sinh\lambda_1 S(x)\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)^2}\right)\cosh\lambda_1 y - \frac{\lambda_2^2\cosh\lambda_2 y}{\cosh\lambda_2 S(x)\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)}+\frac{\lambda_1^2 y\sinh\lambda_1 y}{\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)\sinh\lambda_1 S(x)}\right]\right)$$
$$+\mathrm{Re}\left(c_{76}\sinh d_{15}y + c_{77}\cosh d_{15}y + c_{79}\cosh d_{12}y + c_{83}y^4 + c_{84}y^3 + c_{86}y + c_{87}\right)$$
$$+\mathrm{Re}\,me^{it}\left(\begin{array}{l}c_{60}\sinh d_{11}y + c_{61}\cosh d_{14}y + c_{62}\sinh d_{13}y + c_{63}\cosh d_{13}y +\\ c_{64}\sinh\lambda_1 y + c_{65}\cosh\lambda_1 y + c_{66}\sinh\lambda_2 y + c_{67}\cosh\lambda_2 y +\\ c_{68}\cosh d_{11}y + c_{69}y\sinh\lambda_1 y + c_{70}y^5 + c_{71}y^4 + c_{72}y^3 + c_{73}y^2 + c_{74}y + c_{75}\end{array}\right) \quad (74)$$

where:

$$c_1 = 0 \quad (75)$$

$$c_2 = \frac{1+mie^{it}}{\sinh\lambda_1 S(x)} \quad (76)$$

$$c_3 = \frac{1}{s(x)} \quad (77)$$

$$c_4 = 0 \quad (78)$$

$$c_5 = \frac{1+me^{it}(1+\frac{1}{\lambda_1^2})}{S(x)} \quad (79)$$

$$c_6 = 0 \tag{80}$$

$$c_7 = \frac{S(x)}{2} \tag{81}$$

$$c_8 = 0 \tag{82}$$

$$c_9 = 0 \tag{83}$$

$$c_{10} = -\frac{2E_c \Pr(1+me^{it})\sinh \lambda_1 S(x)}{(\lambda_1^2-\lambda_2^2)S(x)\sinh \lambda_1 S(x)\cosh \lambda_2 S(x)} \tag{84}$$

$$c_{11} = 0 \tag{85}$$

$$c_{12} = 0 \tag{86}$$

$$c_{13} = -\frac{2E_c \Pr(1+me^{it})y}{(\lambda_1^2-\lambda_2^2)S(x)\sinh \lambda_1 S(x)}\sinh \lambda_1 y \tag{87}$$

$$c_{14} = 0 \tag{88}$$

$$c_{15} = 1-\frac{E_c \Pr S^2(x)}{12} \tag{89}$$

$$c_{16} = 0 \tag{90}$$

$$c_{17} = \frac{2Sc_1 E_c \Pr(1+me^{it})}{S(x)Sc(\lambda_1^2-\lambda_2^2)\cosh d_{11}S(x)\sinh \lambda_1 S(x)}\left[\begin{array}{l}\dfrac{S(x)\sinh \lambda_1 S(x)}{\cosh \lambda_2 S(x)}\lambda_2^2 \cosh \lambda_2 S(x) \\ -y\lambda_1^2 \sinh \lambda_1 S(x) - 2\lambda_1 \cosh \lambda_1 S(x)\end{array}\right] \tag{91}$$

$$c_{18} = 0 \tag{92}$$

$$a_{19} = \frac{4\lambda_1 Sc_1 E_c \Pr(1+me^{it})}{ScS(x)\sinh \lambda_1 S(x)(\lambda_1^2-\lambda_2^2)\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)} - \frac{2a_{22}\lambda_1}{\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)} \tag{93}$$

$$c_{20} = 0 \tag{94}$$

$$c_{21} = -\frac{2Sc_1 E_c \Pr(1+me^{it})\lambda_2^2}{Sc\cosh \lambda_2 S(x)\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)(\lambda_1^2-\lambda_2^2)} \tag{95}$$

$$c_{22} = \frac{2Sc_1 E_c \Pr(1+me^{it})\lambda_1^2}{Sc\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)(\lambda_1^2-\lambda_2^2)S(x)\sinh \lambda_1 S(x)} \tag{96}$$

$$c_{23} = 0 \tag{97}$$

$$c_{24} = -\frac{Sc_1 \Pr Ec}{\cosh d_{12}S\left(x\right)(Sc-K_1)} \tag{98}$$

$$c_{25} = 0 \tag{99}$$

$$c_{26} = -\frac{Ec \Pr Sc_1}{S^2(x)(2Sc - K_1)} \tag{100}$$

$$c_{27} = 0 \tag{101}$$

$$c_{28} = -c_{34} - 2c_{36} - c_{38}d_{11} \tag{102}$$

$$\begin{aligned} c_{29} = &\left(c_{34} + 2c_{36} + c_{38}d_{11}\right)\sinh\lambda_1 S(x) - c_{30} - c_{31}\sinh\lambda_1 S(x) - c_{32}\cosh\lambda_1 S(x) - c_{33}\sinh\lambda_2 S(x) \\ &- c_{34}\cosh\lambda_2 S(x) - c_{35}S(x)\sinh\lambda_1 S(x) - c_{36}S(x)\cosh\lambda_1 S(x) - c_{37}S(x)^2 - c_{38}\sinh d_{11}S(x) \end{aligned} \tag{103}$$

$$c_{30} = \frac{(1+me^{it})(\lambda_1^2+1)S'(x)}{\lambda_1^2\left(S(x)\right)^2} + \frac{S'(x)}{2} + \frac{S'(x)}{\left(S(x)\right)^2} \tag{104}$$

$$\begin{aligned} c_{31} = &-\frac{(\Omega + Da + Fs)(1+me^{it})}{\sinh\lambda_1 S(x)} + \frac{2E_c Gr \Pr(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)} + \\ &\left(\frac{2Sc_1 GcE_c \Pr(1+me^{it})\lambda_1}{Sc(\lambda_1^2-\lambda_2^2)S(x)\left(\lambda_2^2 + \frac{1}{Sc}(K_1 - i)\right)\sinh\lambda_1 S(x)}\right)\left(2\lambda_1 Sc - \frac{2\lambda_1^3}{\left(\lambda_2^2 + \frac{1}{Sc}(K_1 - i)\right)} + 1\right) \end{aligned} \tag{105}$$

$$c_{32} = \frac{(1+me^{it})\lambda_1\left(\cosh\lambda_1 S(x) + \frac{S'(x)}{\lambda_1^2}\right)}{\left(\sinh\lambda_1 S(x)\right)^2} \tag{106}$$

$$c_{33} = \frac{2E_c Gr \Pr(1+me^{it})\lambda_2^3}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)}\left(\frac{2\lambda_1\cosh\lambda_1 S(x) + S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\right) \tag{107}$$

$$c_{34} = 0 \tag{108}$$

$$c_{35} = \frac{\lambda_1 2Sc_1 GcE_c \Pr(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)\left(\lambda_2^2 + \frac{1}{Sc}(K_1 - i)\right)\sinh\lambda_1 S(x)} \tag{109}$$

$$c_{36} = \frac{2E_c Gr \Pr(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)} \tag{110}$$

$$c_{37} = -\frac{S'(x)}{2\left(S(x)\right)^2} \tag{111}$$

$$c_{38} = \left(\frac{2Sc_1 GcE_c \Pr(1+me^{it})}{Sc(\lambda_1^2-\lambda_2^2)S(x)} \right) \frac{\left[\frac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)} \lambda_2^2 \cosh\lambda_2 S(x) - y\lambda_1^2 \sinh\lambda_1 S(x) - 2\lambda_1 \cosh\lambda_1 S(x) \right] d_{11}}{\cosh d_{11} S(x) \sinh\lambda_1 S(x)} \quad (112)$$

$$c_{39} = \frac{(\Omega + Da + Fs)S(x)}{2} + Gcc_{24} d_{12}^2 \quad (113)$$

$$c_{40} = \frac{S(x)^4}{24} + \frac{(\Omega+Da+Fs)S(x)^2}{3} + Gr\left(1 + Ec\Pr\left(\frac{S(x)^3}{60} - \frac{S(x)^2}{12}\right)\right) + Gc\left(d_{12}\sinh d_{12}S(x) - c_{32}S(x)^3 - c_{34}S(x)\right) \quad (114)$$

$$c_{41} = 0 \quad (115)$$

$$c_{42} = \frac{S'(x)S(x)^4}{48} + \frac{S(x)^6}{720} + \Omega\left(\frac{S(x)S(x)^2}{12} - \frac{S(x)^5}{120S(x)}\right) + Gr\left(\frac{S(x)^2}{2} + \frac{Ec\Pr S(x)^7}{3240S(x)^2}\right) + Gc\left(\frac{c_{24}d_{12}\sinh d_{12}S(x)}{d_{12}^2} + \frac{c_{26}S(x)^5}{60}\right) - \frac{c_{39}S(x)^3}{6} - \frac{c_{40}S(x)^2}{2} \quad (116)$$

$$c_{43} = 0 \quad (117)$$

$$c_{44} = -\Pr\frac{7S^2(x)S'(x)}{24} - \frac{E_c\Pr S(x)^2 S'(x)}{30} + \frac{\Omega}{8}\left(S(x)^4 S'(x) - \frac{S(x)^4}{3} + \frac{S(x)^4}{15}\right) + Gr\left(\frac{S(x)^2}{2} - \frac{E_c\Pr S(x)^4}{360}\right) + Gc\left(a_{24}\frac{\cosh d_{12}S(x)}{d_{12}^2}\cosh d_{12}S(x) + c_{26}\frac{S(x)^4}{12}\right) + 2E_c\Pr\left(\frac{\left(1+me^{it}\right)}{\lambda_1^2}\right)\left(\begin{array}{l} \frac{S(x)^4}{24} - \Omega\frac{S(x)^2}{6} + Gr\left(\frac{Ec\Pr 4S(x)^3}{240}\right) \\ -Gc\left(c_{24}\sinh d_{12}S(x) + \frac{c_{26}S(x)^3}{3}\right) \end{array}\right) \quad (118)$$

$$c_{45} = \frac{c_{47}\lambda_1 - c_{50}\lambda_2}{d_{13}} \quad (119)$$

$$c_{46} = -\frac{\left(\begin{array}{l} (\frac{c_{47}\lambda_1 - c_{50}\lambda_2}{d_3})\sinh d_{13}S(x) + c_{47}\sinh\lambda_1 S(x) + c_{48}\cosh\lambda_1 S(x) \\ + c_{50}\cosh\lambda_2 S(x) + +c_{52}\cosh d_{11}S(x) + c_{53}S(x)\sinh\lambda_1 S(x) + c_{54}S(x)^5 \\ + c_{55}S(x)^4 + c_{56}S(x)^3 + c_{57}S(x)^2 + c_{59} \end{array}\right)}{\cosh d_{13}S(x)} \quad (121)$$

$$c_{47} = \frac{2Ec\Pr(1+me^{it})c_{40}}{\sinh\lambda_1 S(x)} \tag{122}$$

$$c_{48} = \frac{\Pr(1+me^{it})S'(x)}{\lambda_1 \sinh\lambda_1 S(x)\left(\lambda_1^2-\lambda_2\right)} - \frac{Gr\Pr}{\left(\lambda_1^2-\lambda_2\right)\sinh\lambda_1 S(x)\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)} \left(2\lambda_1 Sc - \frac{2\lambda_1^3}{\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)}\right) \tag{123}$$

$$c_{49} = 0 \tag{124}$$

$$c_{50} = -Gr\Pr\left(\frac{2E_c\Pr(1+me^{it})\left(2\lambda_1\cosh\lambda_1 S(x)+S(x)\sinh\lambda_1 S(x)\right)}{\left(\lambda_2^2-\lambda_2\right)(\lambda_1^2-\lambda_2^2)S(x)\sinh\lambda_1 S(x)\cosh\lambda_2 S(x)}\right) - \frac{\lambda_2^2 Gc\Pr}{\cosh\lambda_2 S(x)\left(\lambda_2^2-\lambda_2\right)\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)} \tag{125}$$

$$c_{51} = 0 \tag{126}$$

$$c_{52} = \frac{-Gc\Pr\left(\frac{S(x)\sinh\lambda_1 S(x)}{\cosh\lambda_2 S(x)}\lambda_2^2\cosh\lambda_2 S(x) - y\lambda_1^2\sinh\lambda_1 S(x) - 2\lambda_1\cosh\lambda_1 S(x)\right)}{\left(d_{11}^2-\lambda_2\right)\cosh d_{11}S(x)\sinh\lambda_1 S(x)} \tag{127}$$

$$c_{53} = -\frac{\Pr}{\sinh\lambda_1 S(x)(2\lambda_1+\lambda_1^2-\lambda_2)} \left(\frac{\lambda_1 Gc}{\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)}\left(\lambda_1^3-2Sc-\frac{2\lambda_1^2}{\left(\lambda_2^2+\frac{1}{Sc}(K_1-i)\right)}\right)+\frac{2E_c Gr\Pr(1+me^{it})}{(\lambda_1^2-\lambda_2^2)S(x)}\right) \tag{128}$$

$$c_{54} = \frac{E_c(\Pr)^2 S'(x)}{S(x)^3}\left(\frac{1}{10}-4E_c c_{30}\right)+\frac{\lambda_1 c_{56}\Pr}{20} \tag{129}$$

$$c_{55} = \frac{-4(Ec\Pr)^2 S'(x)c_{37}}{(12-\lambda_2)S(x)^3} \tag{130}$$

$$c_{56} = \frac{-4(Ec\Pr)^2 S'(x)c_{37}}{(6-\lambda_2)S(x)^3} \tag{131}$$

$$c_{57}=\frac{\Pr}{(2-\lambda_2)}\left(\frac{(1+me^{it})}{\lambda_1^2}\left(\frac{\left(\lambda_1^2+1\right)S'(x)}{S(x)^2}\right)+\frac{S'(x)}{2}-\frac{S'(x)}{S(x)^2}\right) \quad (132)$$

$$c_{58}=0 \quad (133)$$

$$c_{59}=\frac{-\Pr}{\lambda_2}\left(\frac{(1+me^{it})\left(\lambda_1^2+1\right)S'(x)}{\lambda_1^2 S(x)^2}+\frac{S'(x)}{2}\right) \quad (134)$$

$$c_{60}=\frac{-(c_{74}+c_{62}d_{13}+c_{64}\lambda_1+c_{66}\lambda_2)}{d_{14}} \quad (135)$$

$$c_{61}=-\left(\frac{\begin{array}{l}(-\frac{(c_{74}+c_{74}d_{13}+c_{74}\lambda_1+c_{74}\lambda_2)}{d_{14}})\sinh d_{14}S(x)+c_{62}\sinh d_{13}S(x)+c_{63}\cosh d_{13}S(x)+\\ c_{64}\sinh\lambda_1 S(x)+c_{65}\cosh\lambda_1 S(x)+c_{66}\sinh\lambda_2 S(x)+c_{67}\cosh\lambda_2 S(x)+c_{68}\cosh_{11}S(x)\\ +c_{69}y\sinh\lambda_1 S(x)+c_{70}S(x)^5+c_{71}S(x)^4+c_{72}S(x)^3+c_{73}S(x)^2+c_{74}S(x)+c_{75}\end{array}}{\cosh d_{14}S(x)}\right) \quad (136)$$

$$c_{62}=\frac{Sc_1 d_{13}\left(c_{47}\lambda_1-c_{50}\lambda_2\right)}{Scd_{13}^2-K_1-1} \quad (137)$$

$$c_{63}=\frac{Sc_1 d_{13}^2 c_{46}}{Scd_{13}^2-K_1-1} \quad (138)$$

$$c_{64}=\frac{Sc_1\lambda_1^2 c_{47}}{Sc\lambda_1^2-K_1-1} \quad (139)$$

$$c_{65}=\frac{Sc_1\lambda_1^2 c_{48}}{Sc\lambda_1^2-K_1-1} \quad (140)$$

$$c_{66}=0 \quad (141)$$

$$c_{67}=\frac{Sc_1\lambda_2^2 c_{50}}{Sc\lambda_2^2-K_1-1} \quad (142)$$

$$c_{68}=\frac{Sc_1 d_{11}^2 c_{52}}{Scd_1^2-K_1-1} \quad (143)$$

$$c_{69}=\frac{Sc_1\lambda_1^2 c_{53}}{Sc\lambda_1^2-K_1-1} \quad (144)$$

$$c_{70}=\frac{20Sc_1 c_{54}}{20Sc-K_1-1} \quad (145)$$

$$c_{71} = \frac{12Sc_1c_{55}}{12Sc - K_1 - 1} \tag{146}$$

$$c_{72} = \frac{6Sc_1c_{55} + 20c_{54}}{6Sc - K_1 - 1} \tag{147}$$

$$c_{73} = \frac{2Sc_1c_{57} + 12c_{55}}{2Sc - K_1 - 1} \tag{148}$$

$$c_{74} = \frac{-6Sc_1c_{56}}{K_1 + 1} \tag{149}$$

$$c_{75} = \frac{-2Sc_1c_{57}}{K_1 + 1} \tag{150}$$

$$c_{76} = \frac{-c_{83} - c_{78}d_{12}}{d_{15}} \tag{151}$$

$$c_{77} = -\left(\frac{\begin{array}{c}(\frac{-c_{83} - c_{78}d_{12}}{d_{15}})\sinh d_{12}S(x) + c_{78}\sinh d_{12}S(x) + c_{79}\cosh c_2S(x) + c_{80}\sinh \lambda_1 S(x) + \\ c_{81}\cosh \lambda_1 S(x) + c_{82}S(x)^5 + c_{83}S(x)^4 + c_{84}S(x)^3 + c_{85}S(x)^2 + c_{86}S(x) + c_{87}\end{array}}{\cosh d_{15}y} \right) \tag{152}$$

$$c_{78} = 0 \tag{153}$$

$$c_{79} = \frac{GcSc_1c_{24}}{Sc} \tag{154}$$

$$c_{80} = 0 \tag{155}$$

$$c_{81} = 0 \tag{156}$$

$$c_{82} = 0 \tag{157}$$

$$c_{83} = \frac{Sc_1}{4S(x)^2 Sc}\left(\Omega - \frac{GrEc\Pr}{6} \right) \tag{158}$$

$$c_{84} = \frac{2Ec\Pr S'(x)Sc_1}{3S(x)^3 Sc} \tag{159}$$

$$c_{85} = \frac{Sc_1}{Sc}\left(\frac{\Pr S'(x)}{S(x)^2} - 4\Omega - Gcc_{26} \right) \tag{160}$$

$$c_{86} = 0 \tag{161}$$

$$c_{87} = \frac{Sc_1}{Sc}\left(\frac{\Pr S'(x)}{2} - \frac{\Omega S(x)^2 S'(x)}{4} - Gr - \frac{Ec\Pr S(x)^2}{12} \right) \tag{162}$$

$$d_{11} = \sqrt{\frac{1}{Sc}(K_1 - i)} \tag{163}$$

$$d_{12} = \pm\sqrt{\frac{k_1}{Sc}} \tag{164}$$

$$\mathrm{d}_{13} = \pm\sqrt{\alpha \Pr} \tag{165}$$

$$d_{14} = \pm\sqrt{\frac{1+K_1}{Sc}} \tag{166}$$

$$d_{15} = \pm\sqrt{\frac{k_1}{Sc}i} \tag{167}$$

Method of Solution

The problems of consideration are solved analytically which resulted into the graphical results under results and discussion.

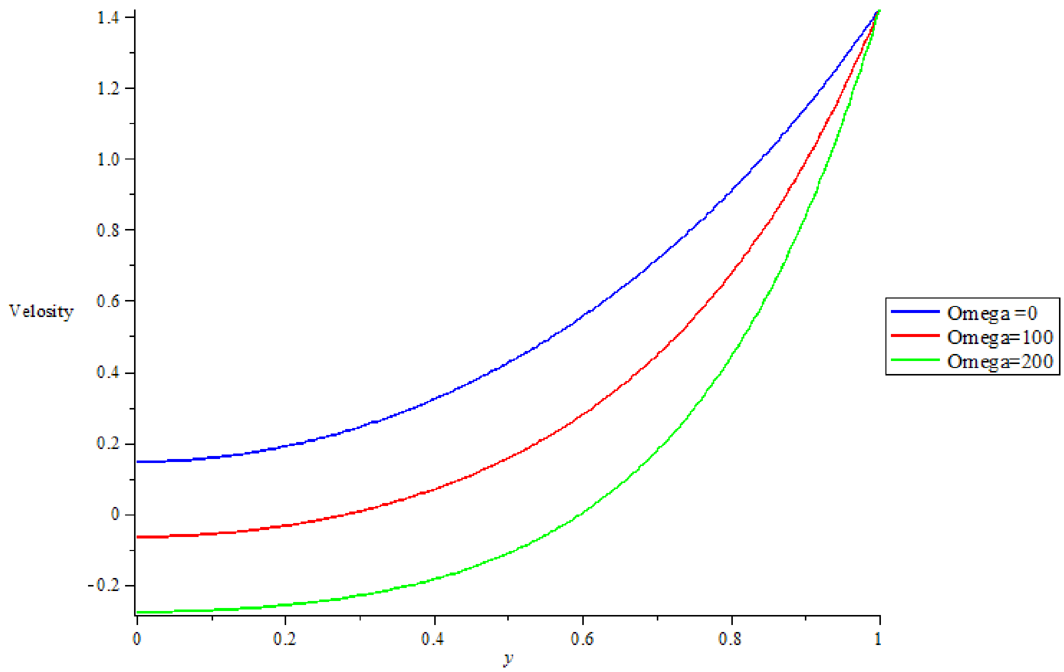

Figure 1: Velocity profile for different value of magnetic field intensity parameters

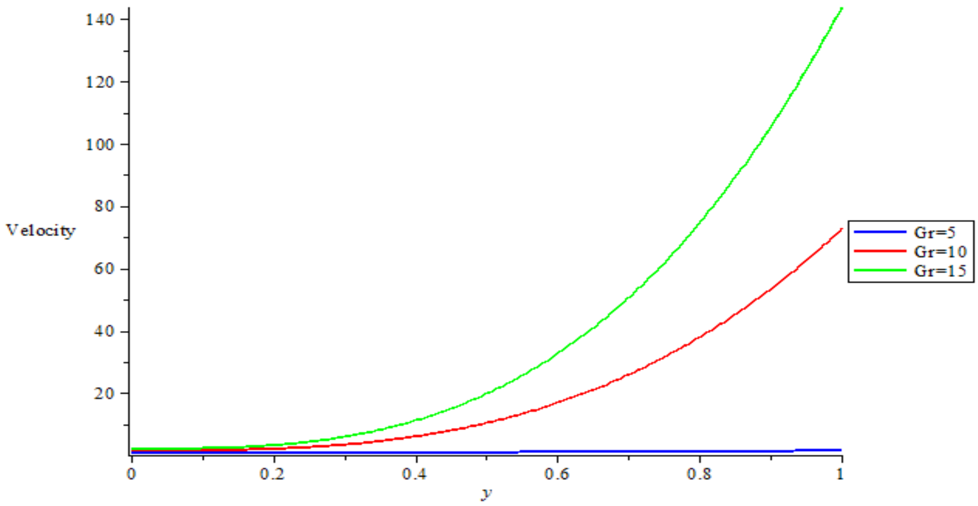

Figure 2: Velocity profile for different value of thermal Grashof numbers

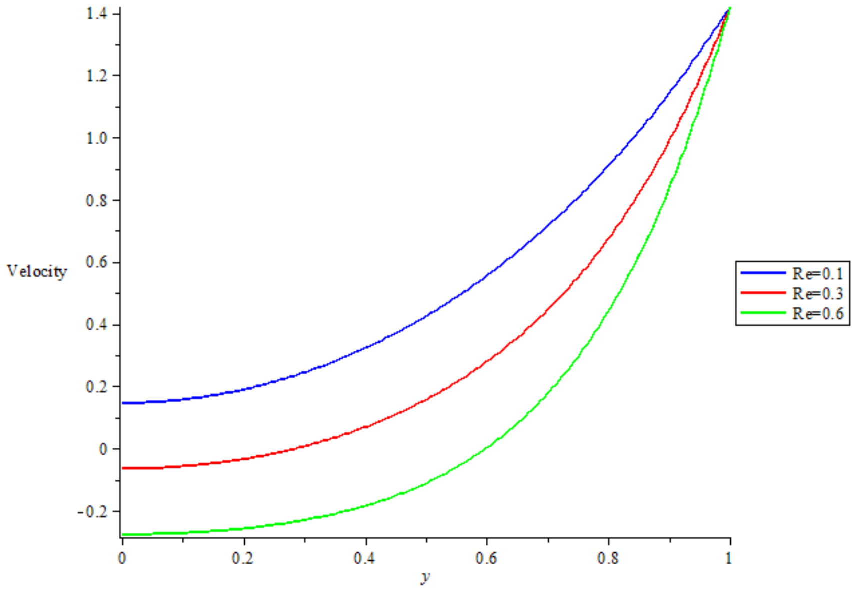

Figure 3: Velocity profile for different value of Reynolds numbers

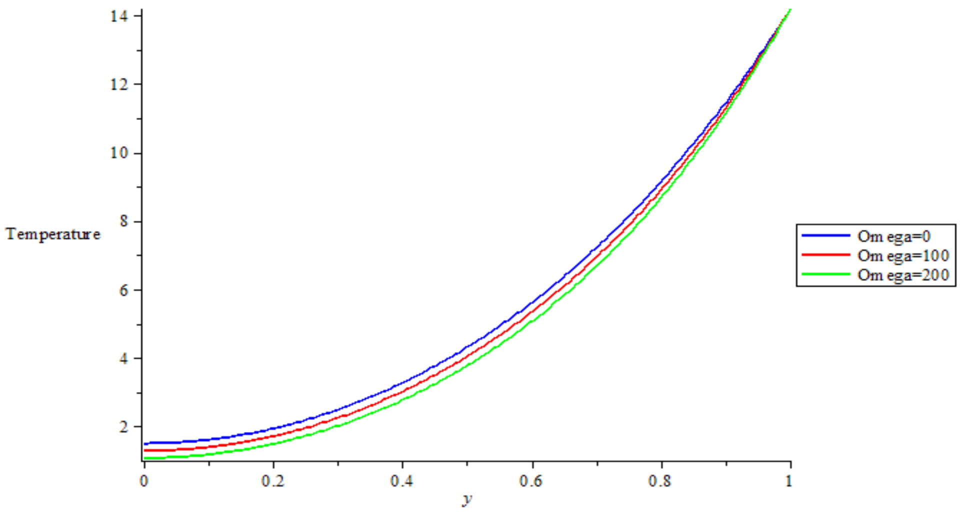

Figure 4: Temperature profile for different value of magnetic field intensity parameters

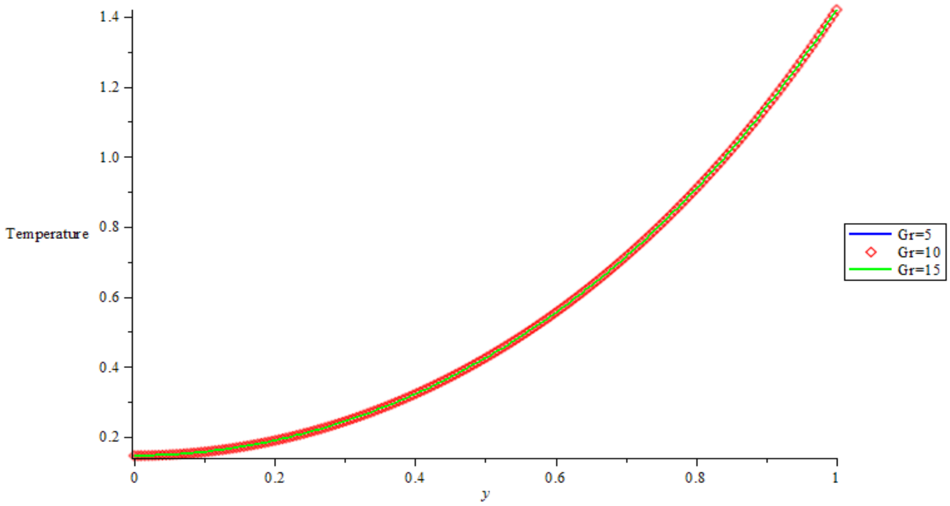

Figure 5: Temperature profile for different value of thermal Grashof numbers

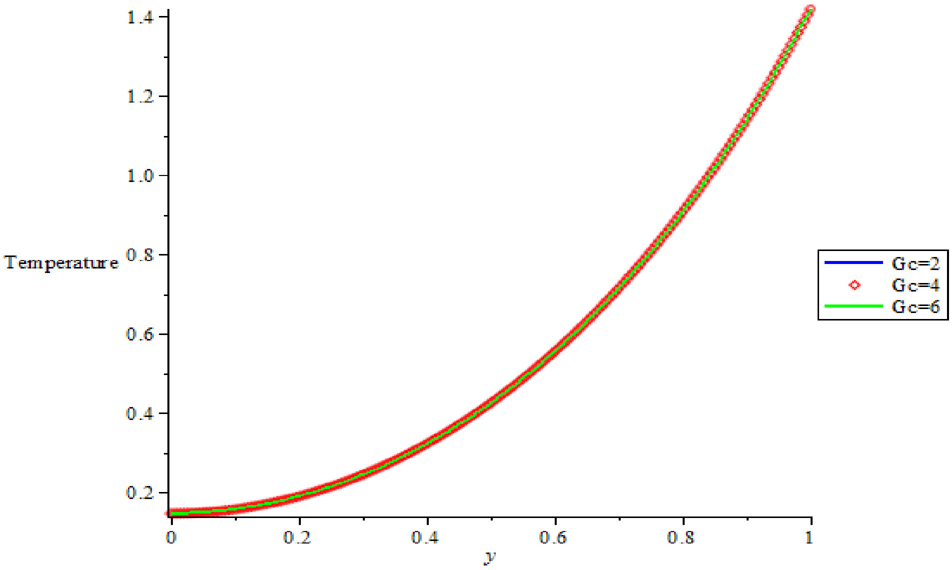

Figure 6: Temperature profile for different value of Mass Grashof numbers

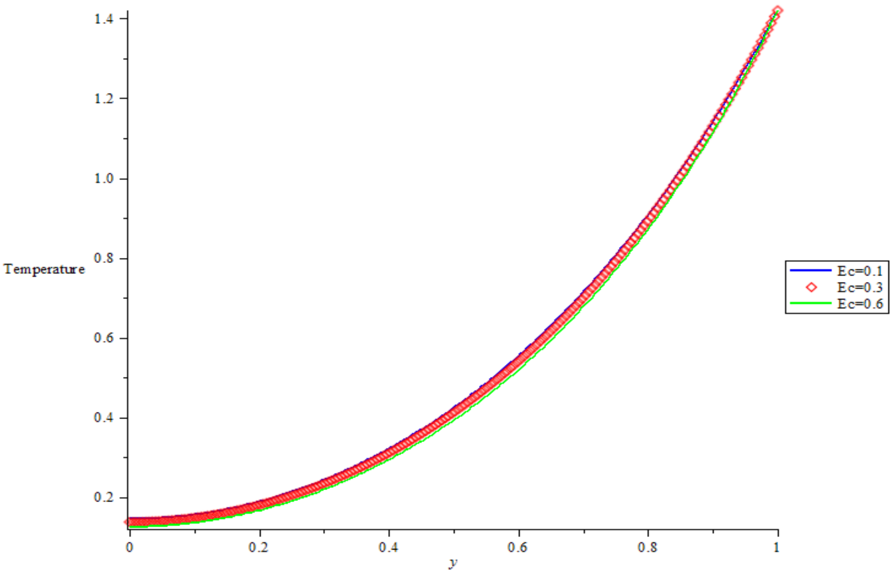

Figure 7: Temperature profile for different value of Eckert numbers

Results and Discussion

In order to study the behaviour of velocity, temperature and concentration profile, a comprehensive numerical computation using mathematical software Maple 12 was carried out for various values of the parameters that describe the flow characteristics, and the results were reported in terms of graphs as shown in figures (1)- (7). When the viscosity was considered, figure 1 shows that Velocity decreases with increase in Magnetic field intensity parameter (omega) numbers and pushed the flow to the wall of the channel which leads to increase in heat along the wall of the channel thereby increasing the velocity in the boundary layer. This is depicted in Figure 2 with increase in thermal Grashof numbers the velocity increase and the flow were pushed away from the wall. This suggests that unsteadiness has the effect of cooling the fluid. For increase in Reynolds number velocity decreases as shown in figure 3. This suggests that in this model, increasing Reynolds number enhances unsteadiness. The pressure gradient, which is trying to accelerate the fluid, is counteracted by the magnetic drag. Figure 4 shows that there is slight decrease in temperature as Magnetic field intensity parameter (omega) increases. The effect of thermal Grashof number and mass Grashof number in figure 5 but not well felt on temperature in figure 6 there is a slight increase in temperature as Eckert number increase in figure 7

Conclusion

A model was formulated with the inclusion of concentration equation. The study added viscosity parameter to explain viscous dissipation in a porous vertical channel. Approximate numerical solutions were found using regular perturbation technique together with their boundary condition. The outcome of study showed that increase in viscous dissipation led to decrease in Temperature and velocity profile but not in concentration. The viscosity effect on heat and mass transfer were clearly exposed with the significance of the parameters introduced.

Overall observations based on the problem formulated and analysed upon which conclusions were drawn and listed as:

i. evaluated the effect of viscosity on heat and mass transfer of natural convection fluid flow in porous media;

ii. assessed the impacts of viscosity on heat and mass transfer of

MHD fluid flow in porous media;

iii. established the influence of MHD fluid flow on vertically porous channel; and

iv. determined the effects of viscosity on heat and mass transfer of MHD fluid flow in a vertically porous channel. Limitation exhibited by the methods used was that: The viscosity effect on heat and mass transfer over a vertical porous channel was considered. The resulting governing equations were simplified and solved using perturbation technique. The results are presented in graphical forms.

The impact of variations of velocity, temperature and concentration parameters on non-dimensional variables of the heat and mass transfer was established to explain viscosity heat and mass transfer of MHD fluid flow in porous vertical channel. The influence of viscosity effects were also noticed along with other listed parameters and these contributed chronologically to MHD fluid flow thereby explaining heat and mass transfer over porous vertical channel.

The study concluded that increase in mass Grashof, thermal Grashof, magnetic parameter; womersley parameters, Reynolds, Eckert, Schmidt and chemical reaction numbers had significant effects on the MHD fluid flow in porous vertical channel.

References

[1]. Hartmann J. and Lazarus F. (1937), Kgl, Danske Videnskab, Selskab Fat.-Tys. Medd 15(6,7)

[2]. Mostafa A.A. Mahmoud, 2007. A note on a variable viscosity and chemical reaction effects on mixed convection heat and mass transfer along a semi-infinite vertical plate, Mathematical Problem in Engineering, Vol. 2007, pp.1-7.

[3]. Motsa, S.S., Gajjar, J.S.B. & Sibanda, P (2002). The effect of heating/cooling on the upper-branch stability of boundary layer flows over a compliant boundary. *CAMES* Vol.? **9**: 163-181

[4]. Mhone, P.Y. and Makinde, O.D (2006). "Unsteady MHD flow with heat transfer in a diverging channel" Rom. journ Phys. Vol.51.Nos.9-10, pp.967-979, Bucharest

[5]. Ganesan, P., Palani, G., (2004).Numerical solution of unsteady MHD flow past semi-infinite isothermal vertical plate, in: Proceedings of the 6th ISHMT/ASME Heat and Mass Transfer Conference and 17th National Heat and Mass Transfer Conference, pp. 184–187

[6]. Palani G. and Ganesan P. (2007), Heat Transfer Effects on Dusty Gas Flow past a Semi-Infinite Inclined plate forsch Ingenieurwes, Vol. 71, pp 223-230

[7]. Sarma U., Hazarika G.C.,(2010) Effects of variable viscosity and thermal conductivity on free convective heat and mass transfer flow with constant heat flux through a porous medium, J. Com. Math. Sci., Vol. 1(2), pp. 163-170

[8]. Sparrow, E. M. and Cess, R. D., (1961). The effect of magnetic field on free convection heat transfer, Int. J. Heat Mass Transfer 3, 267-274