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Hydrodynamic Anisotropy Effects on Thermal Radiation-Natural Convection Interaction in a Vertical Rectangular Porous Cavity

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Abstract This work analytically investigates the problem of natural convection which develops in a vertical rectangular porous saturated cavity subjected to solar radiation. The porous medium is anisotropic in permeability whose principal axes are oriented in a direction that is oblique to the gravity. The generalized Darcy's law within the boundary layer approximations and the Rosseland approximation are used in the formulation of the problem. Scale analysis is applied to predict the orders of magnitudes involved in the boundary-layer regime for which the condition of validity are presented. Effects of anisotropy parameters and the thermal radiation on the heat transfer are analyzed using an integral approach in the limiting case for high Rayleigh numbers. It was found that the anisotropic permeability ratio, the orientation angle of the principal axes of permeability and the radiation parameter affected significantly the flow regime and the heat transfer.

Keywords Natural convection, anisotropic porous medium, vertical rectangular cavity, Nusselt number, thermal radiation parameter

1. Introduction

The problem of natural convection caused by the buoyancy forces coupled with radiation in rectangular cavities filled with porous medium is motivated by a wide range of industrial applications. Examples include electric heaters, solar energy collectors, the cooling of radioactive waste containers, geophysical systems, the migration of moisture through the air contained in fibrous insulation, the underground diffusion of contaminant, heat transfer within cereal storage facilities and regenerative heat exchangers in porous materials. In most of these applications involving natural convection as a mechanism, the transfer of radiative heat within porous cavities is neglected for low temperatures. Recent works by various authors Vafai [1], Nield and Bejan [2]; Bejan [3] have documented research on natural convection in a two-dimensional rectangular enclosure. For example, Baytas and Pop [4] presented detailed numerical calculations of natural convection in an inclined porous cavity and saturated by fluid. They adopted the Alternating Direction Implicit (ADI) method of finite differences and analyzed the effects of the inclination angle and aspect ratio of the cavity on the streamlines, isotherms and the average Nusselt number. These results were compared to those obtained the past by [5], analyzed numerically and analytically the thermal transfers by natural convection in a rectangular cavity filled with an anisotropic porous medium that is permeable and saturated by the fluid. The side walls of the cavity are heated and cooled at constant temperature and the horizontal walls are adiabatic. These two authors adopted the approach based on integral relations [6], to develop a boundary layer solution. They found that the anisotropic properties and orientation of the main axes of the porous medium greatly influence the flow pattern and the rate of heat transfer compared to the results of obtained for isotropic medium. The buoyancy-driven convection in an open-ended cavity with an obstructing medium such as a porous material is analyzed by [7]. They investigated the effects of important variables such as the aspect ratio, the temperature difference, and the Darcy-Rayleigh number on the flow field and the cavity Nusselt number. However, the transfer of radiative heat is still very important, even for small temperature differences. The literature review indicates that natural convection coupled with radiation in a porous cavity received little attention compared to natural convection. Badruddin and al. [8] studied numerically the natural convection with thermal radiation inside a porous cavity using the Darcy model. The authors showed that the average Nusselt number for porous media increases with the increase of the radiation parameter, whereas the average Nusselt number for fluid media decreases with the increase of the radiation parameter. Ahmed and al. [9] investigated the effects of viscous dissipation and radiation on the laminar magneto-hydrodynamic natural convection in a square enclosure filled with a porous medium by numerical two-dimensional analysis using finite difference approach. They showed that the heat transfer increases with the radiation parameter and decreases as the Darcy number decreases. An analysis of the combined effects of thermal radiation and heat source on natural convection was undertaken by [10]. The authors showed that the vertical velocity and the average Nusselt number increases as the radiation parameter increases. Zahmatkesh [11] studied numerically the influence of thermal radiation on natural convection within a porous cavity saturated by fluid. They indicate that, thermal radiation makes the temperature distribution almost uniform near the vertical walls inside the cavity and makes the streamlines almost parallel to the vertical walls. In addition, the average Nusselt number increases almost linearly with the increase of the radiation parameter. Rani and al.[12] analyzed the combine influence of radiation and dissipation on the convective heat and mass transfer flow of a viscous fluid through a porous medium in a rectangular cavity using Darcy model. The Galerkin finite element method with linear triangular elements is adopted for the study. The effects of the different parameters governing the problem are discussed. Mansour and al.[13] studied numerically the influence thermal radiation on steady convection in Wavy porous cavities using thermal non-equilibrium model. These authors examined the effects of thermal radiation, modified conductivity ratio and Rayleigh number on flow and temperature fields. The present work is devoted to the study of coupled fluid flow and heat transfer by natural convection and radiation in a vertical cavity filled with a fluid-saturated porous medium whose horizontal walls are considered permeable. The vertical walls are respectively heated and cooled at a constant temperature. The effects of hydrodynamic anisotropy of the porous medium and the influence of radiation will be investigated, since the physical problem is of significant importance to many engineering-related applications.

2. Physical and Mathematical Modelling

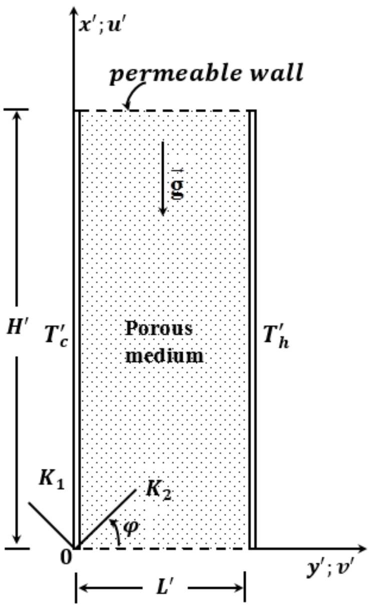


Figure 1: Schematic diagram of the 2-D vertical porous cavity

Fig 1 illustrates a two-dimensional cross-sectional section of the porous cavity saturated by an incompressible fluid. Fluid flow in the vertical cavity is stable, laminar and thermo-physical properties are assumed to be constant except for density in the buoyancy term. The porous medium is hydrodynamically anisotropic in permeability. The main directions of permeability noted $\mathbf{K_1}$ and $\mathbf{K_2}$ form respectively with the axes of horizontal coordinate (y) and vertical coordinate (x) , an angle and rotates around the origin point. The anisotropy ratio $\mathbf{K_1/K_2}$ and orientation angle φ characterize the anisotropy of the porous medium on the fluid flow. The porous medium subjected to a solar thermal radiation, can emit, absorb and diffuse in an isotropic way the radiative energy. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x direction is assumed to be negligible compared to the y direction. The movement of the fluid through the anisotropic porous medium obeys the generalized Darcy law. The vertical walls of the cavity are assumed to be impermeable and one heated $\mathbf{T_h'}$ and $\mathbf{T_c'}$ the other cooled . The horizontal walls are permeable. On the basis of the thermal balance between the porous medium and the fluid and taking into account the approximation of Boussinesq, the equations governing the flow can be written in the following ways:

$$\vec{\nabla}.\vec{V'} = 0 \tag{1}$$

$$\vec{V'} = -\frac{\overline{\overline{K'}}}{\mu}\cdot\left[\vec{\nabla}P' + \rho\beta\vec{g}\left(T' - T_c'\right)\right] \tag{2}$$

$$\left(\rho C_p\right)_p \frac{\partial T'}{\partial t'} + \left(\rho C_p\right)_f \vec{\nabla}\cdot\left(\vec{V'}T'\right) = k\nabla^2 T' - \frac{\partial q^{r'}}{\partial y'} \tag{3}$$

Where the second order permeability tensor is defined in the Cartesian coordinates (O, Ox, Oy) as:

$$\overline{\overline{K'}} = \begin{bmatrix} K_1\cos^2\varphi + K_2\sin^2\varphi & (K_1 - K_2)\sin\varphi\cos\varphi \\ (K_1 - K_2)\sin\varphi\cos\varphi & K_2\cos^2\varphi + K_1\sin^2\varphi \end{bmatrix} \tag{4}$$

The radiant heat flux is simplified using the Rosseland approximation which can be written:

$$q^{r'} = -\frac{4\sigma_s}{3\beta_r}\frac{\partial T'^4}{\partial y'} \tag{5}$$

where σ_s is the Stefan-Boltzmann constant and β_r is the average absorption coefficient.

In the case of the Boussinesq approximation, where the temperature differences inside the cavity during the flow are sufficiently small, the non-linear term T'^4 can be expressed as a linear function of temperature using the Taylor series for T'^4 around T_c' . By neglecting higher order terms, we find:

$$T'^4 \approx 4T'T_c'^3 - 3T_c'^4 \tag{6}$$

It is important to note that the equation (6) is widely used in the fluid flow involving the radiation absorption problem by expressing the non-linear term T'^4 as a linear function.

By introducing the equation (6) into (5), we obtain:

$$q^{r'} = -\frac{16\sigma_s T_c'^3}{3\beta_r}\frac{\partial T'}{\partial y'} \tag{7}$$

By eliminating the pressure in the equation of motion and using the scaling factors L' , α/L' , $\Delta T' = T_h' - T_c'$ and $L'^2\sigma/\alpha$ respectively for length, velocity, temperature and time, dimensionless governing equations can be written as follows:

$$a\frac{\partial^2\psi}{\partial y^2}+2c\frac{\partial^2\psi}{\partial x\partial y}+b\frac{\partial^2\psi}{\partial x^2}=Ra\frac{\partial T}{\partial y} \tag{8}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\gamma\frac{\partial^2 T}{\partial y^2} \tag{9}$$

where

$$\left.\begin{aligned} a &= K^*\sin^2\varphi+\cos^2\varphi \\ b &= K^*\cos^2\varphi+\sin^2\varphi \\ c &= (K^*-1)\cos\varphi\sin\varphi \end{aligned}\right\} \tag{10}$$

and

$$\gamma=1+\frac{4Rd}{3} \tag{11}$$

The continuity equation can be satisfied automatically when the stream function ψ is defined as following:

$$u=-\frac{\partial\psi}{\partial y}, v=\frac{\partial\psi}{\partial x} \tag{12}$$

The dimensionless boundary conditions on the walls of the cavity give:

$$x=0,\frac{\partial\psi}{\partial x}=0, T=0 \tag{13}$$

$$x=A,\frac{\partial\psi}{\partial x}=0,\frac{\partial T}{\partial x}=0 \tag{14}$$

$$y=0,\psi=0,T=0 \tag{15}$$

$$y=1,\psi=0,T=1 \tag{16}$$

Where $A=H^{'}/L^{'}$ is the cavity shape ratio, Ra is the Rayleigh number and Rd is the radiation parameter.

$$\left.\begin{aligned} Ra &= K_1 g\beta\Delta T^{'}L^{'}/\alpha\nu \\ Rd &= 4\sigma_s T_c^{'3}/k\beta_r \end{aligned}\right\} \tag{17}$$

The average Nusselt number measuring the total heat transfer through the porous layer is evaluated on the hot wall by the relation:

$$Nu=-\frac{\gamma}{A}\int_0^A\left.\frac{\partial T}{\partial y}\right|_{y=1}dx \tag{18}$$

The present problem is controlled by the following dimensionless parameters: the Rayleigh number Ra, the aspect ratio A, the anisotropy ratio K^*, the orientation angle φ and the radiation parameter Rd.

3. Boundary Layer Analysis

When the movement of the fluid induced by the buoyancy force inside the cavity is sufficiently strong enough, the movement of the fluid is restricted to a thin boundary layer along each vertical wall oriented on the ascending vertical. The movement of the fluid is limited by a boundary layer of thickness $\delta^{'}$ and height $H^{'}(\delta^{'}\langle\langle H^{'})$. In this region the following conditions are valid.

$$a\frac{\partial u^{'}}{\partial y^{'}}\rangle\rangle\frac{\partial u^{'}}{\partial x^{'}} \tag{19}$$

$$a\frac{\partial u'}{\partial y'}\rangle\rangle\frac{\partial v'}{\partial y'} \quad (20)$$

$$a\frac{\partial u'}{\partial y'}\rangle\rangle\frac{\partial v'}{\partial x'} \quad (21)$$

Scale analysis is used to find the order of magnitude of the parameters of interest of the problem. The temperature variation is of the order of unity $(T \sim 1)$ in the region of the boundary layer, by analyzing the thermal boundary conditions. The analysis gives the following equivalences for motion and energy conservation equations in the boundary layer region.

$$\frac{u}{A} \sim \gamma\frac{v}{\delta} \quad (22)$$

$$a\frac{u}{\delta} \sim Ra\frac{1}{\delta} \quad (23)$$

$$u\frac{1}{A} \sim \gamma\frac{1}{\delta^2} \quad (24)$$

The resolution of equations (22) to (24) in the region of the boundary layer give respectively for δ, u, v and ψ.

$$\left.\begin{array}{l} \delta \sim \gamma^{1/2}a^{1/2}A^{1/2}Ra^{1/2} \\ u \sim a^{-1}Ra \\ v \sim \gamma^{1/2}a^{-1/2}A^{-1/2}Ra^{1/2} \\ \psi \sim \gamma^{1/2}a^{-1/2}A^{1/2}Ra^{1/2} \end{array}\right\} \quad (25)$$

The average Nusselt number that reflects the heat transfer is defined by the quotient of the amount of total heat from one vertical wall to another q' over the amount of heat exchanged in pure conduction q_c' .The order of magnitude of the average Nusselt number is given by:

$$Nu \sim \gamma^{-1/2}a^{-1/2}A^{-1/2}Ra^{1/2} \quad (26)$$

The previously established orders of magnitude are valid when the boundary layer is thin $(Ra \to \infty)$ and distinct. Under these conditions, the coefficients b and c give:

$$\left.\begin{array}{l} b\langle\langle\gamma^{-1}ARa \\ c\langle\langle\gamma^{-1/2}a^{1/2}A^{1/2}Ra^{1/2} \end{array}\right\} \quad (27)$$

The relations (25) allow to define the scales of normalization of the different functions in the region of the boundary layer. When the dimensionless variables used in the boundary layer are:

$$\left.\begin{array}{l} \bar{x} = \frac{1}{A}x, \bar{y} = \frac{L'}{\Lambda}y, \bar{\psi} = \frac{\Lambda}{L'}\psi \\ \bar{u} = \frac{\Lambda^2}{L'H'\gamma}u, \bar{v} = \frac{\Lambda}{L'\gamma}v, \bar{T} = T \\ \Lambda^2 = \frac{L'H'a\gamma}{Ra} \end{array}\right\} \quad (28)$$

The approximate forms of equations (8) and (9) are given by the following differential equations:

$$\frac{\partial\bar{u}}{\partial\bar{y}} = \frac{\partial\bar{T}}{\partial\bar{y}} \quad (29)$$

$$\overline{u}\frac{\partial \overline{T}}{\partial \overline{x}}+\overline{v}\frac{\partial \overline{T}}{\partial \overline{y}}=\frac{\partial^2 \overline{T}}{\partial \overline{y}^2} \tag{30}$$

According to Gill [14], the centro-symmetrical of the studied physical problem leads to considering a single boundary layer along the vertical walls. Thus in the layer region we will introduce the Gill variables which are written:

$$\left.\begin{aligned}\overline{T}&=\overline{T}_\infty(\overline{x})+\theta(\overline{x},\overline{y})\\ \overline{\psi}&=\overline{\psi}_\infty(\overline{x})+\phi(\overline{x},\overline{y})\end{aligned}\right\} \tag{31}$$

where θ and $\phi \to 0$ when $\overline{y}\to\infty$

The associated boundary conditions become:

$$\overline{y}\to 0, \overline{\psi}=0, \overline{T}=0 \tag{32}$$

$$\overline{y}\to\infty, \overline{\psi}=\overline{\psi}_\infty(\overline{x}), \overline{T}=\overline{T}_\infty(\overline{x}) \tag{33}$$

$\overline{\psi}_\infty$ and $\overline{T}_\infty$ represent respectively the adimensional form of the stream function and the temperature distribution in the central cavity region very far from the vertical walls. In the past, Weber [15] and Simpkins and Blythe [6] solved the system of non-linear differential equations by considering the isotropic porous medium ($K^*=1$ and $Rd=0$). Recently Degan and Vasseur [5] continued the work by assuming permeable anisotropic porous medium ($Rd=0$). The analytical solution resulting from the work done in the past by adopting the integral relations is in agreement with the numerical results, provided that the chosen boundary layer profile presents the correct asymptotic behavior [5].

The solutions obtained in the present work, the stream function and the distribution of the temperature in the central region of the cavity by considering the profile of two vertical boundary layers give:

$$\psi_\infty = 2.410\left(\frac{ARa^*\sqrt{K^*}}{a\gamma}\right)^{1/2}[T_\infty(1-T_\infty)]^{0.8587} \tag{34}$$

$$x=2.17AT_\infty^{1.7174}\left(1-0.453T_\infty-4.68\times10^{-2}T_\infty^2-1.580\times10^{-2}T_\infty^3-7.44\times10^{-3}T_\infty^4+\cdots\right) \tag{35}$$

The value of the stream function at the center of the cavity gives:

$$\psi_c = 0.733\left(\frac{ARa^*\sqrt{K^*}}{a\gamma}\right)^{1/2} \tag{36}$$

The average Nusselt number, characterizing the contributions of natural convection and solar radiation and taking into account the two permeabilities is written:

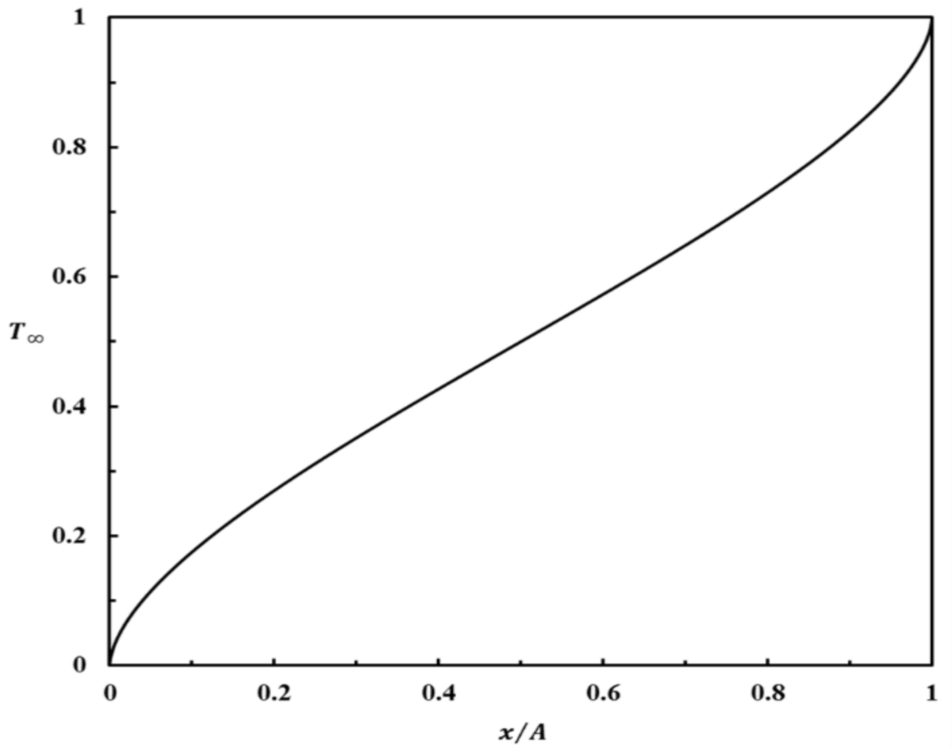
$$Nu = 0.509\left(\frac{Ra^*\sqrt{K^*}}{aA\gamma}\right)^{1/2} \tag{37}$$

In all of these equations, the modified Rayleigh number is given by the following expression:

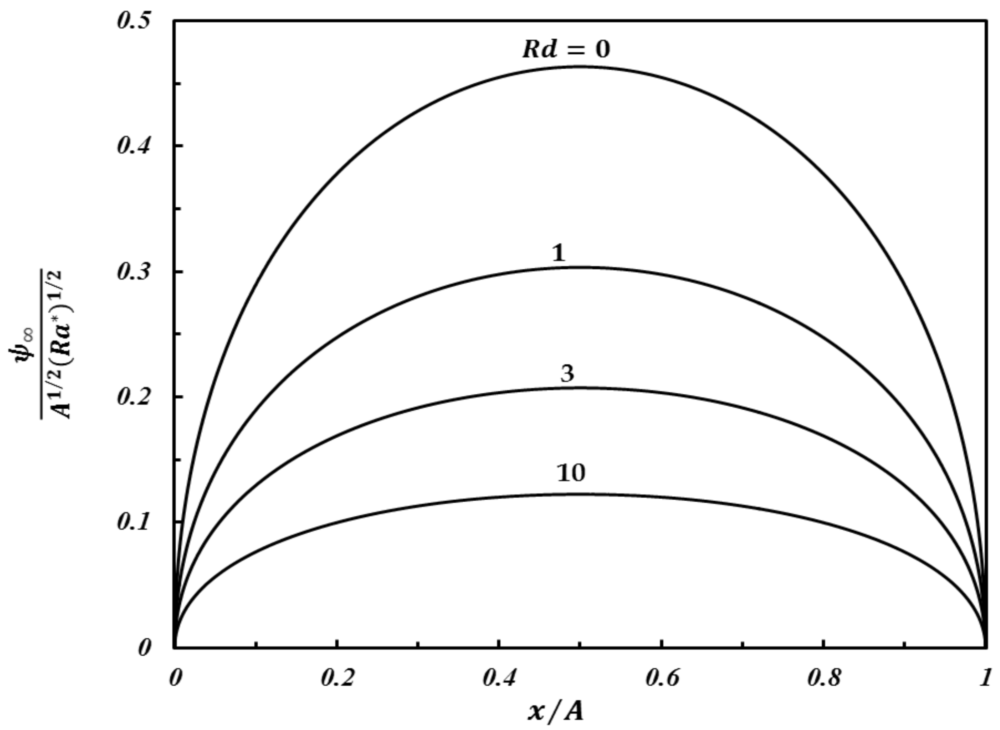
$$Ra^* = \frac{g\beta\Delta T' L'\sqrt{K_1K_2}}{\alpha\nu}=\frac{Ra}{\sqrt{K^*}} \tag{38}$$

4. Results and Discussion

Indeed, to analyze the impact of these parameters on the flow in this cavity, we have evaluated the distributions of the temperature and stream function in the central region of the cavity and the average heat transfer rate, the respective expressions of which are given by equations (34); (36) and (37).



(a)



(b)

Figure 2: The core solution along a vertical plane through the center of the cavity for $K^* = 0.25$ and $\varphi = 45°$; (a) temperature profile; (b) stream function distribution

Figures 2 (a) and 2 (b) show the effects of the radiation parameter Rd respectively on the temperature and stream function distributions in the core structure, for a porous medium with an anisotropic orientation φ and a permeability ratio K^*. The temperature profile in the central region $T_\infty(x)$ shown in figure 2 (a) increases based on the x/A distance from zero to unit.On the one hand, there is a symmetry of the temperature distribution with respect to the center of the cavity. On the other hand, it can be seen that the thermal temperature gradient at the top horizontal wall is not zero contrary to the boundary condition. This behavior is justified by the approximations made in the analysis equation (14). At the lower boundary, the temperature T takes the value zero, which satisfies the boundary condition imposed on the lower horizontal wall.

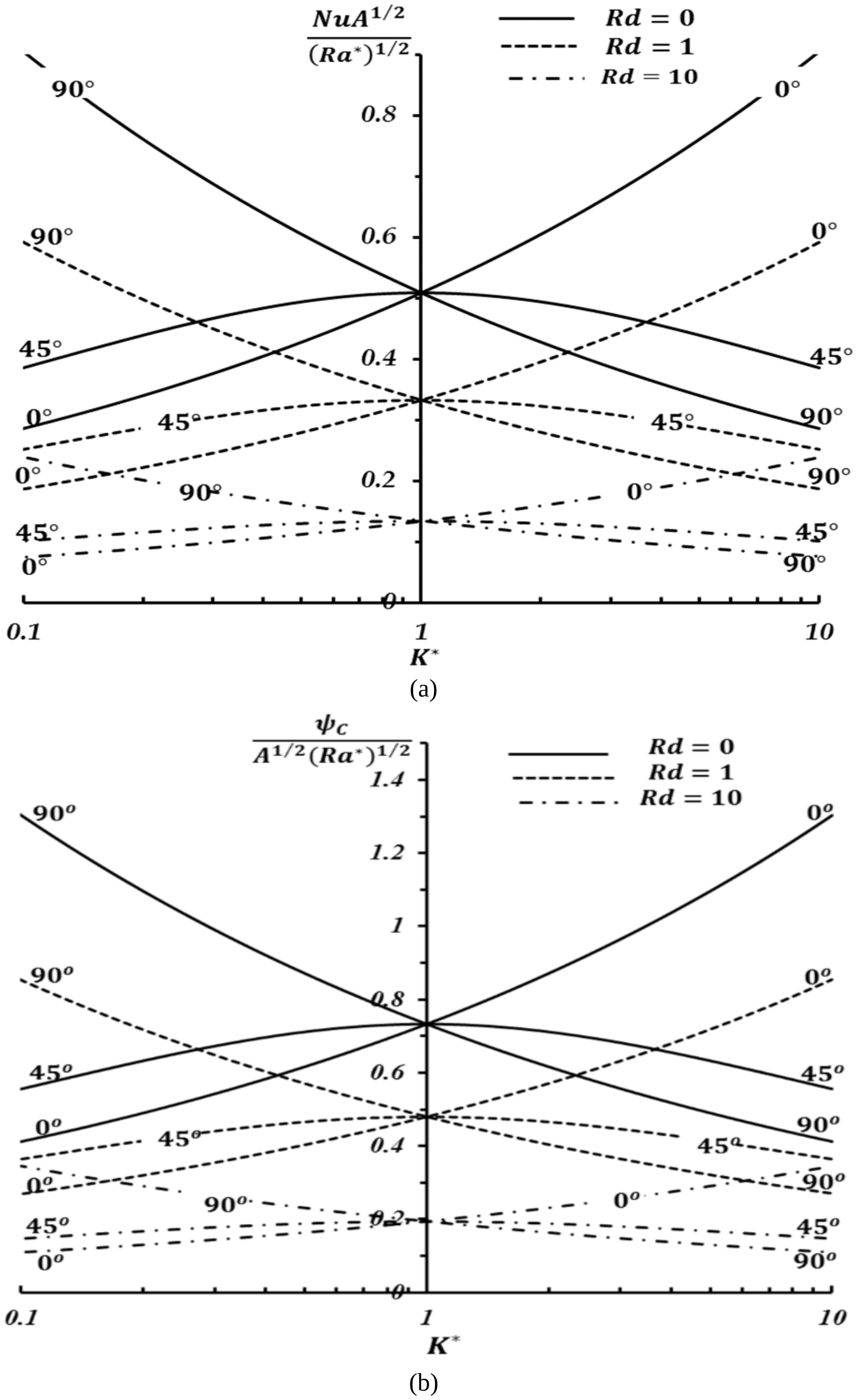


Figure 3: Effects of the permeability ratio K^* and the orientation angle φ on Nusselt number (a) and stream function (b) at the center of the cavity for $Rd = 0;1;10$.

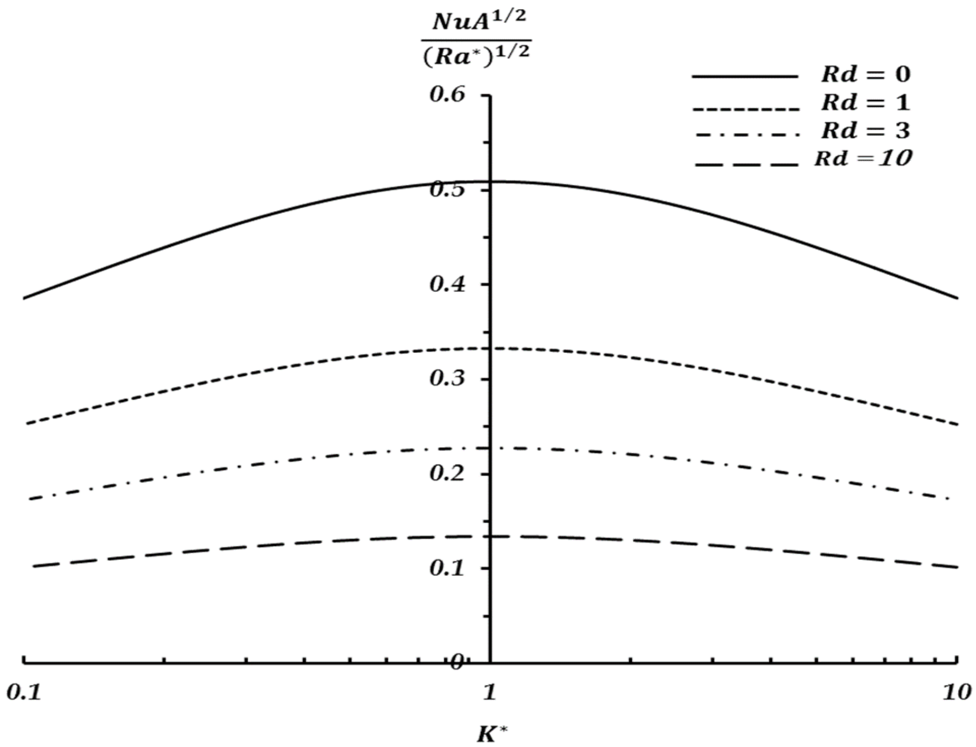
Bejan [16] who the same result is found by carried out a detailed analysis of the importance of this approximation on the solution by considering an isotropic porous medium $(K^* = 1)$. He indicated that the zero flow condition applied to the horizontal walls proposed by Weber [15] does not confirm the argument based on the impermeability and adiabacity of those walls.

The effects of the radiation parameter Rd on the average heat transfer rate Nu and the stream function ψ_c in the center of the cavity through the parameters $NuA^{1/2}/(Ra^*)^{1/2}$ and $\psi_c/(ARa^*)^{1/2}$ varying as a function of the permeability anisotropy ratio of the porous medium K^* for different values of the angle of orientation φ of the main axes, are illustrated in figure 3 (a) and 3 (b).

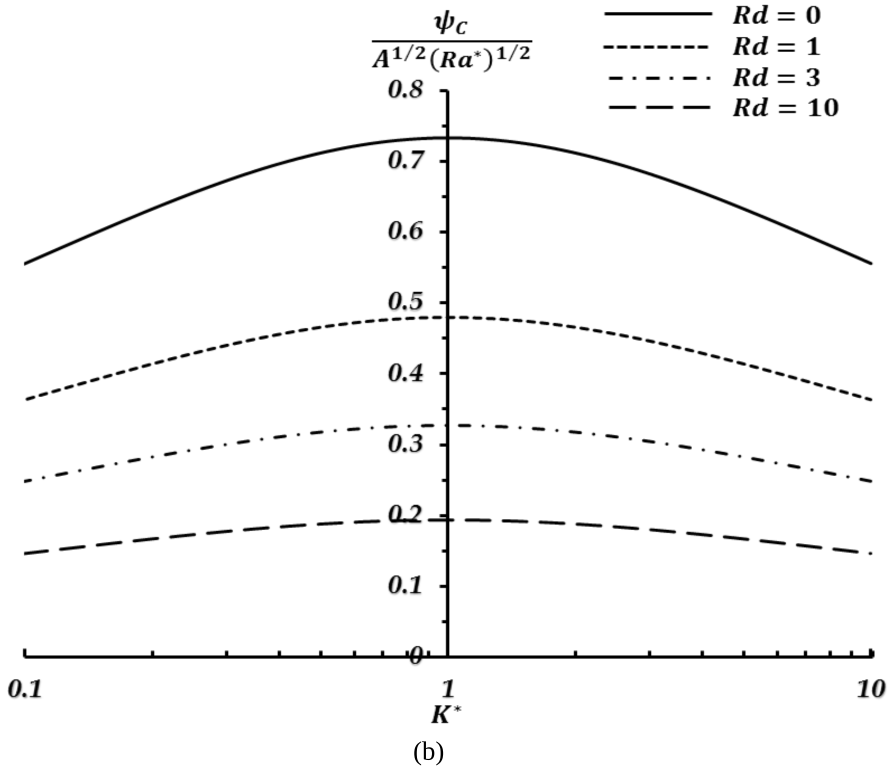
As discussed by Aboubi and al. [17], Degan and Vasseur [5], the modified Rayleigh number defined by equation (38) is more appropriate to describe the present phenomenon, since the extreme permeabilities K_1 and K_2 are among the effects normally associated with any change in Rayleigh number. Indeed from equations (36) and (37), we observe that the parameters $NuA^{1/2}/(Ra^*)^{1/2}$ and $\psi_c/(ARa^*)^{1/2}$ depend only on the radiation parameter Rd given by equation (17) and the anisotropic properties of the porous medium, namely the ratio of anisotropy in permeability K^* and the orientation angle φ across the constant $a = \cos^2\varphi + K^*\sin^2\varphi$. Moreover, for a given value of the radiation parameter Rd and the modified Rayleigh number Ra^*, we can deduce that if $\psi(x, y)$ and $T(x, y)$ are solutions for (Ra^*, A, φ, K^*), then they are also solutions for $(Ra^*, A, \pi/2-\varphi, K^*)$. This remark is illustrated in figure3a and figure3b, where the symmetry obtained with respect to the vertical line at $K^* = 1$ results from the logarithmic scale which was used for the ratio of anisotropy K^*.

When $\varphi = 45^o$, φ equals $\pi/2-\varphi$ and the corresponding curves are perfectly symmetrical to K^* so that the results obtained for a given value of K^*, are identical to those for $1/K^*$. However, when is $varphi$ different from 45^o, the symmetry observed for a given set of anisotropic properties φ, is achieved for and . In addition, figure 3 (a) and 3 (b) show that for a given value of the angle of orientation , an increase in the radiation parameter causes a decrease in the average Nusselt number and the value of the stream function at the center of the cavity.

Figure 4 (a) and 4 (b) show the effects of the anisotropy ratio in permeability K^*, and the radiation parameter Rd respectively on the average Nusselt number Nu and the stream function ψ_c in the center of the cavity for $\varphi = 45^o$.



(a)



(b)

Figure 4: Effects of the permeability ratio K^* *and the radiation parameter* Rd *for* $\varphi = 45^o$ *on Nusselt number (a) and stream function (b) at the center of the cavity.*

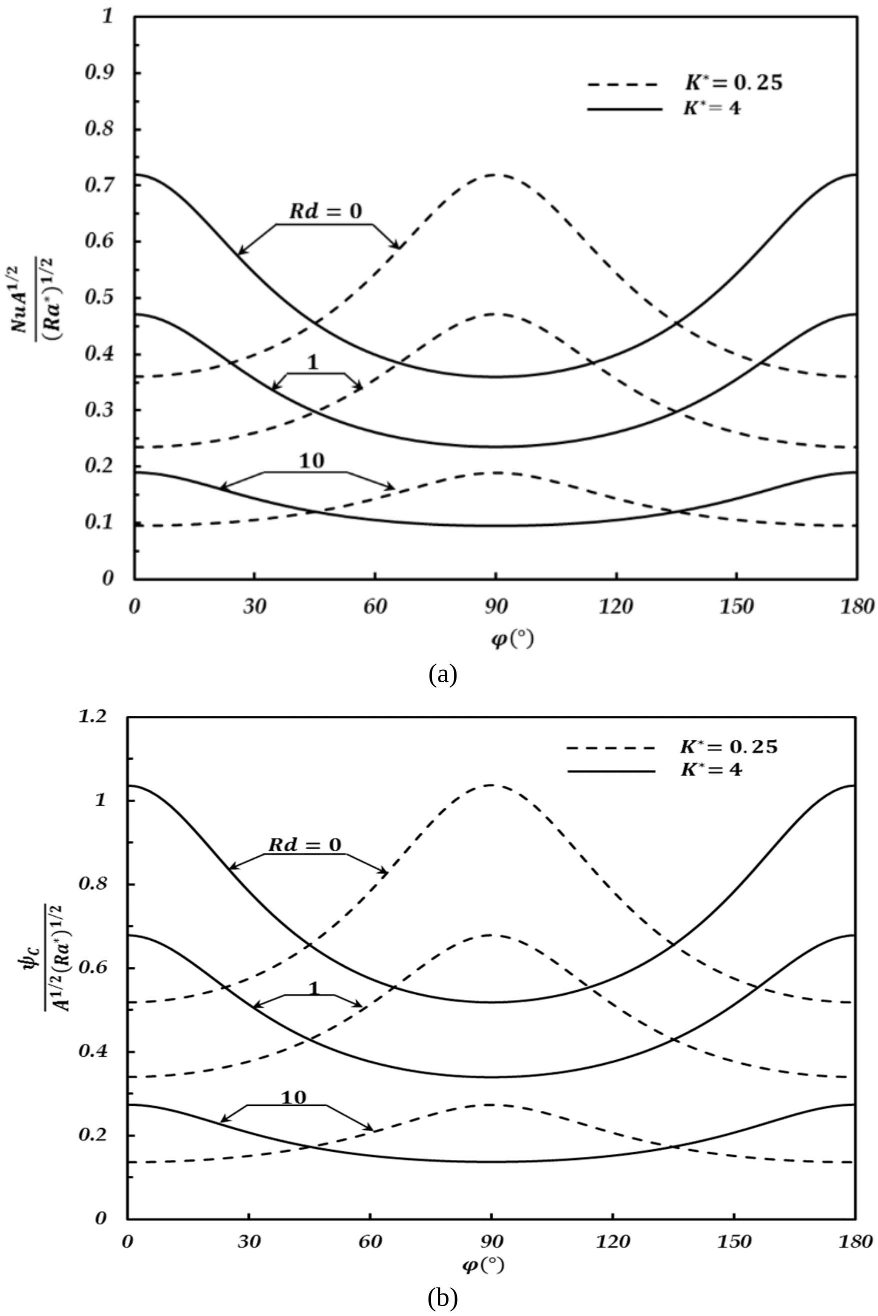


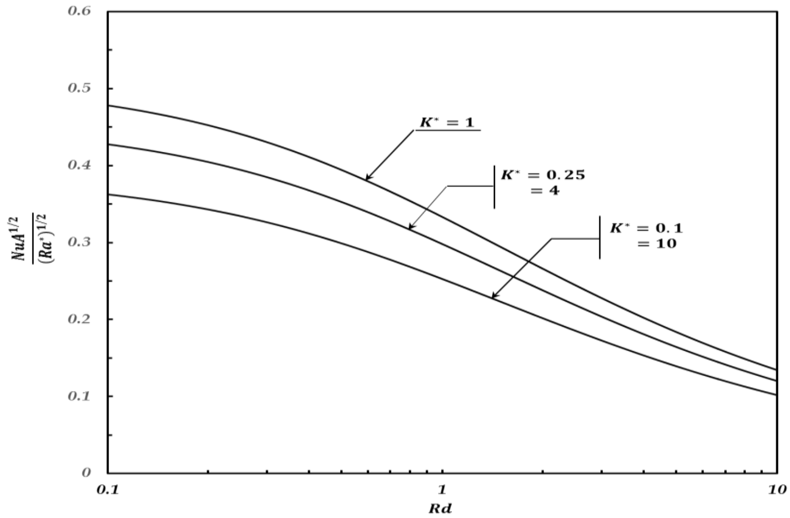
Figure 5: Effects of the radiation parameter Rd and the orientation angle φ on Nusselt number (a) and stream function (a) at the center of the cavity.

For reasons mentioned in the previous paragraph, we note that the obtained curves are symmetrical with respect to the vertical line $K^* = 1$. We can also easily deduce from the governing equations that for a given value of the radiation parameter Rd, the solution obtained for $\varphi = 45^o$ and for a particular value of K^*, corresponds to that obtained for $\varphi = 45^o$ and $1/K^*$. For a given value of the anisotropy ratio in permeability, it is noted that the Nusselt number and the stream function increases with a decrease in the value of the radiation parameter Rd.

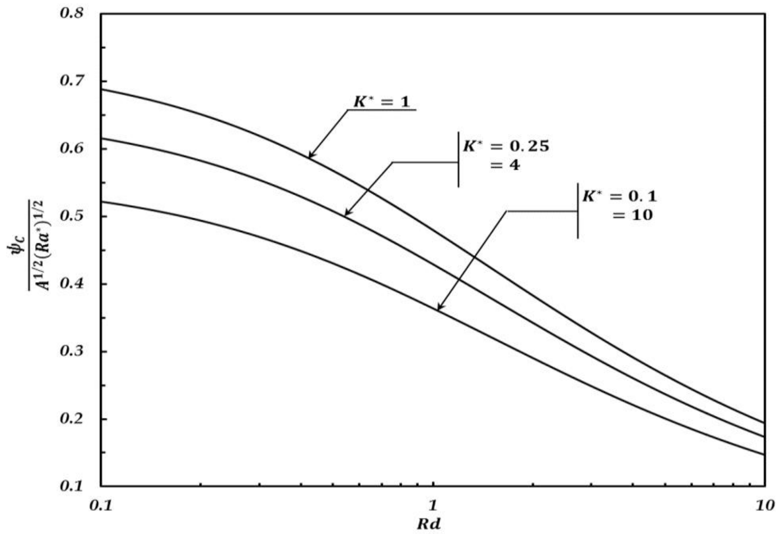
The influence of the orientation angle φ of the permeability axes on the main average heat transfer rate Nu and the stream function at the center of the cavity is illustrated in figures 5 (a) and 5 (b), respectively K^* equal to 0.25 and 4. The results show that Nu and ψ_c both highly depend on the permeability orientation angle φ

of the porous medium. These figures indicate that for $K^* = 0.25$, heat transfer by natural convection is maximum when $\varphi = 90^o$ the angle of orientation for which the permeability in the vertical direction is maximum, but is minimum $\varphi = 0^o$ at $\varphi = 180^o$ and when the permeability in the vertical direction is minimal.

The opposite situation is observed for $K^* = 4$. In this case, the intensity of single-celled convective motion and the resulting heat transfer are minimal at $\varphi = 90^o$, and maximum at $\varphi = 0^o$ and $\varphi = 180^o$. The fact that for $K^* > 1$ ($K^* < 1$), Nu is maximum (minimum) at $\varphi = 0^o$ and $\varphi = 180^o$; minimum (maximum) at $\varphi = 90^o$ can easily be deduced from the first and second derivatives of Nu compared to φ (equation 37). It follows from these observations that heat transfer is maximum (minimal) when the orientation of the principal axis of the medium with the highest permeability is parallel (perpendicular) to the gravitational field. These results are similar to those obtained numerically in the past by Zhang [18]; Degan and al. [19]; Degan and Vasseur [5;20] who have studied the influence of the orientation angle of the main permeability axes on heat transfer in a vertical cavity heated by the vertical walls.



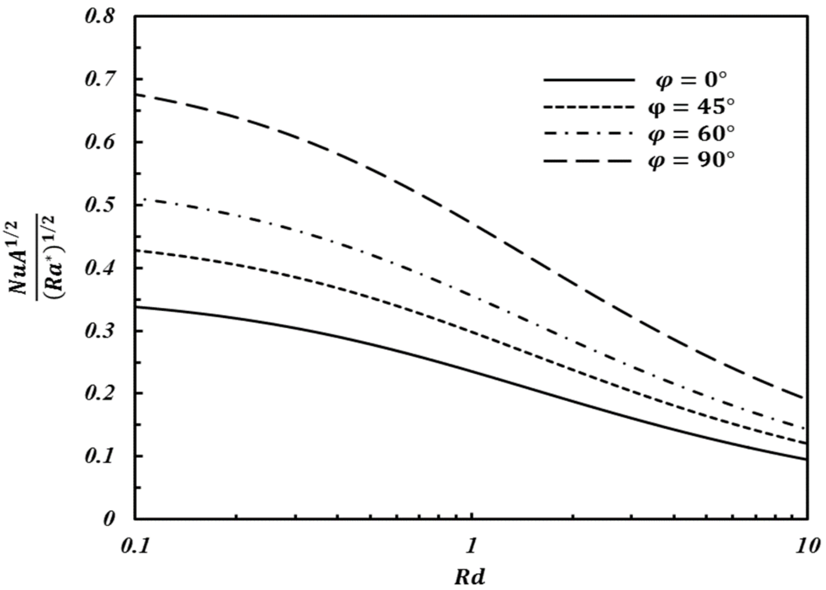
(a)



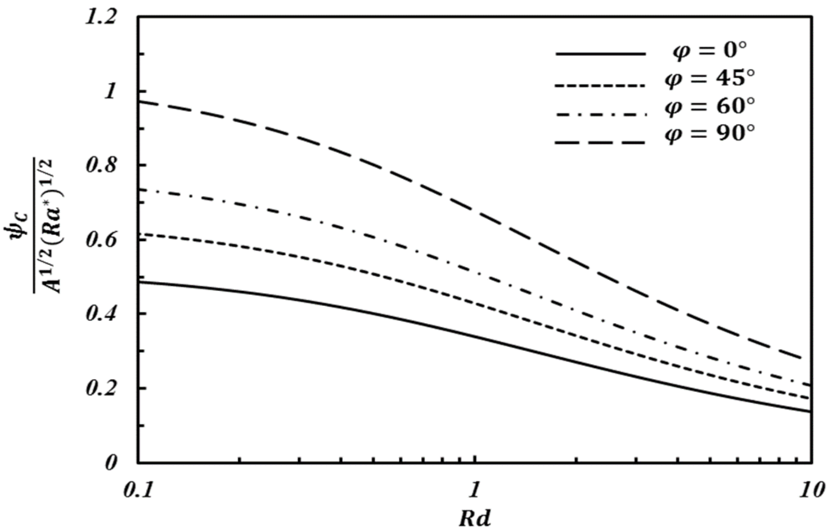
(b)

*Figure 6: Variation of Nusselt number (a) and stream function (b) at the center of the cavity according to the radiation parameter **Rd** for $\varphi = 45°$*

Figures 6 (a) and 6 (b) show the influence of the radiation parameter Rd on the heat transfer rate and the stream function at the center of the cavity for different values of the ratio of anisotropy to permeability, respectively and $\varphi = 45^o$. The curves show that the average Nusselt number and the stream function at the center of the cavity are decreasing functions of the number of radiation parameters Rd . The results found in figures 6 (a) and 6 (b) show that the corresponding curves for $K^* = 0.1$ and 10 on the one hand, $K^* = 0.25$ and 4 on the other hand are confused for $\varphi = 45^o$. This finding, which is a consequence of the standardization adopted in this study, is verified only for $\varphi = 45^o$.



(a)



(b)

Figure 7: Variation of Nusselt number (a) and stream function (b) at the center of the cavity according to the radiation parameter Rd for $K^* = 0.25$

On figures 7 (a) and 7 (b), the variation of the average Nusselt number Nu and the stream function ψ_c at the center of the cavity is represented by the radiation parameter Rd for different values of the orientation angle of anisotropy φ and $K^* = 0.25$. As the radiation parameter Rd becomes increasingly important, the heat transfer rate Nu and the stream function at the center of the cavity ψ_c decreases from its maximum. In addition, an increase of φ , from 0^o to 90^o for $K^* = 0.25$, corresponds to an increase in the value of the heat transfer rate and the value of the stream function at the center of the cavity.

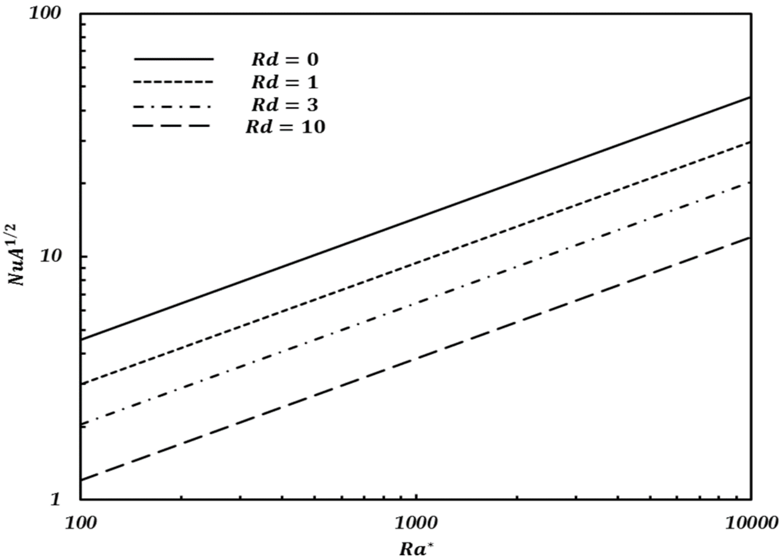


Figure 8: Effects of the Rayleigh number Ra^* and the radiation parameter Rd for $K^* = 0.25$ and on Nusselt number.

Figure 8 illustrates the variation of the average heat transfer rate Nu in porous media as a function of the Rayleigh number modified Ra^* for different values of the solar radiation parameter when $K^* = 0.25$ and $\varphi = 45^o$. It can be seen that when the anisotropic parameters and the radiation parameter Rd are kept constant, the average Nusselt number through the parameter is a linear function of the modified number of Rayleigh Ra . In addition, an Rd increase from 0 to 10 for $K^* = 0.25$ and $\varphi = 45^o$, corresponds to a decrease in the average rate of heat transfer in porous medium. It is important to note that in boundary layer conditions, as reported by Degan and Vasseur [5], heat transfer is proportional to $Ra^{*1/2}$.

5. Conclusion

In this paper, we have studied the problem of the influence of solar radiation on the natural convection heat transfer, in steady state in a vertical cavity confining a porous anisotropic medium and isothermally heated by the side walls. The governing equations were established using generalized Darcy's law taking into account boundary layer approximations. We have considered permeability anisotropy, having its principal axes oriented arbitrarily with respect to the gravitational field. The following conclusions emerge from this study:

• The anisotropic parameters and the solar radiation parameters have a significant influence on the flow structure and on the average rate of heat transfer by natural convection. The intensity of the flow and the transfer rate are improved for the low values of the radiation parameter Rd . The heat transfer rate tends asymptotically towards unity for large values of Rd .

• The intensity of the convective flow and the average rate of heat transfer advance at the same direction. Moreover, for an anisotropy ratio value $K^* < 1$, the latter are maximal when the orientation angle $\varphi = 90^o$ and minimum when $\varphi = 0^o$ and $\varphi = 180^o$. In the case where $K^* > 1$, the intensity of the flow and the Nusselt number are maximum when $\varphi = 0^o$ and $\varphi = 180^o$, and minimum when $\varphi = 90^o$.

• The results found show that the convective flow and the average heat transfer number, for an orientation angle Ï† of the principal axes, the solution for a given set of control parameters (Ra^*, Rd, A, φ and K^*) is equivalent to that corresponding to the parameters($Ra^*, Rd, A, \pi/2-\varphi$ and $1/K^*$). Moreover for $\varphi = 45^o$ the solutions for fixed values of K^* , and are perfectly symmetrical with respect to $K^* = 1$, so that the results for a value of K^* are equivalent to those obtained for $1/K^*$.

References

[1]. Vafai, K. (2000). Handbook of porous media. *Marcel Dekker, New York*.

[2]. Nield, D. A. and Bejan, A. (2013). Convection in Porous Media. *New York, NY: Springer New York*.

[3]. Bejan, A. (2013). Convection heat transfer. . *4. ed. Chichester: Wiley*.

[4]. Baytas A. C. and Pop, I. (1999). Free convection in oblique enclosures filled with a porous medium. *Int. J. Heat Mass Transf.* 42 (6): 1047‑1057.

[5]. Degan, G. and Vasseur, P. (1996). Natural convection in a vertical slot filled with an anisotropic porous medium with oblique principal axes. *Numer. Heat Transf. Part Appl.* 30(4): 397‑412.

[6]. Simpkins, P. G. and Blythe, P. A. (1980). Convection in a porous layer. *Int. J. Heat Mass Transf.* 23(6): 881‑887.

[7]. Ettefagh, J. and Vafai, K. Natural (1988). Convection in open-ended cavities with a porous obstructing medium. *Int. J. Heat Mass Transf.* 31(4): 673‑693.

[8]. Badruddin, I. A., Zainal, Z. A., Aswatha Narayana, P. A. and Seetharamu, K. N. (2006). Heat transfer in porous cavity under the influence of radiation and viscous dissipation. *Int. Commun. Heat Mass Transf.* 33(4): 491‑499.

[9]. Ahmed, S.E., Hussein, A.K., Mohammed, H.A., Adegun, I.K., Zhang, X., Kolsi, L., Hasanpour, A. and Sivasankaran, S. (2014). Viscous dissipation and radiation effects on MHD natural convection in a square enclosure filled with a porous medium. *», Nucl. Eng. Des.* 266: 34‑42.

[10]. Mahapatra, T. R., Pal, D. and Mondal, S. (2012). Combined effects of thermal radiation and heat generation on natural convection in a square cavity filled with Darcy-Forchheimer porous medium. *Int. J. Applied Mathematics and computation*, 4(4):359-368.

[11]. Zahmatkesh, I. (2007). Influence of thermal radiation on free convection inside a porous enclosure. *Emirates Journal for Engineering Research*, 12(2): 41-52.

[12]. Rani, A. V. S., Sugunamma, Dr. V. and Sandeep, N. (2012). Radiation effects on convective heat and mass transfer flow in a rectangular cavity. *International Journal of innovation and applied studies*, 1(1): 118-152.

[13]. Mansour, M. A., Abd El-Aziz, M. M., Mohamed, R. A. and Ahmed, S. A. (2011). Numerical simulation of natural convection in wavy porous cavities under the influence of thermal radiation using a thermal non-equilibrium model. *Transp. Porous Med.* 86: 585-600.

[14]. Gill, A. E. (1966). The boundary-layer regime for convection in a rectangular cavity. *J. Fluid Mech.* 26(3): 515-536.

[15]. Weber, J. E. (1975). The boundary-layer regime for convection in a vertical porous layer. *Int. J. Heat Mass Transfer*, 18:569-573.

[16]. Bejan, A. (1979). On the boundary layer regime in a vertical enclosure filled with a porous medium. . *Letters in Heat and Mass Transfer*, 6(2): 93–102.

[17]. Aboubi, K., Robillard, L., and Bilgen, E. (1995). Convective heat transfer in an annular porous layer with centrifugal force field. *Numerical Heat Transfer, Part A: Applications*, 28(3): 375–388.

[18]. Zhang, X. (1993). Convective heat transfer in a vertical porous layer with anisotropic permeability. *Proceeding 14th Canadian congress of applied mechanics.* 2:579-580.

[19]. Degan, G., Vasseur, P., and Bilgen, E. (1995). Convective heat transfer in a vertical anisotropic porous layer. *International Journal of Heat and Mass Transfer*, 38 (11): 1975–1987.

[20]. Degan, G., and Vasseur, P. (1997). Boundary-layer regime in a vertical porous layer with anisotropic permeability and boundary effects. *International Journal of Heat and Fluid Flow*, 18 (3), 334–343.