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A High-Precision Geoid for Water Resources Management: A Case Study in Menofia Governorate, Egypt

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Abstract Since the mid-1980s, the Global Navigation Satellite Systems (GNSS) have been used as a positioning technique in various water resources management applications in Egypt. However, up till now, there is no published official accurate national geoid model to convert the GNNS-based heights into Mean Sea Level (MSL) elevations. On the other hand, the accuracy level of the global geoid models could not satisfy the requirements of civil engineering projects. Recently, the Survey Research Institute (SRI) has initiated a research project to establish a prototype centimeters-level geoid utilizing heterogeneous geodetic datasets. A new vertical geodetic network has been established in the Menofia governorate using the precise levelling technique, and 47 new Bench Marks (BM) have been established with an average spacing of 4 kilometers apart. The entire network has been positioning by the Global Positioning System (GPS) applying the international geodetic standards and specifications and tied to the national geodetic datum of Egypt. The least-squares adjustment method has been carried out for both networks. The geoidal heights have been achieved with a few centimeters level of precision, and a precise geoid model has been developed for the study area, enabling expanded utilization of GNSS, time reduction, and cost-saving in water resources management projects. It is recommended that the exploited field procedures and processing strategies being followed as national geodetic standards. Additionally, it is highly recommended that a nationwide high-precision geoid model for Egypt has to be established.

Keywords Geoid, GNSS, GPS, MSL, Menofia governorate

Introduction

A geoid model is a three-dimensional (3D) geospatial model that specifies the relationship between the ellipsoid and the geoid surfaces in a given area.

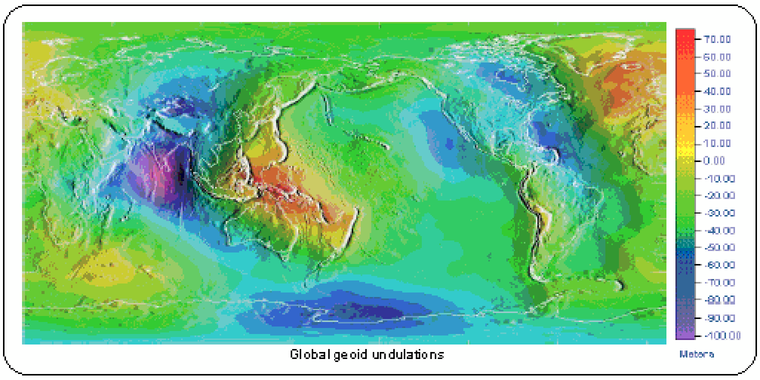

Figure 1: Global Geoid/Ellipsoid Separations

In a sense, geoid models have been one of the main activities of the geodetic community a long time ago. However, a geoid model became an essential task since the spread of Global Navigation Satellite Systems (GNSS) utilizations in the 1980s. Since GNSS-based heights are referred to as the surface of an ellipsoid, a geoid model converts them to the MSL-based orthometric elevations used in civil engineering applications. On a global basis, the separations between the geoid and ellipsoid surfaces are in the range of ± 100 meters as shown (Figure 1).

Geoid models are developed on a global, regional, or national basis. For civil engineering and surveying projects, certain geoid models could be developed at a smaller scale. In the last two decades, several precise geoids have been generated all over the world, e.g. in Italy [1], Brazil [2], Turkey [3], Japan [4], Uganda [5], and Latvia [6].

Since the mid-twentieth century, geoid modelling has been a major concern of Egypt's geodetic and surveying communities. Alnaggar [7] created the first pioneer Egyptian geoid model, and since then, several geoid studies have been conducted. Dawod [8], for example, created a national geoid model based on data from the Egyptian National Standardization Network in 1997 (ENGSN97). In addition, Abd-Elmotaal [9] created a gravimetric geoid model using a high-degree tailored reference geopotential model. Dawod and Abdel-Aziz [10] recently investigated the use of a Geographic Information Systems (GIS) technique in geoid modelling. The vertical geodetic network of Egypt has been established between 1906 and 1945, where thousands of BM lines have been observed to cover the urban areas on a national base. However, it has been found that great damages have occurred to such locations and almost 80% of those BM have been destroyed [11]. SRI has initiated a research study to re-construct and observes the vertical geodetic network in Menofia governorate along with GNSS measurements over BM to develop a prototype high-precision geoid model. This paper discusses such a study in detail and presents the performed processing and adjustments procedures, along with the attained results.

Geoid Modelling

Geoid modelling is performed using several types of geodetic measurements. Gravity observations are applied to develop gravimetric geoid utilizing the well-known Stokes' formula in a spherical harmonic expansion way (e.g. Hofmann and Moritz [12]):

$$N = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g S(\Psi) d_{\sigma} \tag{1}$$

where,N is the geoidal undulation, R is the average radius of the Earth, γ is the normal gravity on the ellipsoid, Δg represents the gravity anomaly, and$S(\Psi)$ is called the Stokes' function computed as:

$$(\Psi) = \frac{1}{\sin\frac{\Psi}{2}} - 6\ \sin\left(\frac{\Psi}{2}\right) + 1 - \cos(\Psi) - 3\ \cos(\Psi) \ln\left[\sin\left(\frac{\Psi}{2}\right) + \sin^2\left(\frac{\Psi}{2}\right)\right] \tag{2}$$

The second method of geoid modelling overcomes the problem of the Stokes' formula that needs gravity measurements all over the Earth to evaluate the double integral in Equation (1). This method concerns the utilization of heterogeneous geodetic datasets to develop a Global
Geopotential or Geoid Model (GGM) as:

$$N = \left(\frac{GM}{r\gamma}\right) \sum_{n=2}^{n\ max} \left(\frac{a}{r}\right)^{n} - \sum_{m=0}^{n} ((\bar{C}_{nm} \cos(m\lambda) + (\bar{S}_{nm} \sin(m\lambda)) P_{nm}\, sin\varphi \tag{3}$$

where: n is the degree of the GGM model, $n\,max$ is the maximum degree of the GGM model, m is the maximum order of the model, γ is the normal gravity of the reference ellipsoid, r is the geocentric radial distance of the computation point projected on the ellipsoid, G is the Newtoniangravitational constant, M is the mass of the Earth, a is the semi-major axis, φ is the geocentriclatitude, $\lambda$is the geocentric longitude, $\bar{C}_{nm}$,$\bar{S}_{nm}$ are the fully normalized harmonic coefficients, and Pnm is the fully normalized associated Legendre polynomial.

GGM development incorporates the integration of heterogeneous geodetic datasets, including terrestrial gravity observations, satellite-based gravity measurements, altimetry measurements, and terrestrial geodetic data. As a result, the coefficients of C's and S's in equation (3) are evaluated based on the required maximum degree and order of the model. GGM models have been developed since the 1960s, and there are about 170 GGM available at the website of the International Center for Global Earth Models (ICGEM) http://icgem.gfz-potsdam.de/home.

Table 1 presents few examples of the most recently released GGM and their characteristics. However, it has been concluded that the precision of utilizing any GGM alone did not reach the two-decimeter level of accuracy (e.g. Al-Krargy [13], Eshagh and Zoghi [14]).

The third geoid modelling strategy breaks the geoid undulation (N) into three components:

$$N = N_{REF} + N_{\Delta g} + N_h \tag{4}$$

where: $N_{\Delta g}$ is the contribution of the reduced gravity anomalies computed by Stokes's integral, N_h is the contribution of the topography, and N_{REF} is the contribution of the reference gravity field from a GGM.

Table 1: Characteristics of Recent Global Geopotential Models

GGM Name	Year	Degree	Data Type
GGM05C	2016	360	S (Goce, Grace, Lageos)
GECO	2015	2190	S (Goce, Grace), G, A
EIGEN-6C4	2014	2190	S (Goce, Grace, Lageos), G, A
EIGEN-6C3stat	2014	1949	S (Goce, Grace, Lageos), G, A
EIGEN-6C	2011	1420	S (Goce, Grace, Lageos), G, A
EGM2008	2008	2190	S (Grace), G, A

where: S = satellite tracking data, G = Gravity data, and A = Altimetry data

This implies that the gravity anomaly $N_{\Delta g}$ is decomposed, too, into three parts:

$$\Delta_g = \Delta_{gREF} + \Delta_{gF} + \Delta_{gh} \tag{5}$$

where: Δ_{gF} represents the residual free-air gravity anomalies, Δ_{gh} is the effect of local topography, and Δ_{gREF} represents the long-wavelength gravity anomalies of a reference gravity field represented by a GGM.

The geometric geoid is the fourth way to develop a geoid model, particularly in small to medium areas. It computes the geoid undulation (N) directly from GPS-based ellipsoidal height (h) and the corresponding orthometric elevation (H) at the same locations as shown in (Figure 2).

$$N = h - H \tag{6}$$

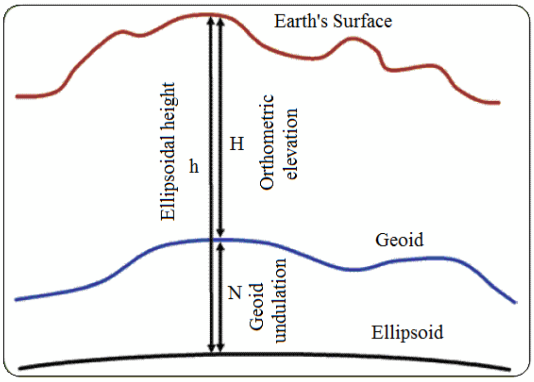

Figure 2: Types of Heights

Due to existing datum inconsistencies, random errors, systematic distortions in the different height datasets in equation (6), a combined least-squares adjustment is needed (e.g. Fotopoulos [15], Kiamehr and Eshagh, [15]). Thus, the last equation is extended, in a matrix form, to:

$$l = Ax + v \tag{7}$$

where, A is the coefficients matrix, x is the vector of unknowns, v is the vector of un-modelled residual effects, and l is the vector of observations: $l = h - H - N$. The solution of equation 7 is obtained as:

$$\hat{x} = (A^T PA)^{-1}\ (A^T PI) \tag{8}$$

where, P represents the weight matrix.

Available Data

SRI has initiated a research study aiming to develop a national high-precision geoid for Egypt for the first time. The Menofia governorate has been selected as the study area that represents a guided primary area in which aprototype geoid will be modelled and, later, could be generalized for the entire Egyptian territories. The study has many objectives including:

- Evaluating the current status of the vertical geodetic (BM) network,
- Designing and establishing a new BM network, applying the international standards and specifications,
- GPS observing of the new BM networks in order to obtain the precise three-dimensional geodetic coordinates,
- Developing and evaluating a high-precision geometric geoid model utilizing the least-squares integrated adjustment technique of all available geodetic observations,
- Building a geospatial database of new geodetic stations using the Geographic Information Systems (GIS).

The study area extends over four urban centres of Menofia governorate; namely: Menouf, Ashmoun, Al-Bagoor, and Sers El-Layan (Figure 3). Its area equals 700 square kilometers approximately. At the first stage of the study, a reconnaissance survey has been carried out. It has been found that only 12 BM (out of 85 BM) exist, which in other words means that the lost BM points constitute about 86%. A new BM network has been designed, established, and observed by the first-order spirit levelling technique fulfilling the international standards and specifications in terms of utilized types of equipment, field measurements, and field computational checks ad quality control measures. This network contains 47 new BM along with 11 old BM (Figure 4). A number of 61 levelling lines have been observed in dual directions, forming 14 closed levelling loops. Dual-frequency GPS receivers have been utilized to observe 3D baselines connecting these BM. A total of 296 vectors have been determined, whose lengths vary between 1.3 km and 33.1 km with an average of 9.2 km. Table 2 summarizes the statistics of both levelling and GPS networks.

Table 2: Statistics of the New BM Network

	Number	Minimum (Km)	Maximum(Km)	Average(Km)
LevellingLines	61	1.1	7.7	3.4
LevellingLoops	14	11.3	34.6	17.0
GPSBaselines	296	1.3	33.1	9.2

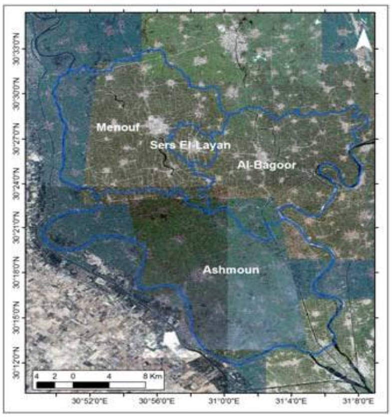

Figure 3: The Study Area

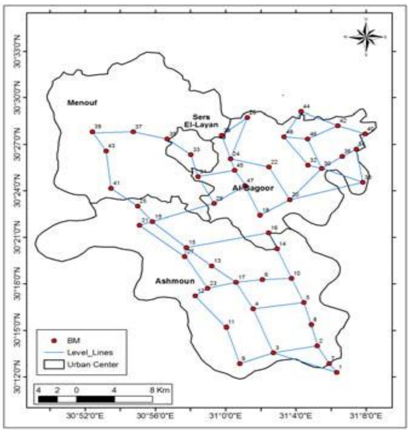

Figure 4: The New Established BM Network

Results & Discussion

The first stage of the data processing concerns the analysis of the systematic errors influencing precise levelling measurements. The main sources of systematic effects are the gravity effect, the astronomical tide effect, and the refraction effect. The most significant error source is the gravity effect, which had not been performed during the computations of the national vertical network of Egypt Saad [17]. Since the determination of that systematic error requires the gravity value at each BM, the gravity field of (ENGSN97) has been utilized to interpolate the gravity values at the established BM stations. It has been found that the gravity at the study area ranges from 979315.2 mGal to 979335.8 mGal (1 mGal = 1x10-5 m/s2). Then, the gravimetric effect on levelling lines, known as the orthometric correction, has been evaluated as:

$$OC_{AB} = \sum_{i=A}^{B-1} \frac{\bar{g}_{ij}-G}{G} dh_{ij} + \frac{\bar{g}_A-G}{G} H_A - \frac{\bar{g}_B-G}{G} H_B \qquad (9)$$

where, G represents the gravity at the mid-latitude of the country, dh_{ij} is the observed height difference between points 1 and j, $\bar{g}_{ij}$ is the average gravity between i and j,H_A and H_B are the elevations of $A$and ,$\bar{g}_A$ and $\bar{g}_B$ are the average gravity along the plumb line at A and B respectively.

Results show that the computed orthometric corrections vary from -0.81 mm to 0.88 mm, with a mean of 0.06 mm. Although such values are small, due to the short observed levelling lines, the orthometric correction is essential in computing the orthometric heights of BM.

A least-square adjustment program has been, then, utilized to adjust the entire levelling network. It should be mentioned that the vertical geodetic network of Egypt has been adjusted loop by loop, which affects its overall quality (e.g. Saad [17]). The results of the least-square adjustment show that the standard deviations of the BM elevations range from ± 2 mm to ± 17 mm, with an average equals ± 9 mm.As a result, it can be concluded that the established BM network meet the accuracy and criteria of a high-precision vertical geodetic network. Regarding the GPS measurements, the Trimble Business Center (TBC) software has been utilized to process and adjust the observed GPS network. The attained findings reveal that thestandard deviations of the horizontal coordinates vary from ± 6 mm and ± 11 mm, with an average of ± 8 mm. The ellipsoidal heights' standard deviation have been found to range from ± 12 mm and ± 27 mm, with an average of ± 22 mm. Figure 5 depicts the precisions of the GPS networks over BM stations.

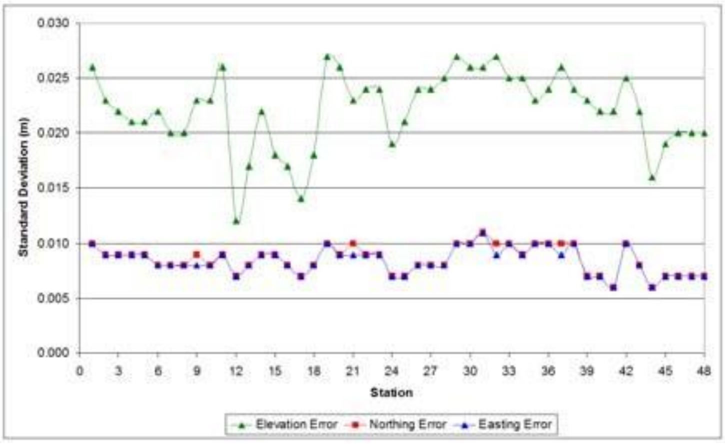

Fig. 5: The Standard Deviations of the GPS Network

Afterward, equation 6 has been performed to compute the geoid undulations at the 48 observed points. The computed geoidal heights range from 14.475 m to 15.092 m, with a mean of14.753 m. firstly, those values have been compared against the Earth Geopotential Model EGM2008-based undulations in order to judge the accuracy of this GGM at the study area. The estimated geoidal differences range from -0.143 m to 0.401 m, with an average of 0.271 m and a standard deviation equals ± 0.131 m (Table 3). Such findings emphasis the fact that the utilization of a GGM alone does not satisfy the required accuracy level for precise orthometric heights determination, and a local precise geoid model is a must in first-order surveying measurements.

Table 3: Statistics of the Estimated Different Heights at BM stations

	Minimum (m)	Maximum (m)	Average (m)
Ellipsoidal heights	26.809	35.020	29.880
Orthometric heights	12.122	20.063	15.075
EGM2008 geoidal heights	14.890	15.325	15.076
Observed geoidal heights	14.475	15.092	14.753
EGM2008 geoidal errors	-0.143	0.401	0.271

Finally, 7 out of the observed 48 GPS/levelling stations have been reserved as check points, and the Arc GIS software has been applied to develop a spatial geoid model over the study area. Next, the geoidal undulations have been interpolated at the check points and compared against their known values. It has been found that the differences range from -0.052 m to 0.049 m, with a mean of 0.0024 m and a standard deviation equals ± 0.038 m. accordingly, it can be concluded that the developed geoid model in the study area (Figure 6) has an accuracy level of almost four centimeters. Such a high-accuracy state-of-the-art geoid model is the first of its kind in Egypt

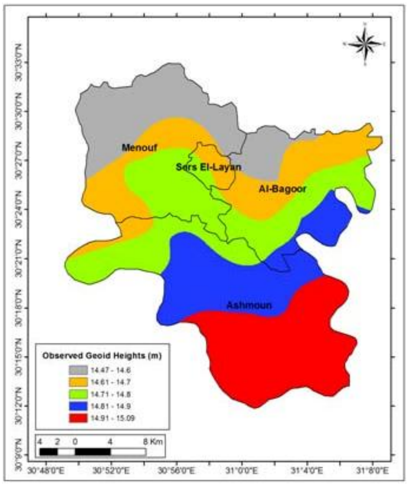

Figure 6: The Developed Final Geoid Model

Conclusion and Future work

Nowadays, satellite-based positioning technologies, particularly the Global Positioning System (GPS), have been extensively utilized in surveying and mapping activities in Egypt. One of the central factors in such applications is the difference between the GPS-based measured height (geodetic height) and that used in civil and surveying engineering projects (orthometric height). A geoid model, describing the spatial relationship between both height systems, is mandatory in order to efficiently utilize GPS in surveying projects. Although geoid modelling has been carried out by several researchers in Egypt, the accuracy level has not satisfied the requirements of precise surveying and mapping.

SRI has carried out a research study aiming to develop a national high-precision geoid for Egypt for the first time. The Menofia governorate has been selected as the study area that represents a guided primary area in which a prototype geoid will be modelled and, later, be generalized for the entire Egyptian territories. It has been found that the lost BM points constitute about 86% in this area, due to urbanization and non-maintenance of the national vertical control network. Then, a new BM network has been established, that contains 47 new BM along with 11 old BM. A number of 61 levelling lines have been observed in dual directions, forming 14 closed levelling loops. Dual-frequency GPS receivers have been utilized to observe 3D baselines connecting these BM. The least-squares adjustment technique has been applied to adjust both control networks, and the attained results have been utilized in determining geoid undulation at all stations. These geoidal heights range from 14.475 m to 15.092 m, with a mean of 14.753 m. Comparing the observed undulations against the EGM2008-based corresponding values reveal that the utilization of any GGM alone does not satisfy the required accuracy level for precise orthometric heights determination. The Arc GIS software has been applied to develop a spatial geoid model over the study area. Validating this model over some observed check points reveals that it has an accuracy level of almost four centimeters. Therefore, it can be concluded that the attained state-of-the-art accurate geoid model is the first of its kind in Egypt.

Finally, is worth mentioning that a national accurate geoid model for Egypt has to be developed as a central component of the currently undergoing project for establishing a national spatial data infrastructure. Economically speaking, such a geoid model will reduce the costs of GPS surveying projects by a considerable amount compared to the traditional surveying techniques. It is a matter of reality that such a national geoid model will advance the surveying and mapping activities in Egypt.

References

[1]. Falchi, U., Parente, C., & Prezioso, G. (2018). Global geoid adjustment on local area for GIS applications using GNSS permanent station coordinates. *Geodesy and Cartography*, *44*(3), 80-88.

[2]. Arana, D., Camargo, P. O., & Guimaraes, G. N. (2017). Hybrid Geoid Model: Theory and Application in Brazil. Anais da Academia Brasileira de Ciências, 89, 1943-1959.

[3]. Karaaslan, Ö., Kayıkçı, E. T., & Aşık, Y. (2016). Comparison of local geoid height surfaces, in the province of Trabzon. Arabian Journal of Geosciences, 9(6), 431.

[4]. Matsuo, K., & Kuroishi, Y. (2020). Refinement of a gravimetric geoid model for Japan using GOCE and an updated regional gravity field model. Earth, Planets and Space, 72(1), 1-18.

[5]. Ssengendo, R., Sjöberg, L., & Gidudu, A. (2015). Computation of the gravimetric quasigeoid model over Uganda using the KTH method. In FIG Working Week 2015 17-21 May, 2015 Sofia Bulgaria (pp. 1-17).

[6]. Balodis, J., Silabriedis, G., Haritonova, D., Kaļinka, M., Janpaule, I., Morozova, K., ... & Liepiņš, I. (2015, October). Development of the One Centimeter Accuracy Geoid Model of Latvia for GNSS Measurements. In IOP Conference Series: Materials Science and Engineering (Vol. 96, No. 1, p. 012037). IOP Publishing.

[7]. Alnaggar, D. (1986). Determination of the geoid in Egypt using heterogeneous geodetic data (Doctoral dissertation, PhD dissertation, Faculty of Engineering, Cairo University, Egypt).

[8]. Dawod, G. (1998). A national gravity standardization network for Egypt (Doctoral dissertation, PhD dissertation, Faculty of Engineering at Shoubra, Zagazig University, Egypt).

[9]. Abd-Elmotaal, H. (2008). Gravimetric geoid for Egypt using high-degree tailored reference geopotential model. *NRIAG Journal of Geophysics special issue*, 507-531.

[10]. Dawod, G. M., & Abdel-Aziz, T. M. (2020). Utilization of geographically weighted regression for geoid modelling in Egypt. Journal of Applied Geodesy, 14(1), 1-12.

[11]. SRI (Survey Research Institute), 2016. Developing a prototype geoid model for Egypt: A guided area in Menofia governorate, unpublished technical report.

[12]. Hofmann-Wellenhof, B., & Moritz, H. (2006). Physical geodesy. Springer Science & Business Media.

[13]. Al-Krargy, E. M., Hosny, M. M., & Dawod, G. M. (2015). Investigating the precision of recent global geoid models and global digital elevation models for geoid modelling in Egypt. RN, 4(1).

[14]. Eshagh, M., & Zoghi, S. (2016). Local error calibration of EGM08 geoid using GNSS/levelling data. Journal of Applied Geophysics, 130, 209-217.

[15]. Fotopoulos, G. (2005). Calibration of geoid error models via a combined adjustment of ellipsoidal, orthometric and gravimetric geoid height data. Journal of Geodesy, 79(1-3), 111-123.

[16]. Kiamehr, R., & Eshagh, M. (2008). Estimating variance components of ellipsoidal, orthometric and geoidalheights through the GPS/levelling Network in Iran. *Journal of the Earth and Space Physics*, *34*(3), 1-13.

[17]. Saad, A. (1993). Towards the redefinition of the Egyptian vertical control network (Doctoral dissertation, PhD Dissertation, Shoubra Faculty of Engineering, Zagazig University, Cairo, Egypt).