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Research Article

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Study of the thermal resistance of heat transfer in a transient dynamic regime in a two-dimensional tow-plaster insulating material: influence of the heat exchange coefficient

Seydou Faye

Laboratory of Semiconductors and Solar Energy, Physics Department, Faculty of Science and Technology, University Cheikh Anta Diop, Dakar, Senegal

Abstract We present in this note a two-dimensional analytical approach for the thermal resistance calculation based on the resolution of the Duhamel method leading to the development of a solution in the form of a rapidly converging series. The objective of this study is to see the behavior of the thermal resistance of the material as a function of the optimum insulation thickness. And the desired goal is to reduce the heat flow passing through this material. To obtain the thermal resistance of the bead-plaster material, it is necessary to add the thermal resistances of the various elements that compose it. After solving the heat equation by the two-dimensional analytical method. We obtain the profiles of Thermal and Relative Resistance. Depending on the depth, the Thermal Resistance has three phases. These are: the positive gradient, the zero gradient and the negative gradient.

Keywords Transitional Regime-Thermal Resistance-Tow Plaster-Gradient

1. Introduction

Thermal resistance [1-4] measures the resistance that a thickness of material opposes to the passage of heat. The thermal resistance depends on the λ (lambda) and the thickness of the material. The natural biodegradable product such as tow [5] is used as a thermal insulator in association with plaster as a binder. Several methods in static regime [6, 7] and in transient [8, 9] or established frequency dynamics [10] are proposed.

In this article we first show the solution of the heat equation [11-14] and then determine the expression of thermal resistance. Then, we evaluate the behavior of the bead-plaster material from the thermal and relative resistance curves under the influence of the depth and the exchange coefficient at the front face. Finally to draw the depths for each maximum

2. Materials and Methods

The tow-plaster material is assumed to be homogeneous and of parallelepipedal shape. The depth of the material is $L = 0{,}05m$; the initial temperature of the material $Ti = 10^0C$ and that of the external ambient environments $Ta1 = Ta2 = 30^0C$. The heat exchange coefficients at the front and rear are and respectively h_1 and h_2. The average thermal diffusivity is $\alpha = 2{,}07.10^{-7} m^2 s^{-1}$ and the thermal conductivity is $\lambda = 0{,}15W.m^{-1 0}C^{-1}$.

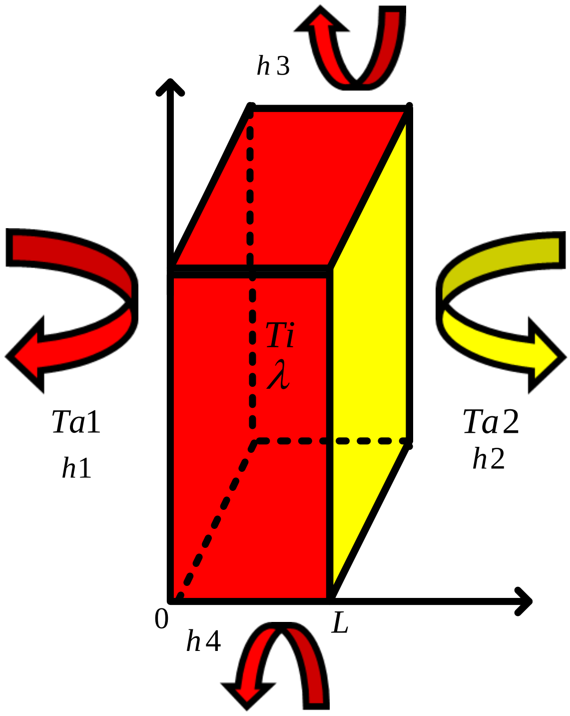

Figure 1: Sample tow-plaster

The thermal resistance Rth is given by the following expression:

$$Rth = \frac{1}{S}\left[\frac{1}{h_1} + \frac{L}{\lambda} + \frac{1}{h_2}\right]; (1)$$

With

$R_{cv1} = \frac{1}{h_1 S}$: The thermal resistance of a flat wall at the front.

$R_{cd} = \frac{L}{\lambda S}$: appears as the thermal resistance of a flat wall of thickness L, thermal conductivity λ and lateral surface S.

$R_{cv2} = \frac{1}{h_2 S}$: The thermal resistance of a flat wall at the back.

La surface S=1m² ;

$Rth = \left[\frac{1}{h_1} + \frac{L}{\lambda} + \frac{1}{h_2}\right]$: The expression of the thermal resistance of a wall subjected to permanent external climatic stresses.

$$Rth = R_{cv1} + R_{cd} + R_{cv2}; (2)$$

3. Theory

The unidirectional heat transfer in the yarn-plaster thermal insulation is governed by equation (1) below:

$$\frac{\partial^2 T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x^2} + \frac{\partial^2 T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y^2} - \frac{1}{\alpha}\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial t} = 0; (3)$$

$T = T(x,y,h_1,h_2,h_3,h_4,t)$ is the temperature inside the material; x the depth and t the time t. Equation (2) gives the expression of the diffusivity α

$$\alpha = \frac{\lambda}{\rho c};(4)$$

α is the coefficient of thermal diffusivity ($m^2.s^{-1}$)

λ is the thermal conductivity ($W.m^{-2}.c^{-1}$)

ρ is the density of the material ($kg.m^{-2}$)

Boundary conditions

$$\begin{cases} \lambda \left.\dfrac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x}\right|_{x\equiv 0} = h_1\left[T(0,y,t)-T_a\right];(5) \\ \lambda \left.\dfrac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x}\right|_{x\equiv L} = -h_2\left[T(L,y,t)-T_a\right];(6) \\ \lambda \left.\dfrac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y}\right|_{y\equiv 0} = h_3\left[T(x,0,t)-T_a\right];(7) \\ \lambda \left.\dfrac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y}\right|_{y\equiv L} = -h_4\left[T(x,L,t)-T_a\right];(8) \\ T(x,y,h_1,h_2,h_3,h_4,t=0) = T_i\,(9) \end{cases}$$

Dimensionless heat equation

$$\theta(u,v,\tau) = \frac{T(x,y,t)-T_a}{T_i-T_a};(10)$$

with $\theta(u,v,\tau)$: reduced temperature;

$u = \dfrac{x}{L}$; is a space reduced variable

$v = \dfrac{y}{L}$; is a space reduced variable

and $\tau = \dfrac{\alpha t}{L^2} = F_0$

F_0 : Reduced time variable or Fourier number

The heat equation (1) becomes:

$$\frac{\partial^2\theta(u,v,\tau)}{\partial u^2} + \frac{\partial^2\theta(u,v,\tau)}{\partial v^2} = \frac{\partial\theta(u,v,\tau)}{\partial \tau};(11)$$

The boundary conditions (5), (6), (7) and (8) become (12), (13), (14) and (15):

$$\begin{cases} \left.\dfrac{\partial\theta(u,v)}{\partial\tau}\right|_{u=0} = \dfrac{h_{1.}L}{\lambda}\theta(0,\tau);(12) \\ \left.\dfrac{\partial\theta(u,v)}{\partial\tau}\right|_{u=1} = -\dfrac{h_{2.}L}{\lambda}\theta(1,\tau);(13) \\ \left.\dfrac{\partial\theta(u,v)}{\partial\tau}\right|_{u=0} = \dfrac{h_{3.}L}{\lambda}\theta(0,\tau);(14) \\ \left.\dfrac{\partial\theta(u,v)}{\partial\tau}\right|_{u=1} = \dfrac{h_{4.}L}{\lambda}\theta(1,\tau);(15) \end{cases}$$

Let us find the solution of equation (13) in the form of reduced variables separable in space and time given by relation (16):

$$\theta(u,v,\tau) = U(u)V(v)W(\tau);(16)$$

Using the relations (13) and (16) we obtain that of (17)

$$\frac{1}{U(u)}\frac{\partial^2 U(u)}{\partial u} + \frac{1}{V(v)}\frac{\partial^2 V(v)}{\partial v} + \frac{1}{W(\tau)}\frac{\partial W(\tau)}{\partial\tau} = -\gamma^2;(17)$$

γ is a positive constant.

From relation (17) we obtain two differential equations:

- The differential equation in time is given by (18):

$$\frac{1}{W(\tau)}\frac{\partial W(\tau)}{\partial\tau} = -\gamma^2;(18)$$

- The differential equation in space (19) is written:

$$\frac{1}{U(u)}\frac{\partial^2 U(u)}{\partial u} = -\beta^2;(19)$$

The boundary conditions space:

$$\begin{cases} \left.\dfrac{\partial\theta(0,\tau)}{\partial\tau}\right|_{u=0} = B_{i1}\theta(0,\tau);(20) \\ \left.\dfrac{\partial\theta(1,\tau)}{\partial\tau}\right|_{u=1} = -B_{i2}\theta(1,\tau);(21) \\ \left.\dfrac{\partial\theta(0,\tau)}{\partial\tau}\right|_{u=0} = B_{i3}\theta(0,\tau);(22) \\ \left.\dfrac{\partial\theta(1,\tau)}{\partial\tau}\right|_{u=1} = -B_{i4}\theta(1,\tau);(23) \end{cases}$$

With $B_{i1} = \dfrac{h_1.L}{\lambda}$; $B_{i2} = \dfrac{h_2.L}{\lambda}$; $B_{i3} = \dfrac{h_3.L}{\lambda}$ et $B_{i4} = \dfrac{h_4.L}{\lambda}$ respectively the Biot numbers on the front face and on the back face.

3.1. Temperature expression:

The general solution of the reduced temperature is in the form

$$\theta(u,v,\tau)=\sum_n\left[(a_n\cos(\beta_n u)+b_n\sin(\beta_n u))\right]\left[c_n\cos(\mu_n v)+d_n\sin(\mu_n v)\right]e^{-\gamma^2\tau};(24)$$

$$\theta(u;v,\tau)=\frac{T(x,y,h_1,h_2,h_3,h_4,t)-T_a}{T_i-T_a};(25)$$

$$T(x,y,h_1,h_2,h_3,h_4,t)=T=T_a+(T_i-T_a)\theta(u;v,\tau);(26)$$

The general solution of temperature:

$$T=T_a+(T_i-T_a)\sum_n\left[a_n\left(\cos\left(\beta_n\frac{x}{L}\right)+\frac{\frac{h_{1x}L}{\lambda}}{\beta_n}\sin\left(\beta_n\frac{x}{L}\right)\right)c_n\left(\cos\left(\mu_n\frac{y}{L}\right)+\frac{\frac{h_{1y}L}{\lambda}}{\mu_n}\sin\left(\mu_n\frac{y}{L}\right)\right)\right]e^{-\frac{\alpha t}{L^2}\gamma^2};(27)$$

3.2. Expression of the heat flux density:

We get the expression of the density of the heat flow (or surface heat flow)

Which is the heat flux per unit area (W.m^{-2}) as follows:

$$\vec{\Phi}(x,y,h_1,h_2,h_3,h_4,t)=-\lambda\,\overrightarrow{grad}T(x,y,h_1,h_2,h_3,h_4,t);(28)$$

$$\vec{\Phi}(x,y,h_1,h_2,h_3,h_4,t)=\overrightarrow{\Phi_x}(x,y,h_1,h_2,h_3,h_4,t)+\overrightarrow{\Phi_y}(x,y,h_1,h_2,h_3,h_4,t);(29)$$

From these two expressions we get, the final expression of the heat flux density

$$\vec{\Phi}_x(x,y,h_1,h_2,h_3,h_4,t)=-\lambda\frac{\partial\vec{T}(x,y,h_1,h_2,h_3,h_4,t)}{\partial x};(30)$$

$$\vec{\Phi}_y(x,y,h_1,h_2,h_3,h_4,t)=-\lambda\frac{\partial\vec{T}(x,y,h_1,h_2,h_3,h_4,t)}{\partial y};(31)$$

$$\vec{\Phi}(x,y,h_1,h_2,h_3,h_4,t)=\sqrt{(\Phi_x(x,y,h_1,h_2,h_3,h_4,t))^2+(\Phi_y(x,y,h_1,h_2,h_3,h_4,t))^2};(32)$$

We obtain the expression of the temperature variation:

$$\Delta T(x,y,h_1,h_2,h_3,h_4,t)=T(0,y,h_1,h_2,h_3,h_4,t)-T(x,y,h_1,h_2,h_3,h_4,t);(33)$$

Thermal resistance expresses its resistance to the passage of a heat conduction flow (W.m^{-1}.^{0}C^{-1}).

The greater the thermal resistance, the more insulating the material.

Thermal resistance depends on thickness and thermal conductivity.

The thermal resistance, which is the ratio between the change in temperature and the heat flux density, is given by expression (34).

$$R_{th}(x,y,h_1,h_2,h_3,h_4,t)=\frac{\Delta T(x,y,h_1,h_2,h_3,h_4,t)}{\Phi(x,y,h_1,h_2,h_3,h_4,t)};(34)$$

$\Delta T(x,y,h_1,h_2,h_3,h_4,t)$ being the temperature difference between the two sides of the material.

$\Phi(x,y,h_1,h_2,h_3,h_4,t)$ the heat flow that passes through the yarn-plaster material.

This expression allows us to draw the curves of the thermal resistance according to the various thermophysical parameters.

4. Results and Discussions

4.1. We are going to plot the evolution of the thermal resistance as a function of the depth under the influence of the exchange coefficient at the front face h1.

For low values of depth the thermal resistance gradually increases until it reaches a maximum, i.e. the positive gradient. This increase signifies the storage of heat within the tow-plaster material.

Then we have a maximum which corresponds to the zero gradient, that is to say the storage of heat. And finally we notice a decrease in thermal resistance corresponds to the negative gradient. This decrease corresponds to the dissipation or restriction of heat in the tow-plaster materiel

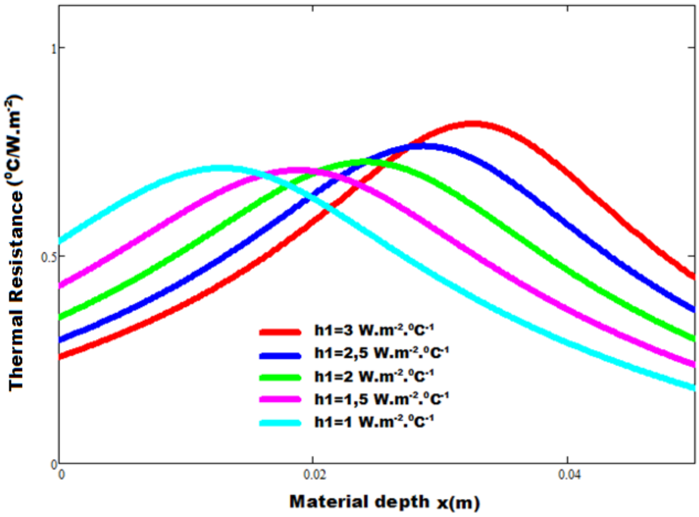

Figure 2: Evolution of thermal resistance as a function of depth

$h_2 = 0.005 \mathrm{W.m^{-2}.^{0}C^{-1}}$ *and* $t = 10s$

Table 1: Depth values for each maximum

$h_1\left(W.m^{-2}.C^{-1}\right)$	**1**	**1.5**	**2**	**2.5**	**3**
Optimal insulation thickness $x(m)$	0.013	0.019	0.025	0.028	0.033

4.2. Evolution of Relative Thermal Resistance as a function of depth under the influence of the exchange coefficient on the front face.

We notice when the depth increases the maximum moves.

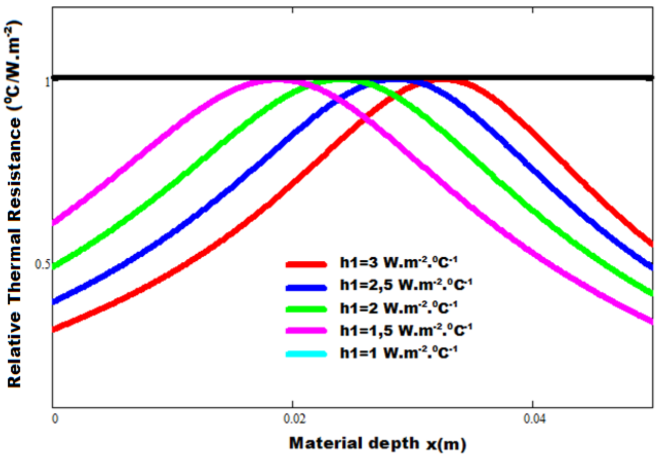

Figure 3: Evolution of Relative Thermal Resistance as a function of depth $h_2 = 0.005 \mathrm{W.m^{-2}.^{0}C^{-1}}$ *and* $t = 10s$

Table 2: Depth values for each maximum

$h_1\left(W.m^{-2}.C^{-1}\right)$	**1.5**	**2**	**2.5**	**3**
Optimal insulation thickness $x(m)$	0.019	0.024	0.029	0.033

4.3. Evolution of thermal resistance as a function of the exchange coefficient on the front face under the influence of depth

We notice when the depth increases the maximum moves.

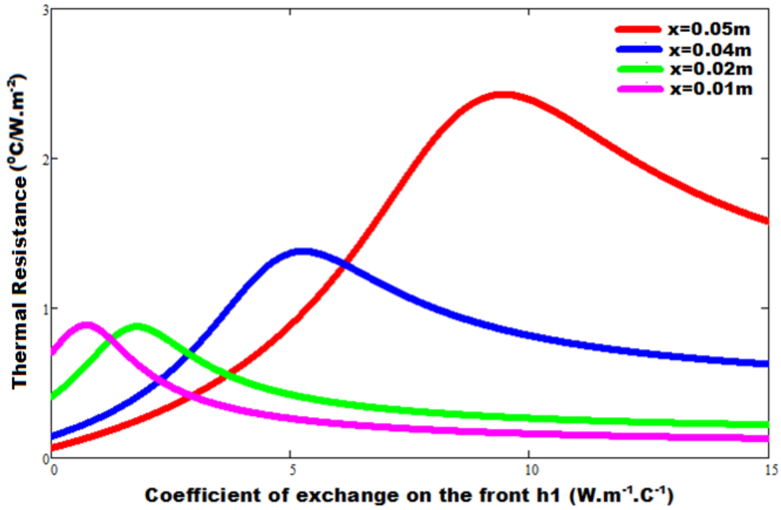

Figure 4: Evolution of thermal resistance as a function of the exchange coefficient on the front face $h_2 = 0.005\text{W.m}^{-2}.^0\text{C}^{-1}$ *and* $t = 10s$

Table 3: Depth values for each maximum

Optimal insulation thickness $x(m)$	**0.01**	**0.02**	**0.04**	**0.05**
$h_1\left(W.m^{-2}.C^{-1}\right)$	0.88	1.79	5.24	9.4

4.4. Evolution of Relative Thermal Resistance as a function of the exchange coefficient on the front face under the influence of depth

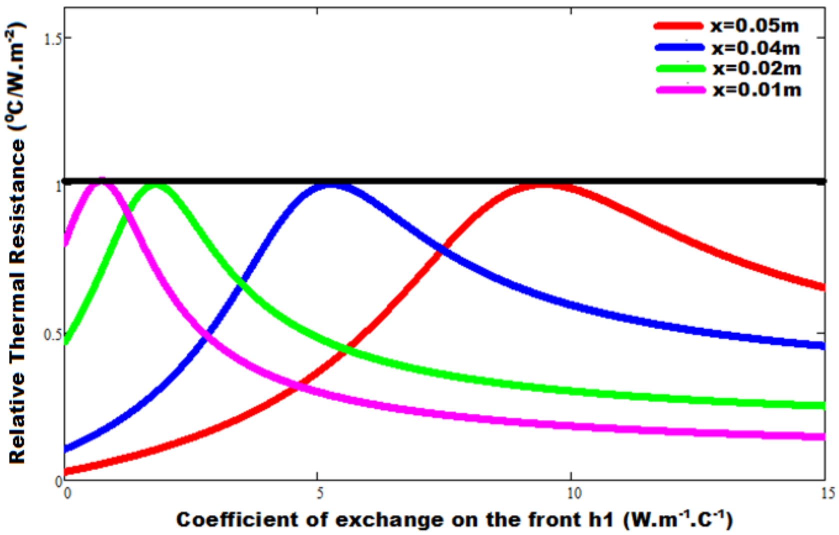

Figure 5: Evolution of the Relative Thermal Resistance as a function of the exchange coefficient on the front face $h_2 = 0.005\text{W.m}^{-2}.^0\text{C}^{-1}$ *and* $t = 10s$

Table 4: Depth values for each maximum

Optimal insulation thickness $x(m)$	**0.01**	**0.02**	**0.04**	**0.05**
$h_1(W.m^{-2}.C^{-1})$	0.7	1.8	5.3	9.5

Conclusion

This work was devoted to the study of the behavior of thermal resistance of plaster tow material. By plotting the thermal and relative resistance as a function of the depth and the exchange coefficient on the front face. And we were able to determine the optimum insulation thickness for each curve. And we notice when the depth increases the maximum moves. Also, we have shown that resistance depends on the thickness of the material.

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