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Research Article

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Effect of Heat Transfer in a Vertical Plate on the MHD Free Convection Flow with Thermally Stratified High Porosity Medium

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Abstract An analysis has been performed to study the free convection fluid flow over a vertical plate in the presence of a magnetic plate in a thermally stratified high porosity medium. The non-linear coupled partial differential equation has been solved numerically by explicit finite difference technique. The results of this study have been discussed for different values of well-known parameters with different time steps. Stability and Convergence Analysis have been used for good accuracy. Results are presented for velocity, temperature distribution, local and average shear stress and Nusselt number for representative values of different controlling parameters.

Keywords MHD flow, Free convection, Stratified high porosity medium

Introduction

Free convection flow over a vertical plate embedded in a high porous medium is a major problem in fluid and heat transfer areas. It has attracted a great deal of attraction because of his presence both in natures and engineering applications. It has enormous applications in various industries and environments such as nuclear power plants, geothermal reservoirs, geothermal extractions, chemical catalytic reactors, and food processing and polymer production. The free convection process arises with temperature stratification in environments. Stratification of fluid occurs due to temperature variations, density differences or the presence of different fluids. Free convection flow past a vertical plate has been studied extensively by Ostrach [1]. Cheng and Lau [2] have obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates in the presence of the withdrawal of the fluid. The natural convection flow over a vertical heated surface in a porous medium has been studied by Bejan and Khair [3]. The effect of the ambient thermal stratification on the problem has been studied by Chen et al. [4] was investigated by Singh and Tiwari [5]. Chamkha [6] has extended the analysis of Chenet al. [4] to include the effects of the magnetic field. Agrawal et al [7] have discussed the effect of stratified viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate. The free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient was studied by Ramanaiah and Malarvizhi [8].

The aim of the work is to study the influence of thermal stratification in a free convection flow past a semi-infinite vertical plate by employing an explicit finite difference scheme. Stratification effects on velocity and temperature distribution are presented graphically. Also the effects of parameters on the rate of heat transfer are discussed.

Mathematical Formulation

Consider an unsteady MHD free convection flow past infinite vertical porous plate which is thermally stratified. Let us consider an unsteady free convective flow of an electrically conducting viscous fluid through a porous

medium along a semi-infinite vertical porous plate $y = 0$ under the influence of applied magnetic field. The flow is assumed to be in the x-direction which is taken along the plate in the upward direction and y-axis is normal to it. The temperature of the plate raised from T_w to T_∞, where T_∞ be the temperature of the uniform flow. A uniform magnetic field B is taken to be acting along the y-axis which is assumed to be electrically non-conducting. The assumption is justified when the magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is negligible and is of the form $B=(0,B_0,0)$ and the magnetic lines of force are fixed relative to the fluid. The equation of conservation of charge $\nabla.J = 0$ gives J_y=constant, where the current density $J = (J_x,\ J_y J_z)$. since the plate is electrically non-conducting, this constant is zero and hence $J_y = 0$ at the plate and hence zero everywhere.
The physical configuration of this study is furnished in the following figure 1;

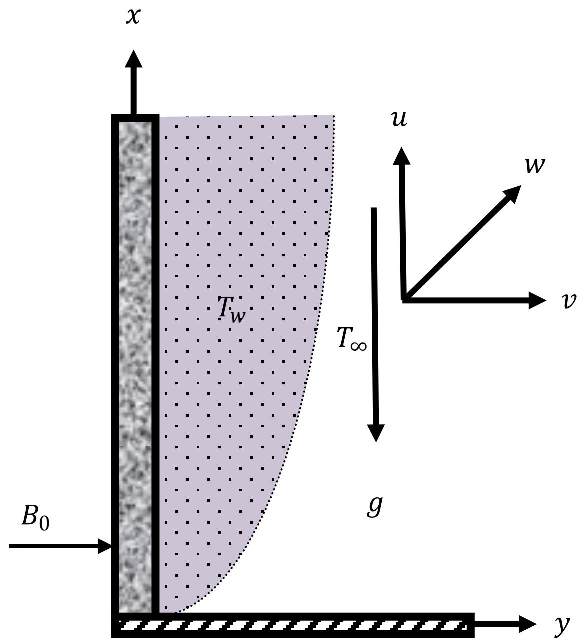

Figure 1: Physical configuration and coordinate system

Under the above assumptions, the governing boundary layer equations for the flow with usual Boussinesq's approximation are follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{1}{\varepsilon^2}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{v}{\varepsilon}\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)-\frac{v}{k}u-c(u^2+w^2)-\frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$\frac{1}{\varepsilon^2}\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}\right)=\frac{v}{\varepsilon}\frac{\partial^2 u}{\partial y^2}-\frac{v}{k}w-c(u^2+w^2)-\frac{\sigma B_0^2 w}{\rho} \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{v}{c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]+\frac{\sigma B_0^2}{\rho c_p}(u^2+w^2) \tag{4}$$

The initial boundary condition are as follows:

$$\left.\begin{array}{lllll} t>0, & u=0, & v=0, & w=0, & T=T_w \quad at \quad y=0 \\ & u\to 0, & v\to 0, & w\to 0, & T\to T_\infty as \quad y\to\infty \end{array}\right\} \tag{5}$$

Equation (1), (2), (3) and (4) reduce the following non-dimensional form;

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{6}$$

$$\frac{\partial U}{\partial \tau}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=\varepsilon^2\theta+\varepsilon\frac{\partial^2 U}{\partial Y^2}-\varepsilon^2\Gamma(U^2+W^2)-\varepsilon^2(D_c^{-1}+M^2)U \tag{7}$$

$$\frac{\partial W}{\partial \tau}+U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}=\varepsilon\frac{\partial^2 W}{\partial Y^2}-\varepsilon^2\Gamma(U^2+W^2)-\varepsilon^2(D_c^{-1}+M^2)W \tag{8}$$

$$\frac{\partial \theta}{\partial \tau}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{P_r}\frac{\partial^2 \theta}{\partial Y^2}+E_c\left[\left(\frac{\partial U}{\partial Y}\right)^2+\left(\frac{\partial W}{\partial Y}\right)^2\right]+M^2E_c(U^2+W^2)-S_TU \tag{9}$$

Thus the boundary condition (5) reduces as follows;

$$\left.\begin{array}{l}\tau>0,\quad U=0,\quad V=0,\quad W=0,\quad \theta=1-S_TX \quad at\ \ Y=0\\ \quad U\to 0,\quad V\to 0,\quad W\to 0,\quad \theta\to 0 \quad as\ \ Y\to\infty\end{array}\right\} \tag{10}$$

where,

$Ec=\dfrac{\nu^2G_r}{C_pL^2(T_w-T_\infty)}$ is the Eckert number , $G_r=\dfrac{g\beta(T_w-T_\infty)L^3}{\nu^2}$ is the Grashof number, $D_c=\dfrac{k\sqrt{G_r}}{L^2}$ is the Darcy number, $\Gamma=CL$ is the dimensionless inertial parameter, $M^2=\dfrac{\sigma\beta_0{}^2L^2}{\rho\nu\sqrt{G_r}}$ is the magnetic force parameter and $S_T=\dfrac{\frac{\partial T_{\infty,x}}{\partial x}}{(T_w-T_{\infty,o})}$ is the ambient thermal stratification parameter.

Shear Stress and Nusselt Number

From the velocity field, the effect of various parameters on the local and average shear stress have been investigated. The following quantities represent the local and average shear stress at the plate.

Local shear stress, $\tau_L=\mu(\frac{\partial u}{\partial y})_{y=0}$ and Average shear stress,

$\tau_A=\mu\int(\frac{\partial u}{\partial y})_{y=0}\ dx$ Which are proportional to $(\frac{\partial U}{\partial Y})_{Y=0}$ and $\int_0^{100}(\frac{\partial U}{\partial Y})_{Y=0}\ dX$ respectively.

From the temperature field, the effects of various parameters on the local and average heat transfer coefficients have been investigated. The following equations represent the local and average heat transfer rate that is well known as Nusselt number.

Local Nusselt number, $N_{uL}=\mu\left(-\frac{\partial T}{\partial y}\right)_{y=0}$ and

Average Nusselt number $N_{u_A}=\mu\int(-\frac{\partial T}{\partial y})_{y=0}\ dx$ Which are proportional to $(-\frac{\partial \overline{T}}{\partial Y})_{Y=0}$

and $\int_0^{100}(-\frac{\partial \overline{T}}{\partial Y})_{Y=0}\ dX$ respectively.

Numerical Techniques

To obtain the difference equations, the region of the flow is divided into a grid of lines parallel to X and Y axes where X-axes is taken along the plates and Y- axes is normal to the plates. The plate of height X_{max} =

100 i.e. X varies from $0\ to\ 100$ and regard $Y_{max} = 20$ has been considered. There are $m = 200$ and $n = 200$ grid spacing in the $X$and Y directions respectively has been shown in the figure

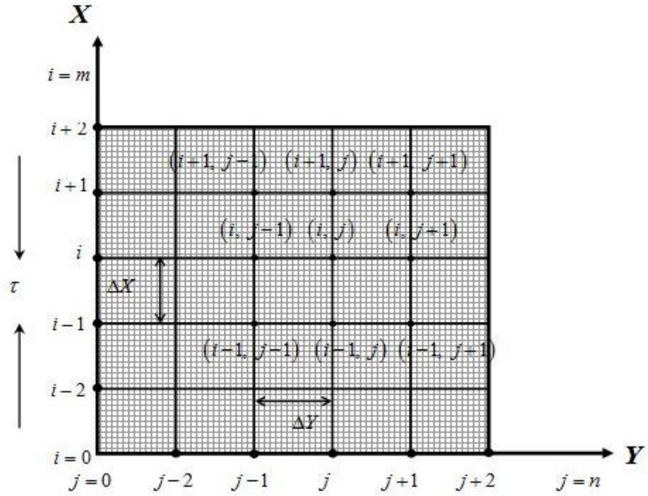

Figure 2: Explicit Finite difference space grid

It has been assumed that $\Delta X, \Delta Y$ are constant mesh sizes along $X$and Y directions respectively are follows;

$\Delta X = 0.5(0 \leq X \leq 100), \Delta Y = 0.1(0 \leq Y \leq 20)$ with the smaller time step, $\Delta\tau = 0.001$.

Now $U^{'}, W^{'}$ and$\theta^{'}$ denote the values of U, W and θ at the end of a time-step respectively. Using the explicit finite difference approximations are as follows;

$$\frac{U_{i,j} - U_{i-1,j}}{\Delta x} + \frac{V_{i,j} - V_{i-1,j}}{\Delta Y} = 0 \tag{11}$$

$$\frac{U^{'}_{i,j} - U_{i,j}}{\Delta\tau} + U_{i,j}\frac{U_{i,j} - U_{i-1,j}}{\Delta x} + V_{i,j}\frac{U_{i,j+1} - U_{i,j}}{\Delta Y} = \varepsilon\frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{(\Delta Y)^2} + \varepsilon^2\theta_{i,j} - \varepsilon^2\Gamma\left(U^2_{i,j} + W^2_{i,j}\right) - \varepsilon^2(D_c^{-1} + M^2)U_{i,j} \tag{12}$$

$$\frac{W^{'}_{i,j} - W_{i,j}}{\Delta\tau} + U_{i,j}\frac{W_{i,j} - W_{i-1,j}}{\Delta x} + V_{i,j}\frac{W_{i,j+1} - W_{i,j}}{\Delta Y} = \varepsilon\frac{W_{i,j+1} - 2W_{i,j} + W_{i,j-1}}{(\Delta Y)^2} - \varepsilon^2\Gamma\left(U^2_{i,j} + W^2_{i,j}\right) - \varepsilon^2(D_c^{-1} + M^2)W_{i,j} \tag{13}$$

$$\frac{\theta^{'}_{i,j} - \theta_{i,j}}{\Delta\tau} + U_{i,j}\frac{\theta_{i,j} - \theta_{i-1,j}}{\Delta x} + V_{i,j}\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta Y} = \frac{1}{P_r}\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{(\Delta Y)^2} - S_T\,U_{i,j} + E_c\left[(\frac{U_{i,j+1} - U_{i,j}}{\Delta Y})^2 + (\frac{W_{i,j+1} - W_{i,j}}{\Delta Y})^2\right] - -M^2E_c\left(U^2_{i,j} + W^2_{i,j}\right) \tag{14}$$

and the boundary condition with the finite difference scheme are as follows;

$$\begin{aligned} \tau > 0 \quad & U^n_{i,0} = 0, \quad V^n_{i,0} = 0, \quad W^n_{i,0} = 0, \quad \theta^n_{i,0} = 1 - S_T X \\ & U^n_{i,L} = 0, \quad V^n_{i,L} = 0, \quad \theta^n_{i,L} = 0 \quad \text{where} \quad L \to \infty \end{aligned} \tag{15}$$

Results and Discussion

For the purpose of discussing the results of the problem, the approximate solutions are obtained for various parameters. In order to analyze the physical significance of the model, the steady state numerical values of the non-dimensional primary velocity, secondary velocity and temperature within the boundary layer for different values of Prandlt number (P_r), Thermal stratification parameter (S_T),High porosity medium$(\varepsilon)$and Darcy number(D_c)has been computed. The displayed figure from figure (3)-(11) are shown the effects of above mentioned parameter. Figure (3), (4), (6), (7) represents the primary and secondary velocity profile for Prandlt number, Thermal stratification parameter, high porosity parameter and Darcy number. From this figure it has been observed that primary velocity decreases with the increase of Prandlt number (P_r) and Thermal stratification parameter (S_T) while the secondary velocity increases with the increase of this parameter. It has also been observed that primary velocity increases with the increase of high porosity number (ε) and Darcy number (D_c) while the secondary velocity decreases with the increase of this parameter. Figure (5) and (8) displays the temperature profile for Prandlt number, Thermal stratification parameter, high porosity parameter and Darcy number. From this figure it has been observed that temperature distribution decreases with the increase of prandlt number (P_r) and thermal stratification parameter (S_T). There has been shown minor effects in the temperature distribution in case of high porosity parameter (ε) and Darcy number (D_c) with the increases of this parameter.

Figure (9), (10), (11) shows the local and average primary and secondary shear stress and nusselt number for different values of Prandlt number and Thermal stratification parameter. From these figure it has been shown that local and average primary shear stress decreases with the increase of Prandlt number (P_r) and Thermal stratification parameter (S_T) while the local and average Secondary shear stress increases with the increase of this parameter. Figure (11) shows the Local and average nusselt number for Prandlt number and Thermal stratification parameter. There has been found a minor change in the nusselt number for the increasing of these parameter.

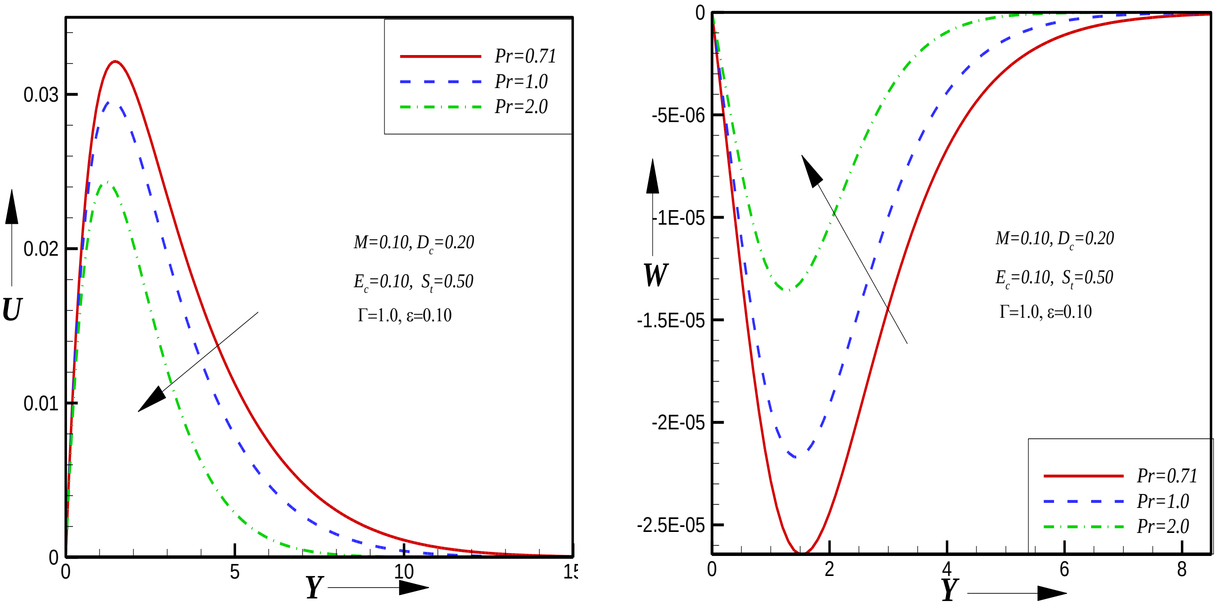

Figure 3: Primary and Secondary velocity profile for different values of Prandlt number

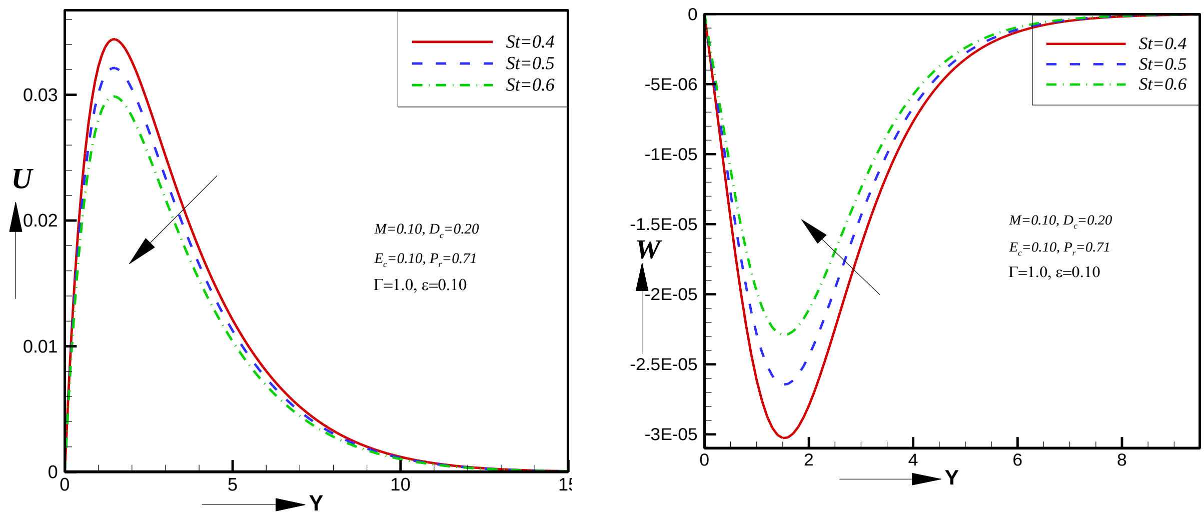

Figure 4: Primary and Secondary velocity profile for different values of thermal stratification parameter

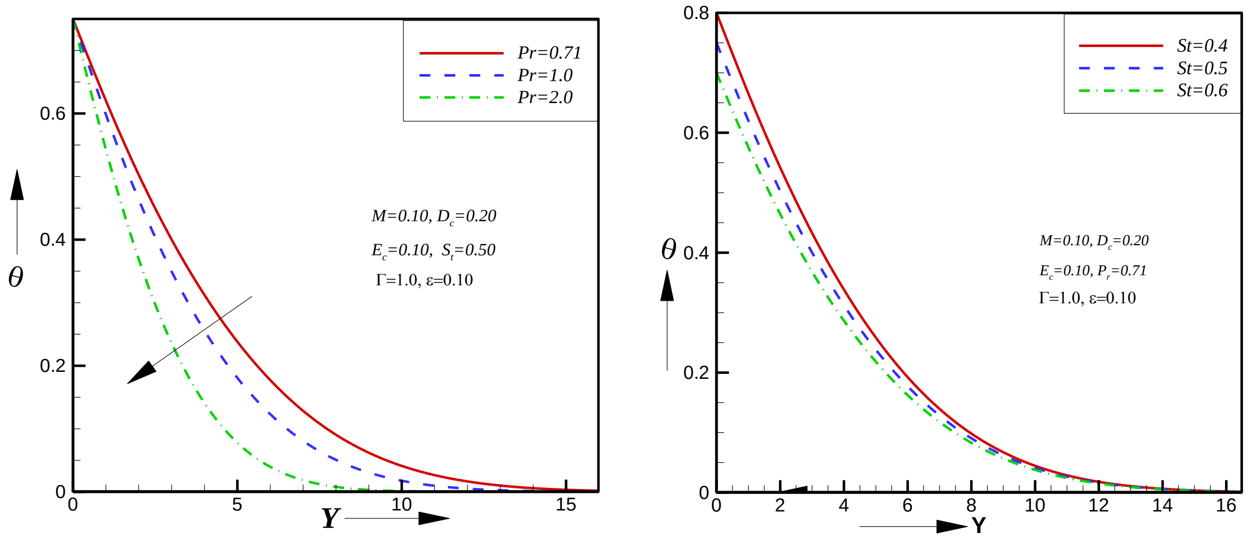

Figure 5: Temperature profile for different values of Prandlt number and thermal stratification parameter

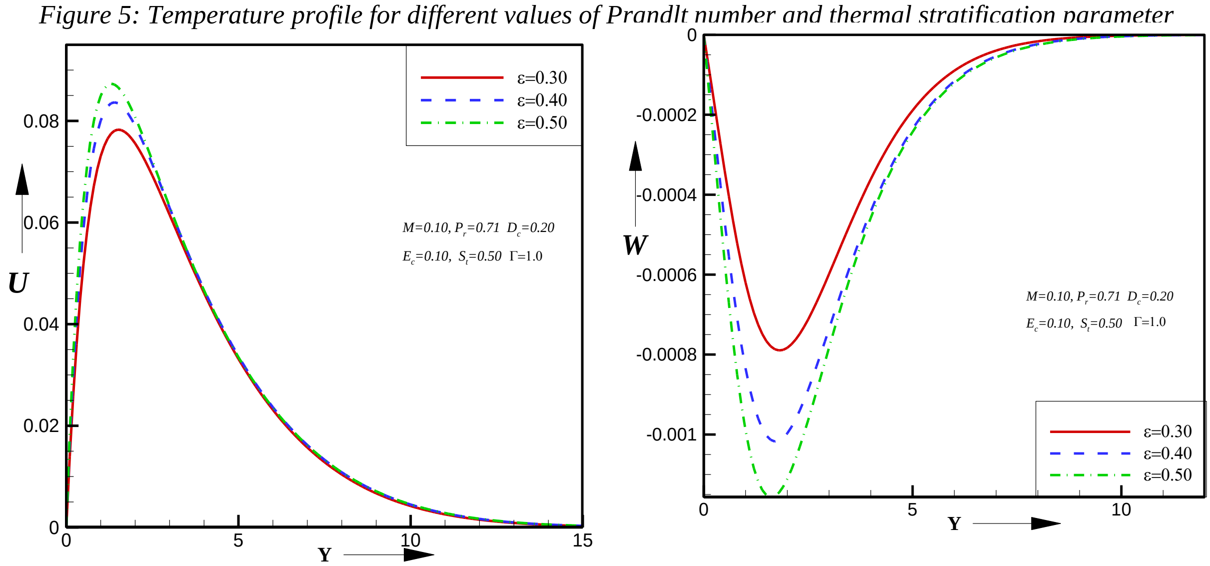

Figure 6: Primary and Secondary velocity profile for different values of high porosity parameter

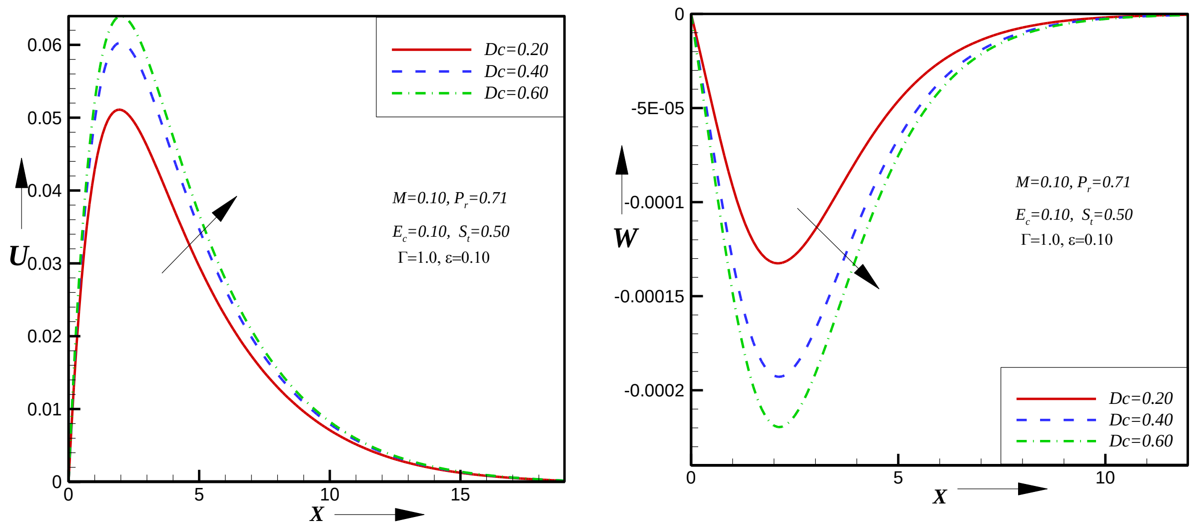

Figure 7: Primary and Secondary velocity profile for different values of Darcy number

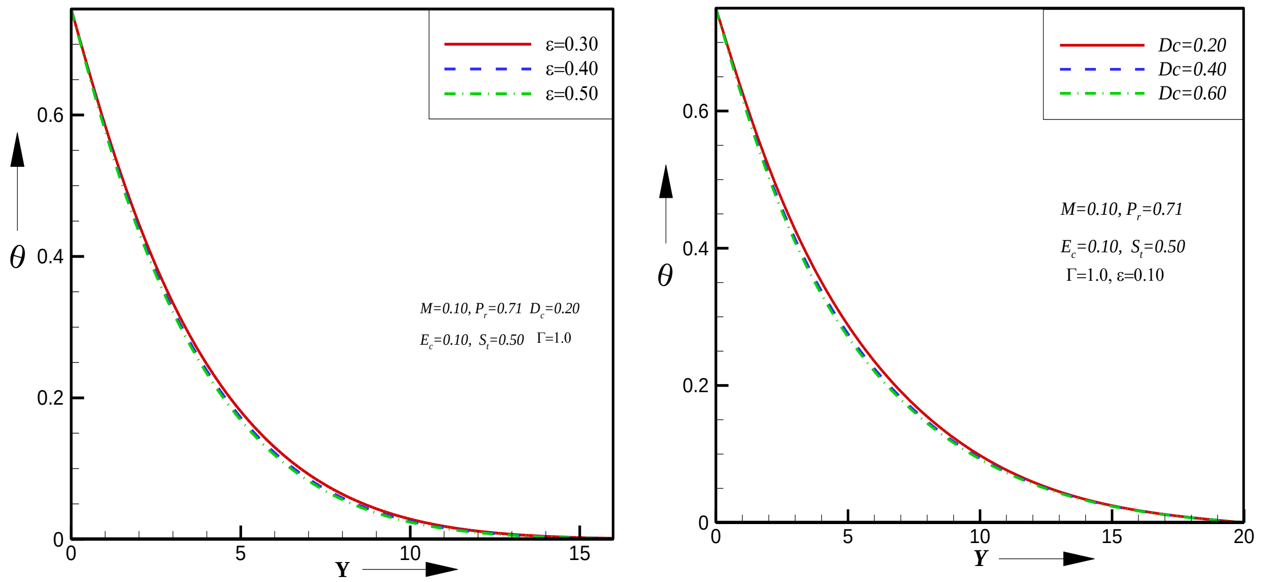

Figure 8: Temperature profile for different values of high porosity number and Darcy number

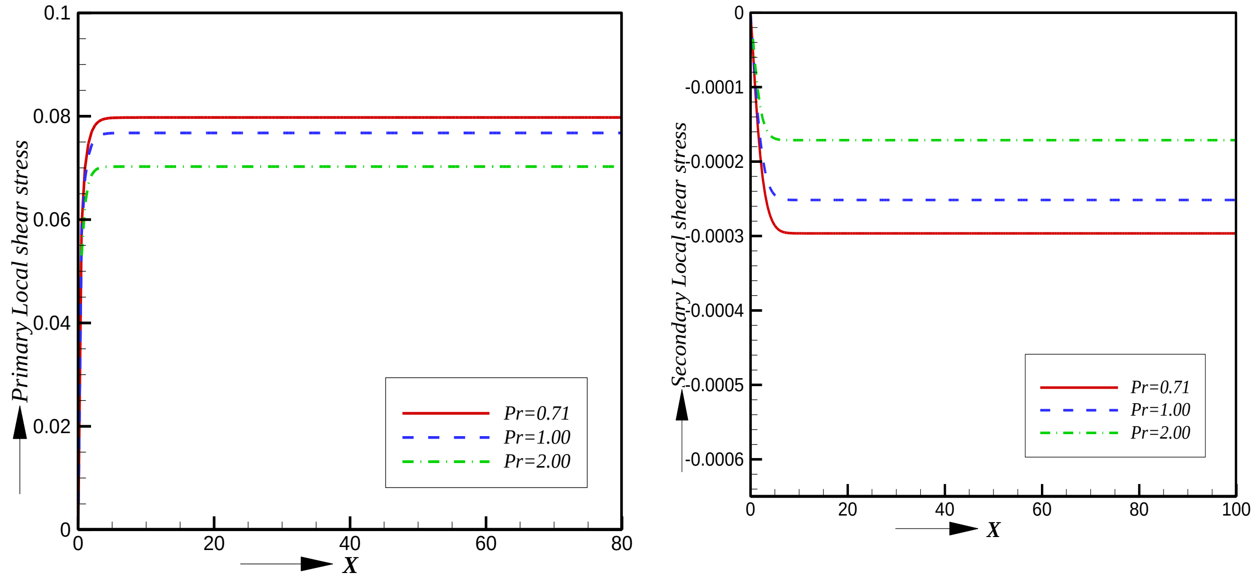

Figure 9: primary and secondary local shear stress for Prandlt number

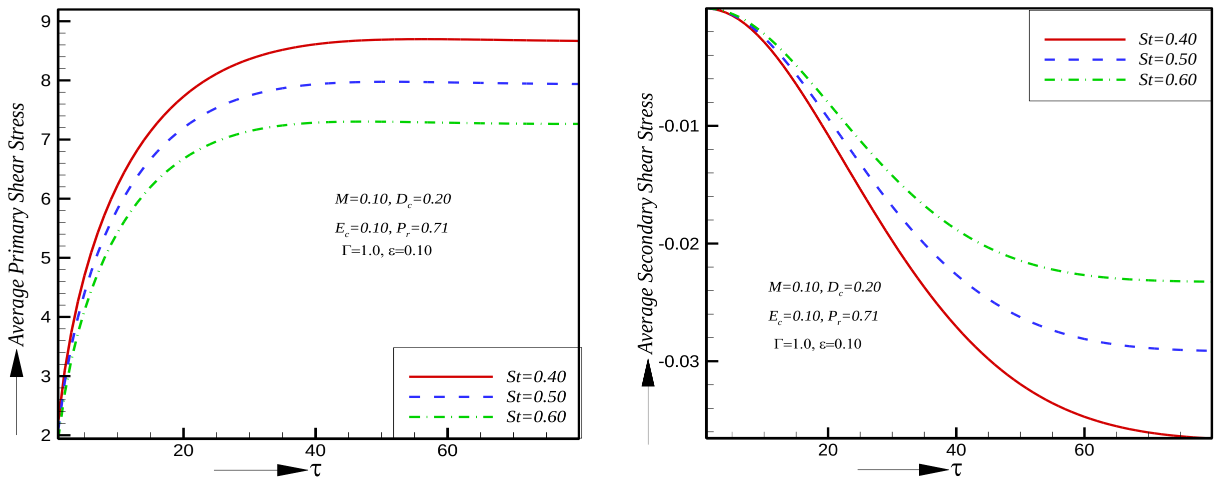

Figure 10: primary and secondary average shear stress for thermal stratification parameter

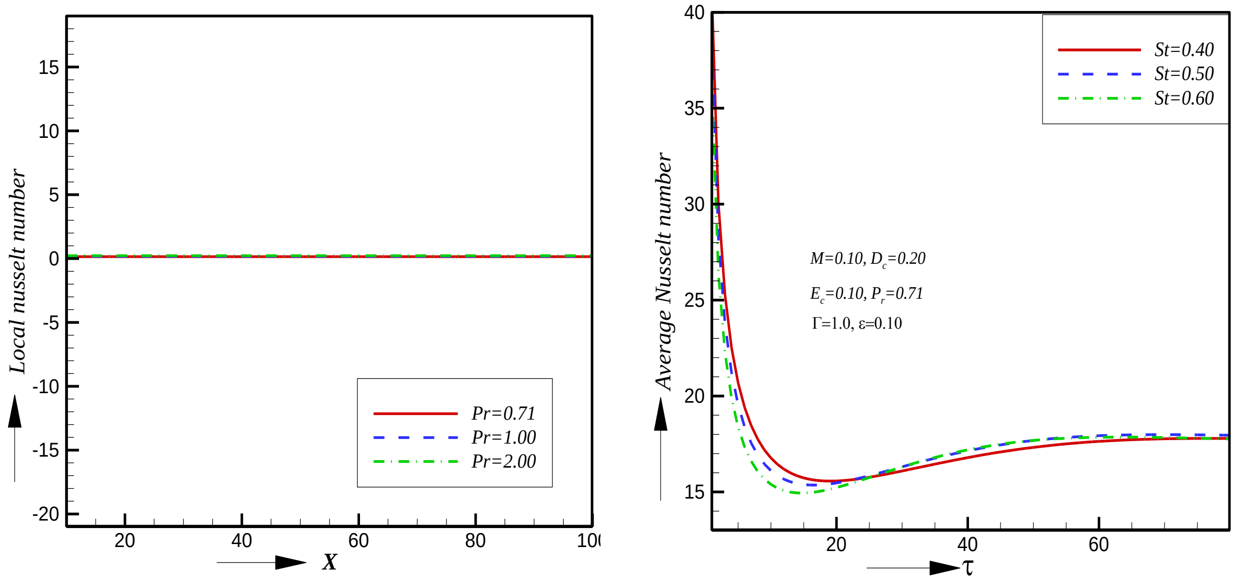

Figure 11: Local and average Nusselt number for Prandlt and thermal stratification parameter

Conclusions

MHD free convection fluid flow with thermally stratified high porosity medium has been considered. The resulting governing systems of dimensionless coupled non-linear partial differential equations are numerically solved by an explicit finite difference method. The results are discussed for different values of important parameters as Prandlt number, Thermal stratification parameter, Darcy number and Porosity parameter. Some of the important findings obtained from the graphical representation of the results are listed below;

1. The primary velocity increases with the increase of D_c ,ε while it decreases with the increase of P_r, S_T.
2. The secondary velocity increases with the increase of P_r, S_T while it decreases with the increases of D_c, ε.
3. The temperature distribution decreases with the increase of P_r, S_T while it shows minor effects with the increase of D_c, ε.
4. The primary local and average shear stress decreases with the increase of P_r, S_T.
5. The secondary local and average shear stress increases with the increase of P_r, S_T.

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