Available online www.jsaer.com

Journal of Scientific and Engineering Research, 2021, 8(7):133-140

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

Study in transient regime by analytical method of heat transfer through a two dimensional tow-plaster insulating material: influence of the heat exchange coefficient

Seydou Faye

Laboratory of Semiconductors and Solar Energy, Physics Department, Faculty of Science and Technology, University Cheikh Anta Diop, Dakar, Senegal

Abstract The study in Cartesian coordinates of a material of the form of a simple wall made of tow-plaster with the following characteristics ($\lambda = 0{,}15 W.m^{-1}.^{0}C^{-1}$ and $\alpha = 2{,}07.10^{-7} m^2.s^{-1}$).

The tow-plaster thermal insulator (the tow is made of fibers) is modeled by the heat equation using the transient dynamic regime analytical method. This article is based on determining different profiles of temperature and heat flux density as a function of material depth, heat exchange coefficient and time.

Keywords Tow Plaster-Transitional regime-Temperature-Heat flux density

Introduction

Artificial thermal [1-3] insulators are very effective, but pose a problem of biodegradability and the environment [4-6], the introduction of natural insulators, such as kapok or tow, reduces environmental problems [7],

A thermal insulator is a material with low thermal conductivity [8-10]. The characteristic of tow-plaster is to slow down the exchange of heat between the interior and the exterior of a building.

Several methods in static regime [11, 12] and in transient [13] or frequency dynamics [14] established are proposed.

In this work, we study the heat transfer [15, 16] in transient regime in a material made of tow-plaster under the influence of the exchange coefficient.

1. Materials and Methods

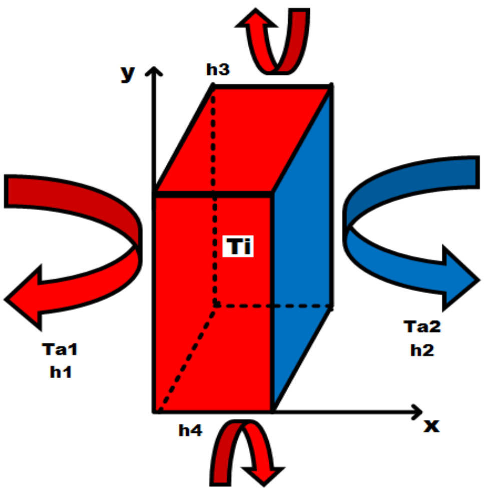

Figure 1: Sample tow-plaster

The tow-plaster material is assumed to be homogeneous and of parallelepipedal shape. The depth of the material is $L = 0{,}05m$; the initial temperature of the material $Ti = 10^0 C$ and that of the external ambient environments $Ta1 = Ta2 = 30^0 C$. The heat exchange coefficients at the front and rear are and respectively h_1 and h_2.. The average thermal diffusivity is $\alpha = 2{,}07.10^{-7} m^2 s^{-1}$ and the thermal conductivity is $\lambda = 0{,}15 W.m^{-1 0} C^{-1}$.

2. Theory

The unidirectional heat transfer in the yarn-plaster thermal insulation is governed by equation (1) below:

$$\frac{\partial^2 T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x^2} + \frac{\partial^2 T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y^2} - \frac{1}{\alpha}\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial t} = 0; (1)$$

$T = T(x,y,h_1,h_2,h_3,h_4,t)$ is the temperature inside the material; x the depth and t the time t. Equation (2) gives the expression of the diffusivity α.

$$\alpha = \frac{\lambda}{\rho c}; (2)$$

α is the coefficient of thermal diffusivity ($m^2.s^{-1}$)

λ is the thermal conductivity ($W.m^{-2}.c^{-1}$)

ρ is the density of the material ($kg.m^{-2}$)

Boundary conditions

$$\begin{cases} \lambda \left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x}\right|_{x=0} = h_1\left[T(0,y,t) - T_a\right]; (3) \\ \lambda \left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial x}\right|_{x=L} = -h_2\left[T(L,y,t) - T_a\right]; (4) \\ \lambda \left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y}\right|_{y=0} = h_3\left[T(x,0,t) - T_a\right]; (5) \\ \lambda \left.\frac{\partial T(x,y,h_1,h_2,h_3,h_4,t)}{\partial y}\right|_{y=L} = -h_4\left[T(x,L,t) - T_a\right]; (6) \\ T(x,y,h_1,h_2,h_3,h_4,t=0) = T_i; (7) \end{cases}$$

Dimensionless heat equation

$$\theta(u,v,\tau) = \frac{T(x,y,t) - T_a}{T_i - T_a}; (8)$$

with $\theta(u,v,\tau)$: reduced temperature;

$u = \frac{x}{L}$; is a space reduced variable

$v = \frac{y}{L}$; is a space reduced variable

and $\tau = \frac{\alpha t}{L^2} = F_0$

F_0 : Reduced time variable or Fourier number

The heat equation (1) becomes:

$$\frac{\partial^2\theta(u,v,\tau)}{\partial u^2}+\frac{\partial^2\theta(u,v,\tau)}{\partial v^2}=\frac{\partial\theta(u,v,\tau)}{\partial\tau};(9)$$

The boundary conditions (3), (4), (5) and (6) become (10), (11), (12) and (13):

$$\begin{cases}\left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=0}=\frac{h_{1x.}L}{\lambda}\theta(0,\tau);(10)\\ \left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=1}=-\frac{h_{2x.}L}{\lambda}\theta(1,\tau);(11)\\ \left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=0}=\frac{h_{1y.}L}{\lambda}\theta(0,\tau);(12)\\ \left.\frac{\partial\theta(u,v)}{\partial\tau}\right|_{u=1}=\frac{h_{2y.}L}{\lambda}\theta(1,\tau);(13)\end{cases}$$

Let us find the solution of equation (9) in the form of reduced variables separable in space and time given by relation (14):

$\theta(u,v,\tau)=U(u)V(v)W(\tau);(14)$

Using the relations (9) and (14) we obtain that of (15)

$$\frac{1}{U(u)}\frac{\partial^2U(u)}{\partial u}+\frac{1}{V(v)}\frac{\partial^2V(v)}{\partial v}+\frac{1}{W(\tau)}\frac{\partial W(\tau)}{\partial\tau}=-\gamma^2;(15)$$

γ is a positive constant.

From relation (15) we obtain two differential equations:

- The differential equation in time is given by (16):

$$\frac{1}{W(\tau)}\frac{\partial W(\tau)}{\partial\tau}=-\gamma^2;(16)$$

- The differential equation in space (17) is written:

$$\frac{1}{U(u)}\frac{\partial^2U(u)}{\partial u}=-\beta^2;(17)$$

The boundary conditions space:

$$\begin{cases}\left.\frac{\partial\theta(0,\tau)}{\partial\tau}\right|_{u=0}=B_{i1x}\theta(0,\tau);(18)\\ \left.\frac{\partial\theta(1,\tau)}{\partial\tau}\right|_{u=1}=-B_{i2x}\theta(1,\tau);(19)\\ \left.\frac{\partial\theta(0,\tau)}{\partial\tau}\right|_{u=0}=B_{i1y}\theta(0,\tau);(20)\\ \left.\frac{\partial\theta(1,\tau)}{\partial\tau}\right|_{u=1}=-B_{i2y}\theta(1,\tau);(21)\end{cases}$$

Avec $B_{i1x}=\frac{h_{1x}.L}{\lambda}$; $B_{i2x}=\frac{h_{2x}.L}{\lambda}$; $B_{i1y}=\frac{h_{1y}.L}{\lambda}$ et $B_{i2y}=\frac{h_{2y}.L}{\lambda}$

respectively the Biot numbers on the front face and on the back face.

The general solution of the reduced temperature is in the form

$$\theta(u,v,\tau)=\sum_n\left[\left(a_n\cos(\beta_n u)+b_n\sin(\beta_n u)\right)\right]\left[c_n\cos(\mu_n v)+d_n\sin(\mu_n v)\right]e^{-\gamma^2\tau};(22)$$

$$\begin{cases}\beta_n b_n=B_{i1x}a_n;(23)\\ -\beta_n a_n\sin(\beta_n L)+\beta_n b_n\cos(\beta_n L)=-B_{i2x}\left(a_n\cos(\beta_n L)+b_n\sin(\beta_n L)\right);(24)\end{cases}$$

$$\sin(\beta_n L)(a_n\beta_n-B_{i2x}b_n)=\cos(\beta_n L)(b_n\beta_n+B_{i2x}a_n);(25)$$

$$\tan(\beta_n L)=\frac{b_n\beta_n+B_{i2x}a_n}{a_n\beta_n-B_{i2x}b_n};(26)$$

The following transcendent equation x:

$$\tan(\beta_n L)=\frac{\frac{h_{1x}L}{\lambda}\beta_n+\frac{h_{2x}L}{\lambda}\beta_n}{\beta_n^2-\frac{h_{1x}h_{2x}L}{\lambda^2}};(27)$$

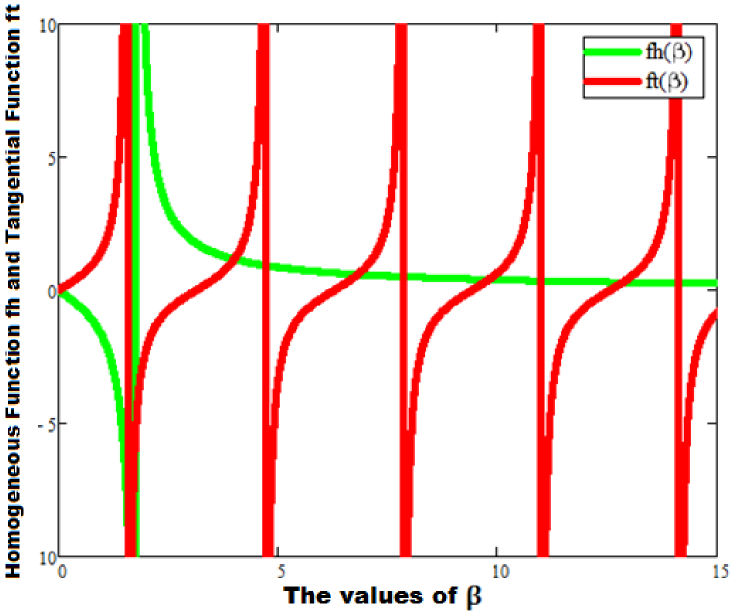

Figure 2: Curve of the transcendent equation following x

The intersection of the two curves fh (β_n) and ft (β_n) corresponds to the solution.
Table 1 summarizes the eigenvalues found of β_n

Table 1: The eigenvalues of β_n of the equation.

n	**1**	**2**	**3**	**4**	**5**
β_n	4.4	7.4	10.3	13.3	16.4

Transcendent equation following y

$$\begin{cases}\mu_n d_n=B_{i1y}c_n;(28)\\ -\mu_n c_n\sin(\mu_n L)+\mu_n d_n\cos(\mu_n L)=-B_{i2y}\left(c_n\cos(\mu_n L)+d_n\sin(\mu_n L)\right);(29)\end{cases}$$

$$\sin(\mu_n L)(c_n\mu_n-B_{i2y}d_n)=\cos(\mu_n L)(c_n\mu_n+B_{i2y}c_n);(30)$$

$$\tan(\mu_n L)=\frac{d_n\mu_n+B_{i2y}c_n}{c_n\mu_n-B_{i2y}d_n};(31)$$

$$\tan(\mu_n L)=\frac{\frac{h_{1y}L}{\lambda}\mu_n+\frac{h_{2y}L}{\lambda}\mu_n}{\mu_n^2-\frac{h_{1y}h_{2y}L}{\lambda^2}};(32)$$

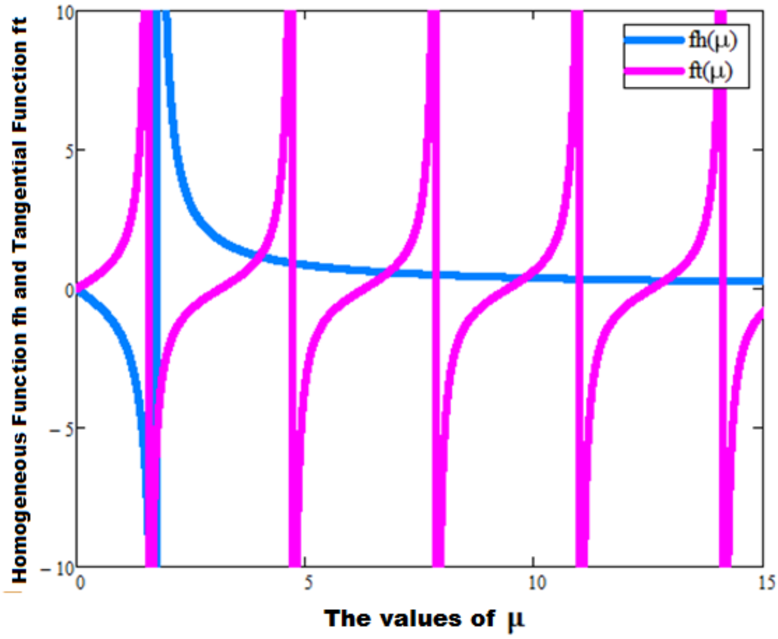

Figure 3: Curve of the transcendent equation following y

The intersection of the two curves fh (μ_n) and ft (μ_n) corresponds to the solution.

Table 1 summarizes the eigenvalues found of μ_n

Table 2: The eigenvalues of μ_n of the equation

n	**1**	**2**	**3**	**4**	**5**
μ_n	4.4	7.4	10.3	13.3	16.4

$$\theta(u;v,\tau)=\frac{T(x,y,h_1,h_2,h_3,h_4,t)-T_a}{T_i-T_a};(33)$$

$$T(x,y,h_1,h_2,h_3,h_4,t)=T=T_a+(T_i-T_a)\theta(u;v,\tau);(34)$$

The general solution of temperature:

$$T=T_a+(T_i-T_a)\sum_n\left[a_n\left(\cos\left(\beta_n\frac{x}{L}\right)+\frac{\frac{h_{1x}L}{\lambda}}{\beta_n}\sin\left(\beta_n\frac{x}{L}\right)\right)c_n\left(\cos\left(\mu_n\frac{y}{L}\right)+\frac{\frac{h_{1y}L}{\lambda}}{\mu_n}\sin\left(\mu_n\frac{y}{L}\right)\right)\right]e^{-\frac{\alpha t}{L^2}\gamma^2};(35)$$

We get the expression of the density of the heat flow (or surface heat flow)

Which is the heat flux per unit area (W.m^{-2}) as follows:

$$\vec{\varphi}(x,y,h_{1x},h_{2x},h_{1y},h_{2y},t)=-\lambda\overrightarrow{grad}T(x,y,h_{1x},h_{2x},h_{1y},h_{2y},t);(36)$$

$$\vec{\varphi}(x,y,h_1,h_2,h_3,h_4,t)=\overrightarrow{\varphi_x}(x,y,h_1,h_2,h_3,h_4,t)+\overrightarrow{\varphi_y}(x,y,h_1,h_2,h_3,h_4,t);(37)$$

From these two expressions we get, the final expression of the heat flux density

$$\varphi(x,y,t)=\lambda(Ti-Ta)\left[\left[\sum_n an\left(-\frac{\beta n}{L}\sin\left(\beta n\frac{x}{L}\right)+\frac{Bi1x}{L}\cos\left(\beta n\frac{x}{L}\right)\right)cn\left(\cos\left(\mu n\frac{y}{L}\right)+\frac{Bi1y}{L}\sin\left(\mu n\frac{y}{L}\right)\right)\right]^2+\left[\sum_n cn\left(-\frac{\mu n}{L}\sin\left(\mu n\frac{y}{L}\right)+\frac{Bi1y}{L}\cos\left(\mu n\frac{x}{L}\right)\right)an\left(\cos\left(\beta n\frac{x}{L}\right)+\frac{Bi1x}{L}\sin\left(\beta n\frac{x}{L}\right)\right)\right]^2\right]^{\frac{1}{2}}e^{-\frac{\alpha t}{L^2}\gamma^2};(38)$$

3. Results and Discussions

3.1. Evolution of the temperature and the density of the heat flow as a function of the depth for different values of the exchange coefficient

Figures 4 and 5 give the temperature and the heat flux density of the samples as a function of the depth under the influence of the heat exchange coefficient at the front face.

We note a decrease in temperature and heat flux density as a function of the depth of the wall l the influence of the heat exchange coefficient at the front face.

The temperature decreases to reach the initial temperature Ti of the material and the transmitted heat flux density gradually decreases before reaching the back side, which means that the material has stored most of the heat. This shows that the material used is a good insulator.

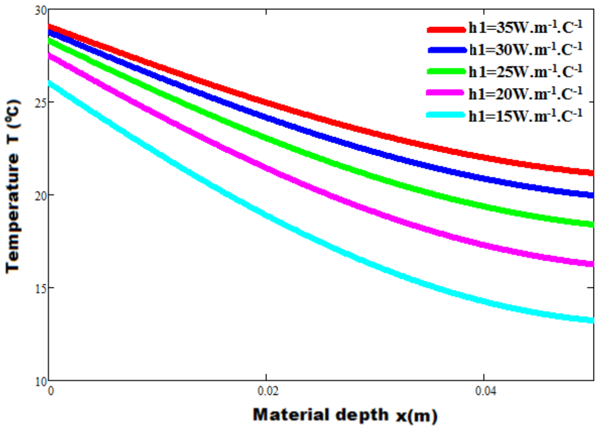

Figure 4: Temperature as a function of the depth of the material; $h2=0.005W.m^{-2}.\ C^{-1}$; t=10s

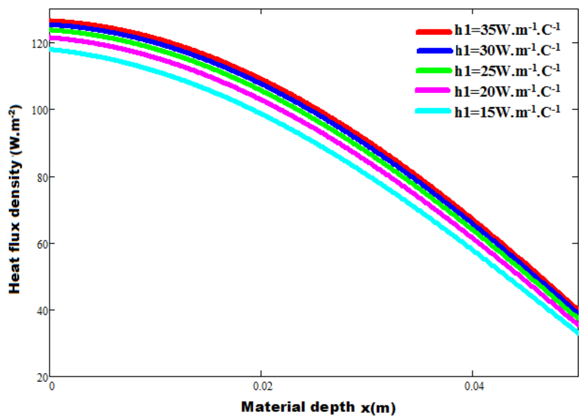

Figure 5: Heat flux density as a function of material depth; $h2=0.005W.m^{-2}.\ C^{-1}$; t=10s

3.2. Evolution of the temperature and the density of heat flow as a function of the exchange coefficient

Figures 6 and 7 give the temperature and heat flux density of the samples as a function of the heat exchange coefficient at the front face under the influence of depth.

The temperature and heat flux density increase very rapidly for low values of the exchange coefficient at the front and after they stabilize. The phenomenon of heat exchange eases to one level. Which shows that there is a threshold exchange coefficient. This is called thermal equilibrium.

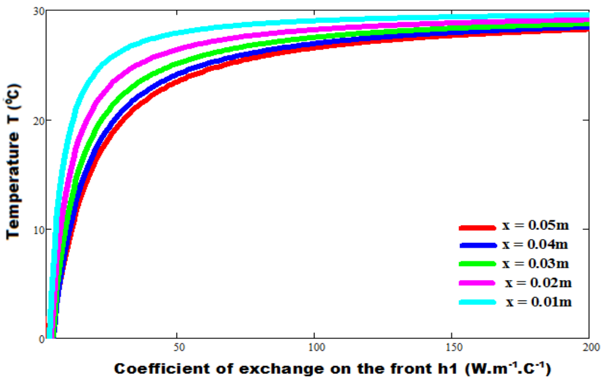

Figure 6: Temperature as a function of the heat exchange coefficient at the front face h1 :$h2=0.005W.m^{-2}.C^{-1}$; t=10s

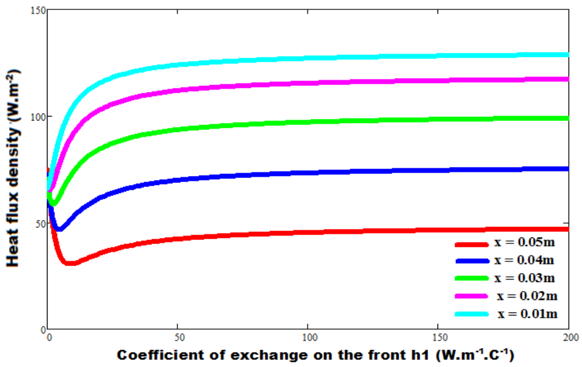

Figure 7: Heat flux density as a function of the heat exchange coefficient at the front face h1: $h2=0.005W.m^{-2}.C^{-1}$; t=10s

3.3. Evolution of temperature and heat flow density as a function of time

The higher the heat exchange coefficient at the front, the higher the temperature. This means that the material stores thermal energy. Unlike the heat flux density which decreases over time. This decrease is due to a loss of heat in the material. These phenomena show that the material used is a good insulator.

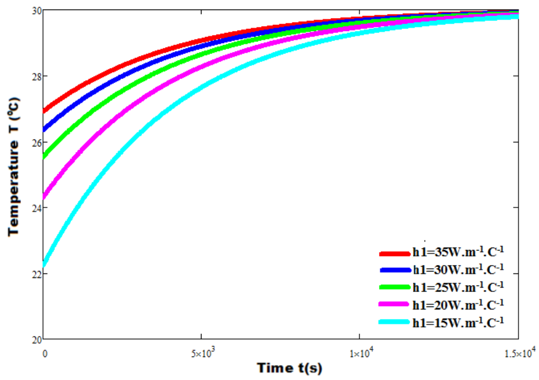

Figure 8: Evolution of the temperature as a function of material time; x=0.01m ; h2=0.005W.m^{-2}.C^{-1}

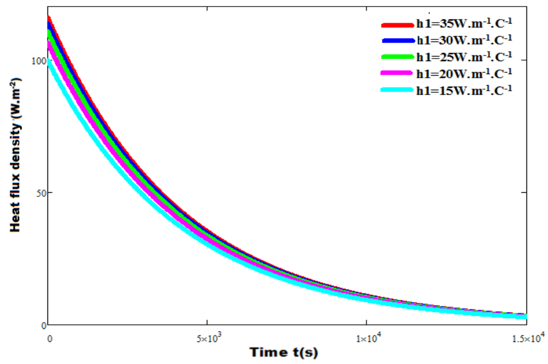

Figure 9: Evolution of the heat flux density as a function of time; x = 0.01m; h2 = 0.005W.m^{-2}.C^{-1}

Conclusion

A two-dimensional analytical model was developed to assess the influence of the exchange coefficient on the evolution of the temperature and the heat flux density of a material made of yarn-plaster. After solving the heat equation by the two-dimensional analytical method. We get the temperature and heat flow density profiles. Depending on the depth, the temperature and the heat flux density gradually decrease, which means that the material has stored most of the heat. This shows that the yarn-plaster used is a good insulator.

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