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Research Article

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Study on Integral Solution of Steady Flow for Oil in Underground Formation

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Abstract To enhance the study on basic problems of oil and gas flow, it is necessary to introduce the integral equation method. Take the steady flow with source as an example, the accurate solution is firstly obtained by using the variable separation method. After that, the first and second grade approximate solution are obtained by introducing the integral equation method. Comparison of approximate and accurate solution shows that the second grade approximate solution can reduce the deviation to very little extent which shows the effectiveness of this integral equation method. This method can be good references for the solution of more and more arising complicated flow problems, especially for the application of boundary element methods and so on to solve these problems.

Keywords linear flow, steady flow, approximate solution, analytical solution, integral solution

1. Introduction

The theory of integral equations develops along with that of mathematical physics. In 1782, Laplace put forward that the substance of inverse transform for Laplace transform is actually the solution of an integral equation [1]. Nowadays, the underground seepage problems for oil or gas development are becoming more and more complicated, it is essential to strengthen the study on basic solution methods. Specially, it is necessary to study the application of integral equations for use in the area of underground oil or gas seepage problems. Differential equations and integral equations are both important tools for studying oil seepage problems, and they have their own advantages respectively: if the seepage problem is studied in terms of "flow field", the disposal by differential equation will become more convenient; if the seepage problem is studied concerning with the "flow source", the disposal of integral equations will have relatively more convenience. When the differential equations in the region are transformed into integral equations on the boundary, the dimension of equation will decrease and the amount of calculation will become less. Furthermore, integral equations are also one of the foundation of boundary element methods, the study on integral equations can also be good reference for broad application of boundary element methods in oil and gas developments. Actually, many scholars have applied boundary element method into the oil and gas seepage problems, and many good results have been obtained [2-6].

Concerning the steady seepage problem with flow source in a two dimensional rectangular region, the theory of integral equations is applied, and the pressure solutions with different approximation degree are obtained. Furthermore, these approximate solutions are compared with the accurate solution and the effectiveness of the integral equation method in solving underground seepage problems is verified. Study can be good reference for solution of more complicated seepage problems, especially for the vast application of related boundary element methods to solve underground oil or gas seepage problems.

Mathematical Model Establishment for Seepage Problem

As demonstrated in figure 1 below, take the steady seepage in a two dimensional rectangular region with flow source as an example, suppose the boundary $x=0$ and the boundary $y=0$ is impermeable, and suppose the boundary $x=L$ and the boundary $y=b$ have constant pressure as the same with initial pressure, suppose the initial pressure is p_i. There is flow source in the region with average strength q, and the target is to find the pressure distribution with time and space $p(x,y,t)$.

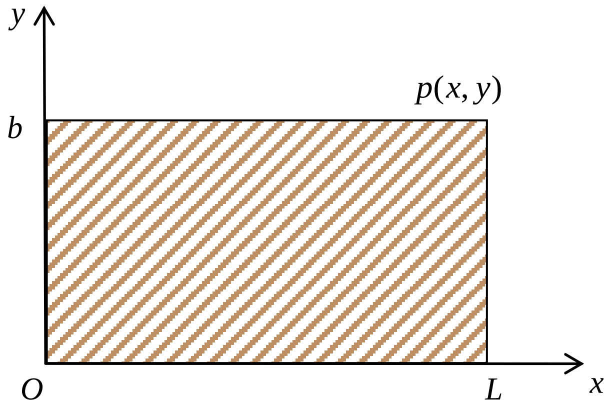

Figure 1: Sketch map of rectangular seepage area with flow source

Based on the principles of model establishment for flow in porous media, in rectangular coordinate system, the differential equations for the steady seepage problem with flow source can be expressed as below [7-8]:

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{q}{\lambda}=0 \tag{1}$$

As for the boundary conditions, there are:

$$\left.\frac{\partial p}{\partial x}\right|_{x=0}=0 \tag{2}$$

$$\left.\frac{\partial p}{\partial y}\right|_{y=0}=0 \tag{3}$$

$$p\big|_{x=L}=p_i \tag{4}$$

$$p\big|_{y=b}=p_i \tag{5}$$

Solution of Seepage Problem by Variable Separation Method

The problem (1)-(5) is not easy to be solved because the boundary conditions (4) and (5) are non-homogeneous. In order to make boundary conditions homogeneous, another variable $\theta(x,y,t)=p(x,y,t)-p_i$ is introduced, and it can be seen as the relative change of pressure. Obviously, $\theta(x,y,t)$ is also a function of time and space. Therefore, problem (1)-(5) can be transformed into another solution determination problem concerning $\theta(x,y,t)$:

$$\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}+\frac{q}{\lambda}=0 \tag{6}$$

$$\left.\frac{\partial\theta}{\partial x}\right|_{x=0}=0 \tag{7}$$

$$\left.\frac{\partial\theta}{\partial y}\right|_{y=0}=0 \tag{8}$$

$$\left.\theta\right|_{x=L}=0 \tag{9}$$

$$\left.\theta\right|_{y=b}=0 \tag{10}$$

For problem (6)-(10), the boundary conditions become homogeneous, but equation (6) in this problem is non-homogeneous. To solve this kind of problem, a special solution of equation (6) has to be found first, and at the same time, the problem (6)-(10) can be converted into a new problem with homogeneous equation and only one non-homogeneous boundary condition. After that, the solution for the new problem can be obtained using traditional variable separation method. To get the solution of problem (6)-(10), just add the solution above to the special solution of equation (6) [9-10], and the final solution will be obtained as:

$$\theta(x,y)=\sum_{m=1}^{\infty}c_m\cosh(\beta_m y)\cos(\beta_m x)+\frac{q(L^2-x^2)}{2\lambda} \tag{11}$$

Among them, β_m is the positive root of equation $\cos\beta_m L=0$. There also is:

$$c_m=\frac{2}{L\cosh(\beta_m b)}\int_0^L\frac{q(x^2-L^2)}{2\lambda}\cos(\beta_m x)dx \tag{12}$$

Solution of Seepage Problem by Integral Equation Method

Considering to solve the problem using integral equation method, integrate each side of seepage equation (1), the following equation will be obtained:

$$\int_0^L\left(\left.\frac{\partial p}{\partial y}\right|_{y=b}-\left.\frac{\partial p}{\partial y}\right|_{y=0}\right)dx+\int_0^b\left(\left.\frac{\partial p}{\partial x}\right|_{x=L}-\left.\frac{\partial p}{\partial x}\right|_{x=0}\right)dy+\int_0^L\int_0^b\frac{q}{\lambda}dxdy=0 \tag{13}$$

In combination with boundary conditions formula (2) and formula (3), the following integral equation will be obtained:

$$\int_0^L\left.\frac{\partial p}{\partial y}\right|_{y=b}dx+\int_0^b\left.\frac{\partial p}{\partial x}\right|_{x=L}dy+\frac{q}{\lambda}Lb=0 \tag{14}$$

In order to solve the solution determination problem, suppose the relative pressure variation is $\theta(x,y)$, and do the separation for $\theta(x,y)$ as:

$$\theta(x,y)=X(x)\cdot Y(y) \tag{15}$$

Among them, $X(x)$ and $Y(y)$ are functions of only one variable respectively. Based on the distribution of pressure, the expression formula of $X(x)$ and $Y(y)$ are at least quadratic polynomial, namely:

$$X(x)=a_0+a_1x+a_2x^2 \tag{16}$$

$$Y(y)=b_0+b_1y+b_2y^2 \tag{17}$$

Make use of the boundary conditions (7)-(10), substitute the corresponding values of x and y, and the undetermined coefficients can be obtained: $a_1 = b_1 = 0$, $a_0 = -a_2 L^2$, $b_0 = -b_2 b^2$. Substitute these coefficients into formula (16) and formula (17), there will be:

$$\theta(x, y) = c_0 (L^2 - x^2)(b^2 - y^2) \quad (18)$$

Among them, $c_0 = a_2 b_2$ 。

Substitute formula(18) into integral equation(14), there will be:

$$-2c_0 b \int_0^L (L^2 - x^2)dx - 2c_0 L \int_0^b (b^2 - y^2)dy + \frac{q}{\lambda} Lb = 0 \quad (19)$$

Finally, the following formula will be obtained:

$$c_0 = \frac{3q}{4\lambda(L^2 + b^2)} \quad (20)$$

Substitute formula(20) into formula(18), the following formula will be obtained:

$$p(x, y) = p_i + \frac{3q}{4\lambda(L^2 + b^2)}(L^2 - x^2)(b^2 - y^2) \quad (21)$$

Therefore, formula (21) is actually the first order integral approximate solution of the seepage problem.

To improve the accuracy of approximate solution, the second order pressure approximate solution can be put into use on basis of formula (18). Generally, in a relatively larger domain, the polynomial with higher order can be better a substitute for the real pressure distribution, therefore, when there is $L > b$, the following polynomial can be settled:

$$\theta(x, y) = (c_0 + c_1 x^2)(L^2 - x^2)(b^2 - y^2) \quad (22)$$

And when there is $L < b$, the following polynomial can be settled:

$$\theta(x, y) = (c_0 + c_1 y^2)(L^2 - x^2)(b^2 - y^2) \quad (23)$$

Without loss of generality, suppose there is always $L > b$ in following discussions. Because formula (22) contains two unknown coefficients c_0 , c_1 , and only one coefficient can be determined by integral equation, another auxiliary condition has to be created. On the other hand, since the settled pressure distribution function can meet boundary conditions, it is not hard to forecast that the largest deviation between approximate solution and real pressure distribution will appear at the coordinate $x = 0$ and $y = 0$, namely the origin of coordinate.

Firstly, suppose the pressure formula (22) can meet the corresponding differential equation (6) at the origin of coordinate (0,0). Therefore, the following formula can be obtained:

$$(L^2 + b^2)c_0 - b^2 L^2 c_1 = \frac{q}{2\lambda} \quad (24)$$

Secondly, substitute formula (22) into seepage integral equation, the following equation can be obtained through operation:

$$(L^2 + b^2)c_0 + L^2(b^2 + \frac{L^2}{5})c_1 = \frac{3q}{4\lambda} \quad (25)$$

Make use of the combination of formula (24) and formula (25), the following coefficients can be obtained:

$$c_1 = \frac{q}{4\lambda b^2 L^2 [2 + 0.2(L/b)^2]} \quad (26)$$

$$c_0 = \frac{q[5 + 0.4(L/b)^2]}{4\lambda b^2 [1 + (L/b)^2][2 + 0.2(L/b)^2]} \quad (27)$$

Substitute formula (26) and formula (27) into formula (22), the second order integral approximate solution of this seepage problem can be obtained.

Results and Discussion

To verify the accuracy of integral solution, without loss of generality, suppose $\lambda = 1$ and $q = 1$ to explore the relative deviation error between accurate solution and approximate solutions. Under the condition of $L > b$, choose different L/b value, calculate the values of θ for accurate solution, first order approximate solution, and second order approximate solution, and the results are shown as that in table 1 below:

Table 1: Accurate and approximate pressure at coordinate (0,0) under different L/b value

L/b	**1**	**2**	**3**
Accurate solution	0.29469	0.45687	0.49807
First order approximate solution	0.375	0.6	0.70584
Second order approximate solution	0.30682	0.47143	0.50921

From the results in table 1, it is easy to see that the first order approximate solution can basically reflect the values of accurate solution, however, there is still relatively large deviation error: with the increase of value L/b from 1 to 3, the deviation error also increases greatly, the percentage increases from 27.3% to 41.7%. Compared with the first order approximate solution, the second order approximate solution is of much higher accuracy. With the increase of L/b value from 1 to 3, the deviation errors from accurate solution become smaller, the percentages range from 4.116% to 2.237%. Therefore, concerning the first order and second order approximate solutions, the value of L/b has different influence trend for the deviation error from accurate solution.

In sum, with the application of integral equation, the approximate solutions of steady seepage for underground oil with flow source are obtained. The effectiveness of integral equation method in solving underground seepage problems are verified. At the same time, accurate solution is compared with approximate solutions. The second order approximate solution can reduce deviation errors to a much smaller extent than first order approximate solution thus improve the accuracy of calculation.

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