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Evaluation of Robust Linear Regression Methods for the Measurement of a Topographic Altimetric Network

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Abstract This work presents the comparison of different adjustment methods for the analysis of observations made to provide heights to a topographic altimetric network. The methods evaluated were the traditional Weighted Least Squares, compared with the results obtained using two other alternative adjustment methods, these being the Robust Regression Methods called M-Estimators and MM-Estimators. The incidence of outliers was assessed with all three methods. It was found that the traditional method presents good results in the range of outliers present. Since the observations and the parameters are functionally related by geometric conditions, the presence of an outlier that distorts a fit is highly unlikely. According to the comparison of the results, under the mentioned conditions, Weighted Least Squares can be considered as reliable, in this case, although the other two methods could provide useful information for the analysis of the results obtained.

Keywords regression methods, ordinary least squares, outliers, robust regression

1. Introduction

The adjustment of topographic altimetric networks is carried out in an extended way by the method of Weighted Least Squares (WLS). For the weighting of each height difference of a topographic altimetric network, a weight of 1 / L is assigned for the adjustment, where L is the length of the path necessary to obtain each measurement. Height differences are the response variables, and the predictor variables are coefficients -1,0,1 because it is a network of Graphs. The topographic altimetric model is valid to determine the differences in heights between points and to be able to solve the direction of fluid runoff in the environment of works at the municipal level. In Geodesy the topographic model is called the System of Geometric Heights. In order to establish the heights of physical marks expressly placed for this purpose, and known as benchmarks, measurements of height differences between them are made. The difference in level between two benchmarks A and B is obtained by adding the individual levels measured along the route or path required to get from one to the other. These height differences obtained will be subjected to an adjustment by WLS to save the inconsistencies due to the random factors that are always present in the measurements. The estimation by WLS allows obtaining indicators of the quality of the work carried out. Measurements were made on the campus of the Faculty of Engineering of the National University of La Plata (UNLP) where there is an altimetric network with benchmarks distributed in almost all the buildings of the different Departments (Figure 1). The aforementioned network shares the same origin or datum of the tide gauge zero in the city of Mar del Plata, Argentina.

Figure 1: Scheme of the altimetric network (Faculty of Engineering campus, UNLP)

Regression analysis is one of the most widely used statistical techniques, within it, the classical Ordinary Least Squares (OLS) method is considered a not robust method of estimating parameters when the observations do not come from a Normal distribution or there are atypical observations. These atypical observations can be caused by rare events or could be the results of a factor not yet considered in a study. Data collected frequently contain one or more atypical observations, known as *outliers* that is, observations that are well separated from the majority of the data, or in some way deviate from the general pattern of the data. There robust regression parameter estimates that provide a good fit of the data when the data contains outliers, else some bias could even be present. There are circumstances in which data can be justifiably removed, but in general, given that there are observations that are not necessarily "bad", it is reasonable to conclude that they should not be excluded. To evaluate the behavior in this situation, a comparison by two robust estimation methods is introduced.

A quantitative measure of the robustness of an estimator, proposed by Donoho and Huber [1], is the finite breakdown point. The finite breakdown point is the smallest fraction of anomalous data that can make the estimator useless. This value can be used as a measure of the robustness of the estimator. If the finite breakdown point for a sample of size n is 1/n, it is equivalent to saying that a single observation can distort the estimator. The finite breaking point of the least squares estimators is 1/n. This has a potentially serious impact on its practical use. When the observations come from a Normal Distribution and there are no outliers, it is correct, as well as safe, to use OLS estimators. The breakdown point is usually expressed in percentage. OLS is 0% which means that one outlier is sufficient to distort the estimation. In addition to the breakdown point, to characterize the robust estimator, we could measure their asymptotic efficiency, which is defined as the rate asymptotic between the asymptotic residual mean square obtained with the OLS and the asymptotic residual mean square obtained with the robust procedure. It is expected that this efficiency measure should be close to 1. For a regression estimator to be robust, it must have high breakdown point and high asymptotic relative efficiency. As it was said the robustness of an estimator is measured by its stability when a small fraction of the observations are arbitrarily replaced by outliers that may not meet the assumed statistical model.

2. Materials and Methods

The classical linear model relates the dependent variables, or responses y_i, with the dependent or explanatory variables $x_{i1}, x_{i2} \ldots\ldots x_{ip}$ for $i = 1,2 \ldots.n$, such

$$y_i = x_i^t \beta + \varepsilon_i \quad i = 1,2 \ldots.n \qquad (1)$$

Where,

$$x_i^t = (x_{i1}, x_{i2} \ldots\ldots x_{ip}), \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \beta_p \end{pmatrix} \qquad (2)$$

And ε_i is the error term, a random variable belonging to a Normal Distribution with mean 0 y variance σ^2. It is necessary to fix $x_{i1} = 1$ for all i so that the first element of β if the linear model include an intercept. In this case we will work without the intercept since we have the information of the Datum or origin of the height references and that allows eliminating the range deficit. The set of all observations together with the β leads to the model of Observation Equations

$$y = X.\beta + \varepsilon \tag{3}$$

Where X is $n\ x\ p$ matrix with x_{ij} elements, vector ε contains errors ε_i and vector y are the observations y_i.

To fit that model to the data, one must use a regression estimator and then estimate the unknown parameters of β, which are denoted by $\hat{\beta}$.

$$\hat{\beta} = \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \cdot \\ \cdot \\ \hat{\beta}_p \end{pmatrix} \tag{4}$$

The fitted value of y_i is $\hat{y}_i = x_i^t \hat{\beta}$and this value can be used to calculate the i-th residual $r_i = y_i - \hat{y}_i$. In the OLS method the value $\hat{\beta}_{MC}$results from minimizing the sum of the squares of the residuals $\sum_{i=1}^{n} r_i^2$.

Although the OLS estimator of $\beta$is easy accessible to calculate, it is also very sensitive to deviations from the model assumptions. Observations that are far from most of the data can drastically affect the estimation result, these values are called outliers. An outlier in the case of a regression is a y_i value that deviates from the linear relationship followed by most of the data (vertical outliers). Another type of atypical data, in the regressions, is that which is far from the set of most of the explanatory variables of the model (horizontal outliers or leverage points). It should be remembered that the explanatory variables consist of -1, 0 and 1 because it is the modeling of a network of Graphs [2], which prevents us from encountering this type of atypical values, saving the case of a mistake. The main objective of robust statistics is to provide methods for data analysis that are reliable even in the presence of outliers. Another objective is to obtain robust estimators that are almost as good as OLS when there are not outliers. A robust estimation method in linear models is given by M-estimation. The concept of an M-estimator for a linear regression model was introduced by Huber [3].

An M-regression estimator of β is defined as the $\hat{\beta}_M$, such that it minimizes.

$$\sum_{i=1}^{n} \rho\left(\frac{y_i - x_i^t \beta}{\hat{\sigma}}\right) \tag{5}$$

We state the following definition of a *ρ-function*: is a continuous, symmetric function called loss function, with a unique minimum at 0 [4, 5]. $\hat{\sigma}$ is a scale estimator of the residuals that can be estimated before or simultaneously. $\hat{\sigma}$ could be the median of the absolute value of the residuals from some initial residual estimator:

$$\hat{\sigma} = \frac{1}{0675} Med_i(|r_i|/r_i \neq o) \tag{6}$$

Not using the scale estimator $\hat{\sigma}$ in (5), it is the same as using the notation to substitute $\hat{\sigma}$ for 1. The fact of using the scale estimator is important since the M-estimator is not necessarily invariant with respect to changes in scale (that is, if the errors $y_i - x_i^t \beta$ were multiplied by a constant, the new solution to the equation might not be the same as the previous one). The OLS estimator is a special less robust case of M-estimators, with loss function $\rho(x) = x^2$. The OLS vulnerability comes from the greater weight that is given to extreme or outlier values by squaring the residuals to be minimized. In the case of the regression M-estimators proposed by Huber [3], the loss function is defined as follows:

$$\rho(x) = \begin{cases} x^2, & |x| < a \\ 2|x|a - a^2, & |x| \geq a \end{cases} \tag{7}$$

Uses M-estimator with a Huber loss function setting $a = 1.345$ such an M-estimator has high asymptotic relative efficiency very close to 0.95 [6].Huber's M estimator is robust to extreme values in the y-direction (vertical outliers) but is not robust to extreme values in the x-direction (horizontal outliers). When the variance of the errors ε_i is not the same for all *i*, the M estimators are more efficient than the OLS.

Another choice of ρ- *function* is the *Tukey bisquare* (also called *biweight*) family of functions:

$$\rho(x) = \begin{cases} 1 - \left(1 - \left(\frac{x}{C}\right)^2\right)^3, |x| \leq C \\ 0, |x| > C \end{cases} \quad (8)$$

The constant C is an adjustment constant, in the case of the present work $C = 1.345$, produces an M-estimator of β more resistant to regression outliers than the Huber M-estimator and the relative asymptotic efficiency is found to be 0.95 [4].

The MM-estimators are a special type of M estimator and was proposed by Yohai [7]. They combine the high asymptotic relative efficiency of M-estimators with the high breakdown point of a class of estimators called S-estimators, and they are based on a ρ- function that determines the robust properties of the estimator [6]. A MM-regression estimator of β is defined as the $\hat{\beta}_{MM}$, such that it minimizes:

$$\sum_{i=1}^{n} \rho(\frac{y_i - x_i^t \beta}{\hat{\sigma}}) \quad (9)$$

Where $\hat{\sigma}$ is a robust scale S- estimator introduced by Rousseauw and Yohai [8]. We describe the stages that define an S- estimator (namely $\hat{\sigma}$) like: A high breakdown estimator is used to an initial estimate of β, which is denoted $\hat{\beta}$. The estimator need not be efficient. Using $\hat{\beta}$ estimate the residual $r_i = y_i - \hat{y}_i$ After a M-estimate of scale, $\hat{\sigma}$, can be computed by:

$$\frac{1}{n} \sum_{i=1}^{n} \rho_1 (\frac{y_i - x_i^t \beta}{\hat{\sigma}}) = \frac{1}{2} \quad (10)$$

Where, ρ_1 is a ρ- *function*. Uses an S-estimator with a Tukey bisquare loss function setting C = 1.547 such an S-estimator has a breakdown point very close to 50 %.

The regression MM-estimator off β is now defined as the $\hat{\beta}_{MM}$, such that it minimizes $\sum_{i=1}^{n} \rho(\frac{y_i - x_i^t \beta}{\hat{\sigma}})$ and the scale estimate $\hat{\sigma}$ obtained from the scale S- estimator. The objective function ρ_1 associated with this ρ- function does not have to be the same as $\rho_1$but, it must satisfy $\rho(u) \leq \rho_1(u)$.

2.1. Weighted Least Squares

The classical linear model assumes that ε_i, the error term of (1), is a random variable, in our case of Normal Distribution with variance σ^2 and zero mean. The assumption that the variance of $\varepsilon_i$is constant is not always true, many times the variance is of the form $\sigma_i^2 = \frac{\sigma^2}{W_i}$, where for each *i*, w_i is a positive number. The fact that the variance of ε_i is constant is a strong assumption, in that case the model is said to be homoskedastic, otherwise it is said to be heteroskedastic. In the latter case, Weighted Least Squares (WLS) are used to estimate β. If the estimator of β by WLS is denoted by$\hat{\beta}_{WLS}$, then $\hat{\beta}_{WLS}$ is the estimator that minimizes the sum of the squares of the weighted residuals:

$$\sum_{i=1}^{n} w_i (y_i - x_i^t \beta)^2 \quad (11)$$

The use of the weighted residual sum of squares recognizes that some of the errors are more variable than others, since cases with large values of w_i will have small variances and therefore will be given more weight in the sum of the squares of the residues [9]. In other words, the WLS method is useful in the presence of heteroskedasticity. Also, the fit model WLS is treated like the ordinary fit OLS, if we write equation (8) as:

$$\sum_{i=1}^{n} (\tilde{y}_i - \tilde{x}_i^t \beta)^2 \quad (12)$$

With $\tilde{y}_i = \sqrt{w_i} y_i$ and $\tilde{x}_i^t = \sqrt{w_i} x_i^t$

In the case of fitting the measurements of a topographic altimetry network with OLS, the variance of $\varepsilon_i$is of the form $\sigma_i^2 = \frac{\sigma^2}{W_i}$ with $w_i = \frac{1}{L_i}$ where L_i is the length of travel required to obtain each measurement. Therefore, WLS is applied to correct heteroskedasticity. But with this, the influence of outliers if any is not resolved.

3. Measurement and adjustment

3.1. Elevation differences measurements

The surveying of the response variables, the height differences, was carried out by means of the geometric leveling method from the middle [10] with automatic levels of 28 magnifications. The measured elevation differences can be seen in Table 1 column 2.

The network consists of 8 benchmarks like those in figure 1 and they have been given the following nomenclature: Agrimensura (AV), Agrimensura Nueva (AV), Partenon (P), Química 1 (Q1), Química 2 (Q2), Hidráulica (H), Decanato (D), Constructiones (C). These benchmarks will be the physical support of the heights of equipotential surfaces whose value will be the result of the adjustment of the unevenness observations made. Agrimensura Vieja (AV) served to establish the reference datum. In Table 1, column 1, the ends of each measured section are shown. The 7x1 vector $\boldsymbol{\beta}$ that has the heights to be adjusted is

$$\beta = \begin{pmatrix} AN \\ Q_1 \\ D \\ Q_2 \\ H \\ P \\ C \end{pmatrix}$$

Table 1: Height differences (m) with their cofactors

Path	Height differences (m)	Cofactor $\boldsymbol{Q_i}$
AV-	2.046	0.14
AN-	1.110	0.15
Q1-D	0.112	0.08
D-Q2	0.381	0.13
H-P	-1.615	0.20
Q2-C	-0.920	0.10
H-AN	0.166	0.10
Q1-C	-0.435	0.10
AV-P	0.267	0.20
H-C	0.842	0.15
AV-	-2.046	0.08
AN-	-1.110	0.13
Q1-D	-0.111	0.08
D-Q2	-0.375	0.13
H-P	1.616	0.20
Q2-C	0.920	0.10
H-AN	-0.167	0.10
Q1-C	0.436	0.10
AV-P	-0.265	0.20
H-C	-0.842	0.15

The variance of each of the observations is directly proportional to the length of the path necessary to obtain them [11]. Although statistically they are independent. let us remember that functionally they are not. This is because they must meet closing conditions for being part of a network of Oriented Graphs [2]. Consequently the sum of the residuals will be different from zero. For this circumstance too the entry of notable outliers can be inspected before adjusting.

The conventional adjustment of a height network is carried out by WLS. With this adjustment we will seek to explain the observations y_i with the values of the adjusted bounds $\hat{\beta}_{WLS}$ through the expression (12). The weight

w_i is obtained from the inverse of the cofactors Q_i where Q_i is the length of travel required to obtain each measurement (See Table1column 3).The parameters $\hat{\beta}_{WLS}$ of the WLS adjustment are in Table 2.

3.2. Outlier detection

To identify possible outliers in the WLS adjustment. The residuals were used:

$$r_i = y_i - \widehat{y_i} \quad (13)$$

Where, y_i are the measured slopes, and $\hat{y}_i$, are the adjusted values of said slopes by WLS. There were also calculated the external studentized residuals $t_{(i)}$ or residuals by the Leave-One-Out method [12] where the influence of an atypical data y_i on the residuals is measured. Eliminating this observation from the model and so defining the remainder leave-one–out $r_{(i)}$ computing:

$$r_{(i)} = y_i - \hat{y}_{(i)} \quad (14)$$

Where $\hat{y}_{(i)}$ is the adjusted slope without taking into account the vector x_i and the measured value y_i

$$\hat{y}_{(i)} = x_i^t \hat{\beta}_{(i)} \quad (15)$$

It can be shown that:

$$r_{(i)} = \frac{r_i}{1-v_{ii}} \quad (16)$$

Where v_{ii} are the elements from the diagonal of the matrix H defined as:

$$H = X(X^tX)^{-1}X^t \quad (17)$$

The matrix H is the so-called *Hat Matrix*.

Then the studentized form for $r_{(i)}$ is $t_{(i)}$:

$$t_{(i)} = \sqrt{1-v_{ii}}\frac{r_{(i)}}{S_{R(i)}} = \frac{r_i}{S_{R(i)}\sqrt{1-v_{ii}}} i=1\ldots n \quad (18)$$

Where r_i is the residual of the *i-th* observation and $S_{R(i)}^2$ is the estimate of the residual variance without taking into account the i-th observation in the regression model.

$$S_{R(i)}^2 = \frac{(n-p)S_R^2 - r_i^2/\sqrt{1-v_{ii}}}{n-p-1} \quad (19)$$

Whit S_R^2 as the assessed variance of residuals involving all observations

$$S_R^2 = \frac{\sum_{i=1}^n r_i^2}{n-p} \quad (20)$$

The studentized residuals $\boldsymbol{t_{(i)}}$ have a *t de Student* distribution with *n-p-1* degrees of freedom.

Finally, we have $t_{Max} = \max_{1\le i\le n} t_{(i)}$. For a significance level α we will say that the observation corresponding to the maximum studentized residual is atypical if:

$$|t_{Max}| > t_{n-p-1}^{1-\alpha/2} \quad (21)$$

Where $t_{n-p-1}^{1-\alpha/2}$ is the $\left(1-\frac{\alpha}{2}\right)x100\%$ percentile from a T de Student with *n-p-1* degrees of freedom.

This network resulted in a variance of the fit of $S_R^2 = 0.00002304$m².For a value $\alpha = 0.05$ the percentile $\left(1-\frac{\alpha}{2}\right)x100\%$ from a T de Student distribution with *n-p-1=12* degrees of freedom is $t_{n-p-1}^{1-\alpha/2} = 2.18$ y $|t_{Max}| = 7.0212768$ (See Table 2). Therefore it is considered to be an outlier.

Table 2: Observations. raw and external studentized residuals.

DH measured	DH adjusted WLS	r_i	$t_{(i)}$
2.046	2.046423	-0.0004231	-0.2946033
1.110	1.1100164	-0.0000164	-0.0103331
0.112	0.1105502	0.0014498	1.4412095
0.381	0.3764464	0.0045536	7.0212768
-1.615	-1.6148721	-0.0001279	-0.0699119
-0.920	-0.9212172	0.0012172	1.0040679

0.166	0.1661877	-0.0001877	-0.1433856
-0.435	-0.4342206	-0.0007794	-0.5861182
0.267	0.265363	0.0016367	0.9170405
0.842	0.8419835	0.0000165	0.0103408
-2.046	-2.046423	0.0004231	0.2946033
-1.110	-1.1100164	0.0000164	0.0103331
-0.111	-0.1105502	-0.0004498	-0.4158049
-0.375	-0.3764464	0.0014464	1.0294347
1.616	1.6148721	0.0011279	0.6262164
0.920	0.9212172	-0.0012172	-1.0040679
-0.167	-0.1661877	-0.0008123	-0.6300562
0.436	0.4342206	0.0017794	1.4269477
-0.265	-0.265363	0.0003633	0.1971074
-0.842	-0.8419835	-0.0000165	-0.0103408

The residuals $\boldsymbol{t}_{(i)}$ with external studentization or Leave-One-Out can only be determined in the case of a network with duplicate observations.since if being the network of simple measurements it is impossible to extract an observation since the section of the network containing that observation would disappear.

3.3. Adjustment by M and MM estimators

To apply the WLS, M and MM estimators the MASS package [13] was used for Language R. R is an environment and programming language with a focus on statistical analysis. R was born as a free software implementation of the S language. and is part of the GNU system and is distributed under the GNU GPL license. It is available for Windows. Macintosh. Unix. and GNU / Linux operating systems. It was initially developed by Robert Gentleman and Ross Ihaka of the Department of Statistics at the University of Auckland in 1993.

The following functions respectively perform the adjustments for WLS. estimator M and MM

lm (formula = y ~x - 1. weights = w)
rlm (formula= y ~ x-1. weights = w)
rlm (formula = y ~ x - 1. weights = w. method = c("MM"))

In all functions the value -1 is used to extract the independent term from the classical regression model. The weights are introduced with "*weights = w*", remembering that *w* is a vector that is formed by the inverse of each cofactor Q_i. The values of Q_i, corresponding to each measured height difference are in the third column of Table 1.

4. Results and Discussion

Figure 2 shows a graph of residuals $r_i = y_i - \hat{y}_i$ versus the values of the adjusted height differences corresponding to the three methods. An extreme residue for WLS is also highlighted in red and the extreme residues of the two robust methods highlighted in light blue all corresponding to the same adjusted value. In the plot the residual corresponding to this value is lower for WLS than in the robust methods (marked in light blue). This is because the outlier distorts the regression fit by WLS in its direction. The robust procedure tends to leave the residuals associated with outliers large thus facilitating identification.

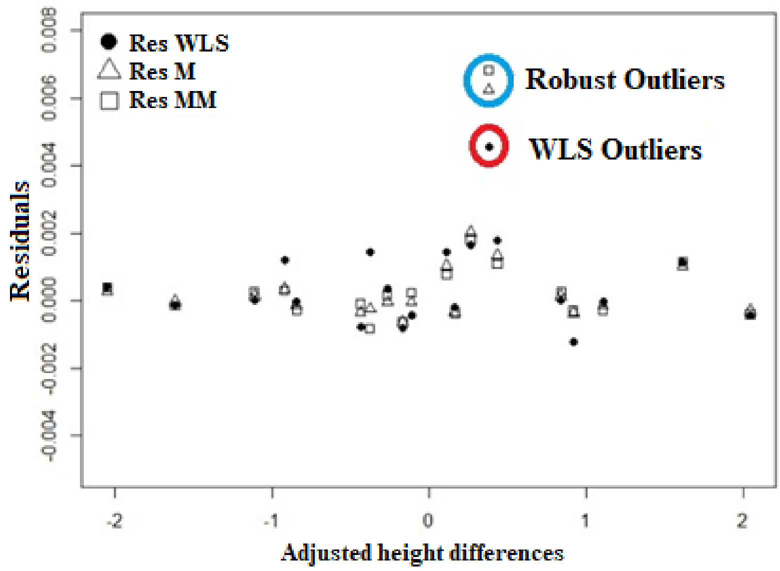

Figure 2: Residuals vs. Adjusted Height Differences

The robust estimation produces, in this case, almost the same adjusted values of the parameters obtained by the WLS method since the residuals have a Normal distribution, and the outliers are not influential. Table 3 shows the values of the residuals $r_i = y_i - \hat{y}_i$ for the three adjustment methods WLS, M-estimator with the Huber loss function and MM-estimator with the Tukey bisquare loss function. In Table 4 the values of the adjusted levels (also for the three methods) in which a very little difference in some cases.

Table 3: Adjusted Heights by all three methods

Benchmark	WLS	M	MM
AN	17.960423	17.960300	17.960000
Q1	19.070440	19.070400	19.070000
D	19.180990	19.181400	19.175000
Q2	19.557436	19.556100	19.556000
H	17.794235	17.793900	17.794000
P	16.179363	16.179000	16.181000
C	18.636219	18.635800	18.636000

Table 4: Observational residual for each method

Observations	WLS	M	MM
AV-AN	-0.0004	-0.0004	-0.0004
AN-Q1	0.0000	-0.0002	-0.0003
Q1-D	0.0014	0.0010	0.0008
D-Q2	0.0046	0.0062	0.0068
H-P	-0.0001	0.0000	-0.0001
Q2-C	0.0012	0.0004	0.0003
H-AN	-0.0002	-0.0004	-0.0004
Q1-C	-0.0008	-0.0004	-0.0001
AV-P	0.0016	0.0020	0.0018
H-C	0.0000	0.0002	0.0003
AV-AN	0.0004	0.0003	0.0004
AN-Q1	0.0000	0.0002	0.0004

Q1-D	-0.0004	0.0000	0.0002
D-Q2	0.0014	-0.0002	-0.0008
H-P	0.0011	0.0010	0.0011
Q2-C	-0.0012	-0.0004	-0.0003
H-AN	-0.0008	-0.0006	-0.0006
Q1-C	0.0018	0.0014	0.0011
AV-P	0.0004	0.0000	0.00018
H-C	0.0000	-0.0002	-0.0003

5. Conclusions

The study of outliers should include strategies both for their detection and for measuring their influence on the estimated parameters.

We consider the comparison between the WLS, M and MM estimators appropriate given the current computational possibility with software such as R. The WLS method allowed us to deal with the heteroskedasticity. We consider that adding the information of the M and MM estimators to the adjustments made by WLS will provide more elements of analysis to the professional, allowing them to study the relevance or not of the observations that represent outliers in the conventional adjustment. Like the WLS estimator, M-estimators assuming that there are no outliers in the independent variable x_i. The M estimation introduced by Huber [3] is a simpler approach than the MM adjustment. Although it is not robust in outlier observations in the direction of X, it is widely used in data analysis, when it can be assumed that the outlier contamination is mainly in the direction of the response.

Robust regression methods are a great help in dealing with highly influential outliers. Robust analysis can be used as confirmation of WLS. Whenever a WLS is done, it would be appropriate to do a robust fit as well. If they agree, or the difference is small, the results of the WLS estimators should be used. However, if they differ, the reasons for such differences should be identified.

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