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Journal of Scientific and Engineering Research, 2021, 8(6):8-13

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

Solution of Underground Oil Seepage Problem in Semi-infinite Rectangular Region

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Abstract Due to the limitation of oil and gas reserves on the earth, it is important to improve the extraction efficiency for oil and gas resources. To realize this goal of high efficiency, it is necessary to make appropriate oil development plans and put this plans into action. The core issue of the oil development plan is to study the oil flow in underground porous rocks. As a traditional method for solving partial differential equations, variable separation method has played an important role for these studies. Take the oil seepage problem in semi-infinite rectangular region as an example, the complete solution is finally got using the solution technique of variation separation method, and this can be good reference for oil development in semi-infinite rectangular areas in the oilfield.

Keywords natural gas engineering, variable separation, porous media, gas seepage, integral transform

Introduction

With the development of modern world, the demand for oil resources have been increasing very fast. Reservoir engineers in each oil company face the problem of extracting oil with the highest efficiency. However, high efficiency of development requires the familiarity of characteristics of oil reservoirs. Among these characteristics, one of the most important is the oil seepage in underground reservoir rocks. Many reservoirs are developed by water injection, and after several years, the oil extracted from the well will be of very high water cut. This will surely decrease the efficiency of oil production as well as the economic profits [1-2]. Some reservoirs have formation rocks of very low permeability, and this will greatly increase the difficulty for oil development. Many studies have been launched and many measures have been adopted to improve the effect for exploitation of these low permeable reservoirs [3-5]. Actually, the oil resources are not reserved in one kind of rocks, and they are even found out and extracted in formation rocks of complex lithologic characters, and this will require the study and knowledge for improving the flow in these unconventional formation rocks [6-8]. To optimize the development plan of oil resources, many methods have been applied to study the oil seepage problems. Finite element method has unique advantages to study the oil seepage problems with complex boundaries, and it is good at dealing with the complex boundary conditions in reservoir modeling [9-11]. Finite difference method is the earliest developed and the mostly used method for reservoir flow modelling, and it can convert the solution of differential equations into the solution of algebraic equations. Many researchers have obtained many good results using this method [12-14]. As a traditional solution method, variable separation method has also played an important role for study of oil seepage problems. In following paragraphs, pressure distribution for oil seepage in a semi-infinite rectangular region will be studied by the variable separation method.

Establishment of Mathematical Model for the Seepage Problem

The seepage area is as that demonstrated in Figure 1 below, and this is a 2-D semi-infinite rectangular region. Variable p is used to represent the pressure, therefore, it is the function of both space and time variables. The coordinate interval for seepage in x direction is from 0 to ∞, and the coordinate interval for seepage in y direction is from 0 to b, so the rectangular area is actually in a strip type. At time $t=0$, The initial pressure in this strip-type area is a function of coordinate x and y as $F(x,y)$, and suppose the boundary conditions on boundary $y=0$ are the linear combination of pressure and its derivatives. When the time $t>0$, suppose the pressure on boundary $y=b$ remain constant as 0 and suppose the pressure on boundary $x=0$ also remain constant as 0.The intent is to find the pressure distribution when time t is larger than 0, that is to find the mathematical expression of $p(x,y,t)$.

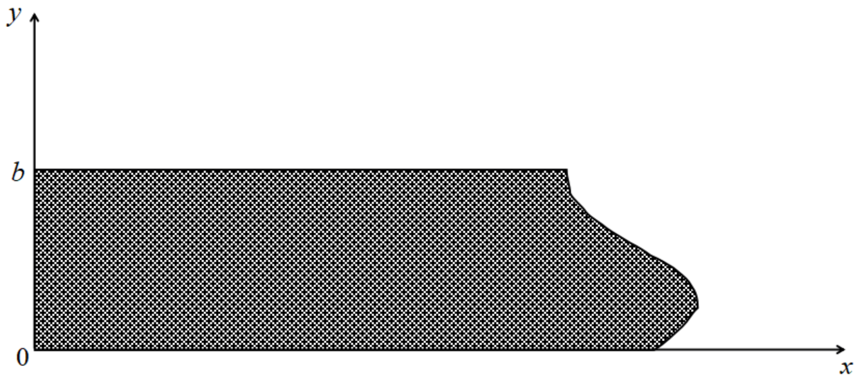

Figure 1: Sketch map of seepage in semi-infinite rectangular region

On basis of the mathematical model establishment theories for flow in reservoir rocks [15-16], suppose η as the coefficient of pressure transmission, the mathematical model of this seepage problem can be expressed with equations in the following:

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=\frac{1}{\eta}\frac{\partial p}{\partial t} \quad 0<x<\infty, 0<y<b, t>0 \tag{1}$$

$$p=0 \quad x=0, t>0 \tag{2}$$

$$-\frac{\partial p}{\partial y}+h_1 p=0 \quad y=0, t>0 \tag{3}$$

$$p=0 \quad y=b, t>0 \tag{4}$$

$$p=F(x,y) \quad 0<x<\infty, 0<y<b, t=0 \tag{5}$$

Introduction of the Variable Separation Method

The solution-determination problem (1)-(5) is related to the solution of partial differential equations, and this is because there are 3 independent variables in the unknown function $p(x,y,t)$. This problem will become relatively easier to be solved if the partial differential equations can be converted into equivalent ordinary differential equations. Variable separation method is just such a method for this conversion process, and it is a basic method for solving partial differential equation problems [17].

Generally, there are four steps to follow for application of variable separation methods. The first step is about variable separation, and the special solution is supposed to be the multiplication of several functions of independent variables. Second, there will be eigenvalue problems of ordinary differential equations after variable separation is executed, and the solution of eigenvalue problems should be found out. Third, finding out the special solutions of seepage problem by making use of the initial conditions, and do superposition on these

special solutions to obtain the general solution. Fourth, determining the coefficient of each item in the superposition by making use of the orthogonality of eigenfunctions concerning the eigenvalue problems.
There are three basic requirements for the successful application of variable separation method. First, there should be solution existed for the eigenvalue problem. Second, the collective eigenfunction should be complete, that is, the solution of seepage problem should be able to be expanded by eigenfunctions. Third, the eigenfunctions should be of orthogonality.

Problem Solving by Variable Separation Method

Based on the solution process of variable separation method, the general solution of pressure can be firstly expressed in the following format:

$$p(x,y,t)=\sum_{n=1}^{\infty}\int_{\beta=0}^{\infty}c_n(\beta)e^{-\eta(\beta^2+\gamma_n{}^2)t}X(\beta,x)Y(\gamma_n,y)d\beta \qquad (6)$$

Among them, the function $X(\beta,x)$ can satisfy the auxiliary problem below:

$$\frac{d^2X(x)}{dx^2}+\beta^2X(x)=0 \quad 0<x<\infty \qquad (7)$$

$$X(x)=0 \quad x=0 \qquad (8)$$

And $Y(\gamma_n,y)$ can satisfy the eigenvalue problem below:

$$\frac{d^2Y(y)}{dy^2}+\gamma^2Y(y)=0 \quad 0<y<b \qquad (9)$$

$$Y=0 \quad y=b \qquad (10)$$

$$-\frac{dY(y)}{dy}+h_1Y(y)=0 \quad y=0 \qquad (11)$$

Here, since the seepage region is semi-infinite in the x direction, the superposition of fundamental solutions $X(\beta,x)$ is carried out by doing the integral towards the separation variable β.
Substituting the initial condition into the formula (7):

$$F(x,y)=\sum_{n=1}^{\infty}\int_{\beta=0}^{\infty}c_n(\beta)X(\beta,x)Y(\gamma_n,y)d\beta \qquad (12)$$

To obtain the unknown coefficient $c_n(\beta)$, do operation on both sides of formula(12) with operator $\int_0^b Y(\gamma_n,y)dy$, and make use of the orthogonality of eigenfunction $Y(\gamma_n,y)$.

Suppose an function $f^*(x)$ defined at interval $0<x<\infty$ as:

$$f^*(x)=\int_{\beta=0}^{\infty}c_n(\beta)X(\beta,x)d\beta \quad 0<x<\infty \qquad (13)$$

Based on operations above, there is:

$$f^*(x)=\frac{1}{N(\gamma_n)}\int_{y=0}^{b}Y(\gamma_n,y)F(x,y)dy \qquad (14)$$

$$N(\gamma_n)=\int_0^b Y^2(\gamma_n,y)dy \qquad (15)$$

Actually, formula (13) can be seen as any arbitrary function $f^*(x)$ defined at interval $0<x<\infty$ which is expressed by the solution $X(\beta,x)$ of auxiliary problem (7)-(8). Based on the method in references [18-19], the unknown coefficient $c_n(\beta)$ can be obtained as:

$$c_n(\beta)=\frac{1}{N(\beta)}\int_{x=0}^{\infty}X(\beta,x)\cdot[\frac{1}{N(\gamma_n)}\int_{y=0}^{b}Y(\gamma_n,y)F(x,y)dy]dx \tag{16}$$

Substituting formula (16) into formula (7), and the solution of will be obtained as:

$$p(x,y,t)=\sum_{n=1}^{\infty}\int_{\beta=0}^{\infty}e^{-\eta(\beta^2+\gamma_n{}^2)t}\frac{1}{N(\beta)N(\gamma_n)}X(\beta,x)Y(\gamma_n,y)$$
$$\cdot[\int_{x'=0}^{\infty}\int_{y'=0}^{b}X(\beta,x')Y(\gamma_n,y')\cdot F(x',y')dx'dy']d\beta \tag{17}$$

As for the separation variable y, the eigenfunction $Y(\gamma_n,y)$, norm $N(\gamma_n)$ and eigenvalue γ_n, can be calculated out as:

$$Y(\gamma_n,y)=\sin\gamma_n(b-y) \tag{18}$$

$$\frac{1}{N(\gamma_n)}=2\frac{\gamma_n{}^2+h_1{}^2}{b(\gamma_n{}^2+h_1{}^2)h_1} \tag{19}$$

and γ_n is the positive root of equation below:

$$\gamma_n\cot\gamma_n b=-h_1 \tag{20}$$

The function $X(\beta,x)$, norm $N(\beta)$ can be calculated out as:

$$X(\beta,x)=\sin\beta x \tag{21}$$

$$\frac{1}{N(\beta)}=\frac{2}{\pi} \tag{22}$$

Substituting formula (18), (19), (21), and (22) into formula (17), and change the sequence of integral, the following expression will be obtained:

$$p(x,y,t)=\frac{4}{\pi}\sum_{n=1}^{\infty}e^{-\eta\gamma_n{}^2t}\cdot\frac{\gamma_n{}^2+h_1{}^2}{b(\gamma_n{}^2+h_1{}^2)+h_1}\sin\gamma_n(b-y)$$
$$\cdot[\int_{x'=0}^{\infty}\int_{y'=0}^{b}F(x',y')\cdot\sin\gamma_n(b-y')dx'dy'] \tag{23}$$
$$\cdot\int_{\beta=0}^{\infty}e^{-\eta\beta^2t}\sin\beta x\sin\beta x'd\beta$$

Among them,

$$\sin\beta x\sin\beta x'=\frac{1}{2}[\cos\beta(x-x')-\cos\beta(x+x')] \tag{24}$$

$$\int_{\beta=0}^{\infty}e^{-\eta\beta^2t}\cos\beta(x-x')d\beta=\sqrt{\frac{\pi}{4\eta t}}\exp[-\frac{(x-x')^2}{4\eta t}] \tag{25}$$

$$\int_{\beta=0}^{\infty}e^{-\eta\beta^2t}\cos\beta(x+x')d\beta=\sqrt{\frac{\pi}{4\eta t}}\exp[-\frac{(x+x')^2}{4\eta t}] \tag{26}$$

So there is:

$$\int_{\beta=0}^{\infty}e^{-\eta\beta^2t}\sin\beta x\sin\beta x'd\beta=\frac{\pi}{4\sqrt{\pi\eta t}}[\exp(-\frac{(x-x')^2}{4\eta t})-\exp(-\frac{(x+x')^2}{4\eta t})] \tag{27}$$

Substituting formula (27) into formula (23), the final results will be obtained:

$$p(x,y,t)=\frac{1}{\sqrt{\pi\eta t}}\sum_{n=1}^{\infty}e^{-\eta\gamma_n{}^2 t}\cdot\frac{\gamma_n{}^2+h_1{}^2}{b(\gamma_n{}^2+h_1{}^2)+h_1}\sin\gamma_n(b-y)$$
$$\cdot\int_{x'=0}^{\infty}\int_{y'=0}^{b}F(x',y')\cdot\sin\gamma_n(b-y') \quad (28)$$
$$\cdot[\exp(-\frac{(x-x')^2}{4\eta t})-\exp(-\frac{(x+x')^2}{4\eta t})]dx'dy'$$

Conclusion

As a fundamental solution method, variable separation method can be a good tool to solve underground oil seepage problems in reservoir. Concerning the seepage problem in semi-infinite rectangular regions, the mathematical model with partial differential equations is first established. The complete solution is finally obtained with the method of variable separation. Since the region is infinite in one direction, the superposition of corresponding fundamental solutions is done through the operation of integration. This is different with the disposal of seepage problems in finite regions. Study in this paper can be good illustration for the effectiveness of variable separation method to solving seepage problems in oil reservoir rocks.

Acknowledgment

This work is supported by Science and Technology Project of Hebei Education Department (No.: QN2018158); and is supported by Science and Technology Fund of Hebei Normal University (No.: L2017B21).

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