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Magnetic Field Effect on the Onset of Darcy-Brinkman Convection in a Thin Porous Layer Induced by Concentration Based Internal Heating

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Abstract The effect of magnetic field on the onset of Darcy-Brinkman convection in a thin porous layer induced by concentration based internal heating using linear stability analysis technique is investigated. The onset criterion for both stationary and oscillatory convection on the stability of the system is obtained. The result shows that, Vadasz and Darcy numbers have a stabilizing effect on the system for both stationary and oscillatory modes, while the internal heat parameter destabilizes the system.

Keywords Darcy-Brinkman convection, porous layer, concentration-based, Magnetic field, compressible fluid, reference system

1. Introduction

The concept of convective instability in a fluid saturated porous layer driven by buoyancy has gained prominence among scholars the world over in the last decade and beyond since Lord Rayleigh's groundbreaking discovery in 1900. When two dense components with differing diffusivities are made to exhibit convection rolls due to temperature and concentration, this process is called double-diffusive or thermosolutal convection. This interest from scholar's stem from its wide range of applications both in the physical, engineering and sciences. Among the numerous applications include: Coal combustors, insulation engineering, magma chambers, petroleum reservoir, crystal growth, electrochemistry, geophysical systems, earth's ocean, liquid gas storage, contaminant transport and others. [1-4]. exhaustive account of double diffusive studies can be found in the following: [1-4], Ingham and Pop [5], Vafai [6], Pop and Ingham [7].

Earlier studies on the thermal instabilities of double diffusive convection (thermosolutal convection) in a porous layer saturated with fluid using the linear stability analysis was done by Nield [8]. In this study, he established the Rayleigh criterion for the onset of both the stationary and oscillatory convection under temperature and specie boundary conditions. Taunto et al [9] extended [8] to include the weakly nonlinear stability analysis where only the heat and mass transfer significantly affected the configuration of the system. The linear and nonlinear stability analysis of a binary fluid mixture in a porous medium heated from below was studied by Lombardo et al [10]. The analysis showed that the parameters are unchanged for both the linear and nonlinear at the exchange of stabilities. Israel-Cookey et al [11] analytically studied the effect of magnetic field and concentration based internal heat source. It was found that the presence of magnetic field significantly imparted the system. Also, the effect of internal heat source on the onset of double diffusive convection in a rotating nanofluid with feedback control has been studied by Khalid et al [12]. The result showed that internal heat source, Dufour and Soret destabilized the system by decreasing the Rayleigh number, whereasthe feedback control delayed the onset of instability. Malashetty et al [13] considered the double diffusive studies in a Darcy porous medium with the effect of rotation. In this study, the advective and Forchheimer inertia effects are ignored due to small velocities in the fluid and the solid phase of the fluid are assumed be in equilibrium with impermeable boundaries. It was found that Taylor number, couple stress parameter and solutal Rayleigh number all stabilized the system while the onset of instability for both the stationary and oscillatory modes are delayed by Lewis number.

Tan Shao [14] conducted a nonlinear stability analysis of a Soret driven double diffusive convection in a porous medium of Maxwell fluid. The analysis showed that, relaxation time leads to a destabilization of the system, while both negative and positive Soret parameter destabilizes the system only for the oscillatory mode. Additionally, for the nonlinear analysis, relaxation time and Soret parameter both decreases the heat transfer in the system. Zhixin et al [15] analytically examined double diffusive convection in a viscoelastic fluid saturated porous medium using the non-equilibrium model. The study assumed that the porous media is slow so momentum balance equation can be linearized, and the porous matrix and saturating fluid determines the nature of heat transfer at the interface. The study showed, increase in relaxation time hastens the onset of instability in the system, while the heat parameter is affected by both the non-equilibrium model and relaxation time. Quite recently, Israel-Cookey et al [16] holistically studied thermosolutal convection in a porous medium with concentration based internal heat source in the presence of Soret and magnetic field. In this study, the onset of instability was hastened by Soret parameter while magnetic field stabilizes the system. Agarwal and Bhargava [17] examined the internal heat source effect on the onset of Darcy-Brinkman convection in a nanofluid saturated porous layer using the Galerkin weighted residual method. the study shows that Lewis number, internal heat source, Rayleigh number, modified diffusivity, and nanoparticles triggers instability in the system while Darcy number and porosity stabilized the system. The onset of Darcy-Brinkman double diffusive convection in a fluid saturated porous layer was studied by Shao et al [18]. The result obtained is accurate and worthy reference for studies using Darcy-Brinkman convection. Mansour [19] investigated the unsteady double diffusive convection with the effects of chemical reaction and thermal reaction. The resulting nonlinear partial differential equation was solved using Finite Difference method. It was found that the thermal Rayleigh number increased with increase in the Nusselt, Sherwood, Darcy number and Buoyancy ratio. Also, the heat and mass transfer significantly increased, whereas increase in thermal radiation increased the Nusselt number. Kumar et al [20] investigated the effects of rotation and magnetic field on the thermosolutal stability of a couple-stress fluid.

In this present study, we study magnetic field induced double diffusiveconvection in a thin porous layer with concentration based internal heat source using the Darcy-Brinkmann model. The conditions leading to the onset of both stationary and oscillatory convection are obtained using the linear stability analysis technique. Our objective therefore in this work is to investigate how the onset criteria is affected by the following parameters viz: magnetic field parameter, Solutal Rayleigh number, Internal heat parameter, Coefficient of viscosities, Darcy number and Vadasz number.

2. Mathematical Formulation

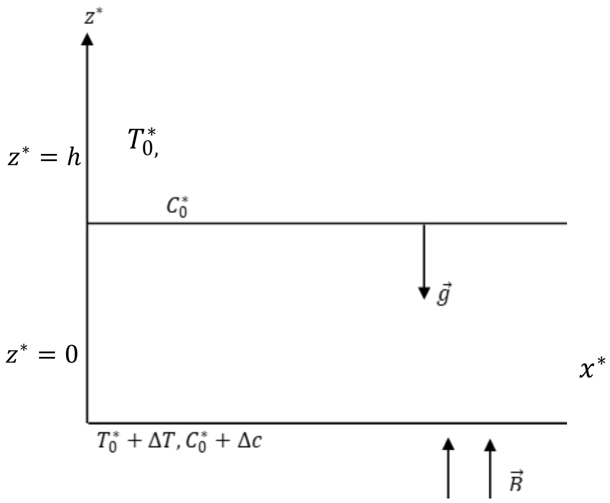

Figure 1: Schematic diagram of the problem

Consider the convection induced by concentration based internal heating in a medium consisting of a horizontal layer of an incompressible but electrically conducting fluid-saturated of height h and placed between two parallel plates located at z^*= 0 and z^*= h. A coordinate system is chosen such that the origin at the lower plane and the z^*-axis acts vertically upwards as shown in the Figure below.

The gravitational force, $\vec{g}$ act vertically downwards. Temperature and concentration gradients applied across the porous layer, such that the lower plate have temperature and concentration at $T^*(= T_0^* + \Delta T)$ and $c^*(= c_0^* + \Delta c,$ while the upper plate have temperature, $T0$ and concentration, $c0$, respectively.A external uniform magnetic field of strength $\vec{B} = (0, 0, B_0)$ is applied such that the induced magnetic field relative to it is negligible (Muller & Buhler, 2001).Assuming that, the internal heating is linearly proportional to solute concentration of the form $Q_0(c^* - c_0)$ where $Q_0$is a sign of proportionality. Following the usual Boussinesq approximation, $\rho = \rho_0[1 - \beta_T(T^* - T_0) + \beta_c(c^* - c_0)]$. Taking couple-stress into account, viscosity and neglecting viscous dissipation together with the heat transfer between the sides of wall, the governing equations are as follows

$$\vec{\nabla}^*.\vec{V}^* = 0 \quad (1)$$

$$\frac{\rho_0}{\phi}\frac{\partial \vec{V}^*}{\partial t^*} = -\vec{\nabla}^* P^* + \rho_0 g[\beta_T(T^* - T_0) - \beta_c(c^* - c_0)]\vec{k} - \frac{\mu}{\kappa}\vec{V} + \vec{F}_L \quad (2)$$

$$A\frac{\partial T^*}{\partial t^*} + \left(\vec{V}^*\vec{\nabla}^*\right)T^* = \alpha_T {\vec{\nabla}^*}^2 T + \gamma(c^* - c_0) \quad (3)$$

$$\Phi\frac{\partial c^*}{\partial t^*} + \left(\vec{V}^*\vec{\nabla}^*\right)c^* = \kappa_c {\vec{\nabla}^*}^2 c^* \quad (4)$$

The boundary conditions are

$$\left.\begin{array}{l} w^* = 0,\ T^* = T_0 + \Delta T, c^* = c_0 + \Delta c \ on\ z^* = 0 \\ w^* = 0, T^* = 0, c^* = 0\ on\ z^* = h \end{array}\right\} \quad (5)$$

3. Method of Solution

Non-dimensionalization

Using $F_L = -\sigma_c B_0^2(u^*, v^*, 0)$ and the scales $h, \frac{h^2 A}{\alpha_T}, \frac{\alpha_T}{h}, \frac{kh}{\alpha_T u}$for length, time, velocity and pressure respectively.

While, $T = \frac{(T^* - T_0)}{\Delta T_\alpha}, c = \frac{(c^* - c_0)}{\Delta c}$ represents temperature and solute concentration.

The governing equations in dimensionless form become

$$\vec{\nabla}.\vec{v} = 0 \quad (6)$$

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1 - Da\Lambda\nabla^2\right)\vec{v} + Ha^2(u, v, 0) = -\nabla P + RaT\vec{k} - Rsc\vec{k} \quad (7)$$

$$\frac{\partial T}{\partial t} + (\vec{v}.\nabla)T = \nabla^2 T + Ric \quad (8)$$

$$Le\varepsilon\frac{\partial c}{\partial t} + Le(\vec{v}.\nabla)c = \nabla^2 c \quad (9)$$

The boundary conditions are

$$\left.\begin{array}{l} w = 0, T = 1, c = 1 \text{ on } z = 0 \\ w = 0, T = 0, c = 0 \text{ on } z = 1 \end{array}\right\} \quad (10)$$

Where the dimensionless parameters are given as

$Va = \frac{\phi Pr}{Da}$is Vadasz number, $Da = \frac{\kappa}{h^2}$ is Darcy number, $Ha = \sqrt{\frac{\sigma_c B_0^2 \kappa}{\mu}}$ is the Hartman number, $Ra = \sqrt{\frac{\rho_0 g \beta_T \kappa h \Delta T}{\mu \kappa_T}}$ is Thermal Rayleigh number, $Rs = \frac{\rho_0 g \beta_T \kappa h \Delta T}{\mu \alpha_T}$ is solutal Rayleigh number, $Le = \frac{\alpha_T}{\alpha_c}$ is Lewis number, $Ri = \frac{h^2 Q_0 \Delta c}{\alpha_T \Delta T}$ is internal heat parameter, $\Lambda = \frac{\mu_e}{\mu}$ is the Brinkman number.

3.1. Basic State

Assuming the basic state of the system is time- independent and described as

$$\vec{V_b} = 0, T = T_b(z),\ c = c_b(z) \text{and}\ p_b(z) \quad (11)$$

And satisfy the equation

$$\left.\begin{array}{l}\frac{dp_b}{dz} = RaT_b(z) - Rsc_b(z)\\ \frac{d^2T_b}{dz^2} + Ric_b = 0\\ \frac{d^2c_b}{dz^2} = 0\end{array}\right\} \tag{12}$$

Together with the boundary conditions

$$\begin{array}{l} T_b = c_b = 1 \text{ on } z = 0\\ T_b = c_b = 0 \text{ on } z = 1\end{array} \tag{13}$$

Solution of Eqs. (12) on integration subject to condition (13) are

$$\begin{array}{l} T_b(z) = \frac{1}{6}[6(1-z) + z(1-z)(2-z)Ri]\\ c_b(z) = 1 - z\end{array} \tag{14}$$

3.2. Linear Stability Analysis

Linearization

To study the stability of the basic state, we superimpose small perturbations of the form

$$\vec{V} = \vec{v}_b + \vec{v}, T = T_b(z) + \theta, c = c_b(z) + \varphi, p = p_b(z) + p \tag{15}$$

where $p << p_b$, $\theta << T_b(z)$, $\varphi << c_b$ and $\vec{v}$ are the perturbed quantities

On substituting Eq.(15) into Eqs. (6) − (7), we obtain

$$\vec{\nabla}.\vec{V} = 0 \tag{16}$$

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1 - \Lambda Da\nabla^2\right)\vec{V} + Ha^2(u, v, 0) = -\nabla P + Ra\theta\vec{k} - Rs\varphi\vec{k} \tag{17}$$

$$\frac{\partial\theta}{\partial t} + (\vec{V}.\nabla)\theta + f(z)w = \nabla^2\theta + Ri\varphi \tag{18}$$

$$Le\varepsilon\frac{\partial\varphi}{\partial t} + Le(\vec{V}.\nabla)\varphi - Lew = \nabla^2\varphi \tag{19}$$

where $f(z) = \frac{\partial T_b}{\partial z} = -1 + \frac{Ri}{6}(2 - 6z + 3z^2)$ is the basic temperature gradient

Subject to

$$w = \theta = \varphi = 0 \text{ on } z = 0,1 \tag{20}$$

Next, we take double curl on Eq. (7), using Eq. (6) to eliminate the pressure term in the momentum equation together with the identity $\vec{\nabla} \times \vec{\nabla} \times \vec{v} = \nabla(\nabla.\vec{v}) - \nabla^2\vec{v}$ gives the z −component as

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1 - \Lambda Da\nabla^2\right)\nabla^2 w + Ha^2D^2w = Ra\nabla_h^2\theta - Rs\nabla_h^2\varphi \tag{21}$$

where $\nabla_h^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}$is the Laplacian in the horizontal plane; and $D = \frac{\partial}{\partial z}$.

The boundary conditions for Eq. (13) is

$$w = D^2w = 0 \text{ on } z = 0, 1 \tag{22}$$

To ascertain the thresholds for stationary and oscillatory convections, the eigenvalue problem in Eqs. (17), (18) and (20) is solved, by assuming time dependent periodic disturbances of the form. Drazin and Reid [22].

$$(w, \theta, \varphi) = [W(z), \Theta(z), \Phi(z)]f(x, y)e^{\sigma t} \tag{23}$$

where $\sigma(= \sigma_r + i\sigma_i)$ is the growth rate of disturbances and (σ_r and $\sigma_i$are real) and $f(x, y)$ is a horizontal plane tiling (x, y)in periodic form

On substituting Eq.(22) into Eqs.(17), (18)and (20) yields the eigenvalue problem

$$\left.\begin{array}{l} -f(z)w + (D^2 - a^2 - \sigma)\Theta + R_i\Phi = 0\\ Lew + (D^2 - a^2 - Le\varepsilon\sigma)\Phi = 0\\ \left[\frac{\sigma}{Va} + 1 - \Lambda Da(D^2 - a^2)\right](D^2 - a^2)w + Ha^2D^2w + aRa\Theta - a^2\end{array}\right\} \tag{24}$$

$$w = D^2w = \Theta = \Phi = 0 \text{on } z = 0,1$$

where $\nabla_h^2 f + a^2 f = 0$, and a is the wave number (Christopherson, 1940)

Now we further assume the solution of Eq. (23) as

$$[W(z), \Theta(z), \Phi(z)] = [W_0, \Theta_0, \Phi_0] \sin \pi z \quad (25)$$

Where W_0, Θ_0, Φ_0 are constants. Substitution of Eq. (24) into the eigenvalue problem (23) yield in matrix form the following system.

$$H\bar{X} = 0 \quad (26)$$

where

$$H = \begin{pmatrix} \left(\frac{\sigma}{Va} + 1 + Da\Lambda J\right)J + Ha^2\pi^2 & -a^2Ra & a^2Rs \\ 2F(z) & J + \sigma & -Ri \\ -Le & 0 & J + Le\varepsilon\sigma \end{pmatrix}$$

$\bar{X} = (W_0, \Theta_0, \Phi_0)^T$, $J = a^2 + \pi^2$ and $F(z) = \int f(z) \sin^2 \pi z dz$.

The solvability condition for non-trivial solution of the eigenvalue problem given by Eq.(25) requires that the determinant of H vanish, i.e $|H| = 0$, from which we obtain the thermal Rayleigh number, Ra as

$$Ra = \Delta_1 + i\sigma\Delta_2 \quad (27)$$

where

$$\Delta_1 = \frac{4\pi^2}{{b_0}^2 + \sigma {b_1}^2}(Jb_0 + \sigma^2 b_1)LeRs + \frac{4\pi^2}{a^2({b_0}^2 + \sigma {b_1}^2)}\Bigg[\Bigg[J^2(J + Ha^2\pi^2) + Da\Lambda J^2(J^2 - Le\varepsilon\sigma^2) - (J + Ha^2\pi^2)Le\varepsilon\sigma^2 - \frac{J^2}{Va}(1 + Le\varepsilon)\sigma^2\Bigg] b_0 + [J(J + Ha^2\pi^2)(1 + Le\varepsilon) + Da\Lambda J^3(1 + Le\varepsilon) - \frac{J}{Va}(J^2 - Le\varepsilon)\sigma^2] + \frac{J}{Va}(J^2 - Le\varepsilon)\sigma^2]b_1\sigma^2 \quad (28)$$

$$\Delta_2 = \frac{4\pi^2}{b_0^2 + \sigma^2 b_1^2}\left(b_0 - Jb_{1)}LeRs + \frac{4\pi^2}{(b_0^2 + \sigma^2 b_1^2)}[J(J + Ha^2\pi^2) + J(J + Ha^2\pi^2)(1 + Le\varepsilon) + Da\Lambda J^3(1 + Le\varepsilon\right) + \frac{J^2}{Va}(J^2 - Le\sigma^2)]b_0 + \frac{[\frac{J^2}{Va}(1 + Le\varepsilon)\sigma^2 - Da\Lambda J^2(J^2 - Le\varepsilon\sigma^2) + (J + Ha^2\pi^2)Le\varepsilon\sigma^2 - J^2(J + Ha^2\pi^2)]b_1]}{(J + Ha^2\pi^2)(Le\varepsilon\sigma^2 - J^2)} \quad (29)$$

The system is stable whenever $\sigma_r < 0$ and unstable when $\sigma_r > 0$. For neutral (marginal) stability $\sigma_r = 0$.

3.3. Stationary State

The validity of the principle of exchange of stabilities and marginal stationary convection occur when $\sigma = 0$, (i.e. $\sigma_r = \sigma_i = 0$). Setting $\sigma = 0$ in Eq. (26) the stationary Rayleigh number, Ra^{st} at which stationary convection occurs becomes

$$Ra^{st} = \frac{4\pi^2 LeRsJ}{b_0} + \frac{4\pi^2}{a^2 b_0}[J^2(J + Ha^2\pi^2) + Da\Lambda J^4] \quad (30)$$

$$= \frac{4\pi^2}{4\pi^2 J + (J + 4\pi^2 Le)Ri}\left[LeRsJ + \frac{J^2(J + Ha^2\pi^2) + Da\Lambda J^4}{a^2}\right]$$

The critical Rayleigh number Ra^{st}_{cri} for the stationary convection occurs at the critical wave number $a = a_c$ obtained by minimizing Eq.(29) according to $\frac{\partial Ra^{st}}{\partial a_c^2} = 0$. Chandrasekhar [24], Wooding, [27]

Minimizing Eq.(29) yields the following $5th$ order polynomial in $p = a_c^2$, whose coefficients depends on the physical parameters Λ, Ha, Da, Ri, Le and Rs and given by

$$a_5p^5 + a_4p^4 + a_3p^3 + a_2p^2 - a_1p_1 - a_0 = 0 \quad (31)$$

where

$a_5 = 8Da\Lambda\pi^2(4 + Ri)$

$a_4 = 4\pi^2(4 + Ri) + 4Da\Lambda\pi^4[28 + (7 + 3Ha^2)Ri]$

$a_3 = 8\pi^4[4 + (1 + Ha^2)Ri] + 32Da\Lambda\pi^6[4 + (1 + Ha^2)Ri]$

$a_2 = 4Ha^2\pi^4[(2\pi^2 + Ha^2\pi^2 + LeRs)Ri - 4\pi^2] + 8Da\Lambda\pi^8[4 + (1 + 3Ha^2)Ri]$

$a_1 = 8\pi^8[4(1 + Ha^2) + (1 + Ha^2)Ri + Da\Lambda\pi^2(4 + Ri)]$

$= 8\pi^8[(4 + Ri)(1 + Ha^2) + Da\Lambda\pi^2(4 + Ri)]$

$= 8\pi^8[(4 + Ri)(1 + Ha^2 + Da\Lambda\pi^2)]$

$a_0 = 4\pi^{10}[4(1 + Ha^2) + (1 + 2Ha^2)Ri + Ha^4Ri + (4 + Ri(1 + Ha^2))Da\Lambda\pi^2]$

Eq.(3.140) is solved numerically for various values of Ri, Rs, Ha, Da and Λ

for minimum values of $a_c\left(= \sqrt{p}\right)$each time using the Software Mathematica version 12.10. For every value of the critical wave number, a_c we calculate the stationary critical Rayleigh number Ra_{cri}^{st} using Eq.(29) above which instability sets in

3.4. Oscillatory Convection

For the onset of marginal oscillatory convection to occur, we set $\sigma_i \neq 0, \Delta_2 = 0$ in Eq. (26) and obtain the oscillatory Rayleigh number, Ra^{Os} as

$Ra^{os} = \frac{4\pi^2(Jb_0+\sigma_i^2 b_1)LeRs}{b_0^2+b_1^2\sigma^2} + \frac{4\pi^2}{a^2(b_0^2+b_1^2\sigma^2)}\{[J + Ha^2\pi^2)(J^2 - Le\varepsilon\sigma^2) + Da\Lambda J^2((J^2 - Le\varepsilon\sigma^2) - \frac{J^2}{Va}(1 +$

$Le\varepsilon\sigma 2]b0+J J+Ha2\pi 2+Da\Lambda J31+Le\varepsilon+JVa(J2-Le\varepsilon)\sigma 2]b1\sigma 2$ (32)

where the frequency of oscillation σ_i^2 is given by

$$\sigma_i^2 = \frac{[J(Va(1+Le\varepsilon)(Ha^2\pi^2+J)+J^2(1+DaVa\Lambda(1+Le\varepsilon))b_0+Va(a^2LeRs-J^2(J+Ha^2\pi^2)}{[J^2Le\varepsilon b_0-Le\varepsilon Va\left(J+\pi^2Ha^2+J^2(1+Le\varepsilon+Da\Lambda Le\varepsilon Va)\right)b_1]} \quad (33)$$

4. Analysis of Results

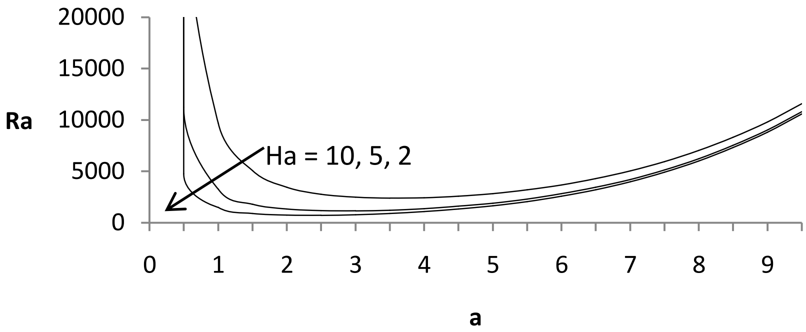

*Figure 1: Influence of magnetic field parameter, **Ha** thermal Rayleigh number, **Ra** for fixed values of $\boldsymbol{Rs} = \boldsymbol{10}, \boldsymbol{Ri} = \boldsymbol{2}, \boldsymbol{Le} = \boldsymbol{1}, \boldsymbol{Da} = \boldsymbol{1}, \boldsymbol{\Lambda} = \boldsymbol{1}$ for stationary convection.*

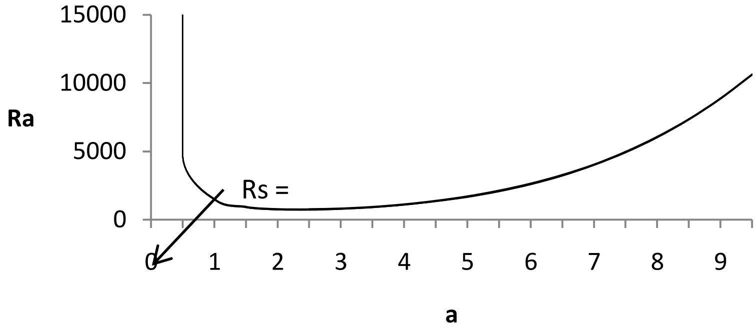

*Figure 2: Influence of solutal Rayleigh number, **Rs** on thermal Rayleigh number, **Ra** for fixedvalues of $\boldsymbol{Ha} = \boldsymbol{2}, \boldsymbol{Ri} = \boldsymbol{2}, \boldsymbol{Le} = \boldsymbol{1}, \boldsymbol{Da} = \boldsymbol{1}, \boldsymbol{\Lambda} = \boldsymbol{1}$ for stationary convection.*

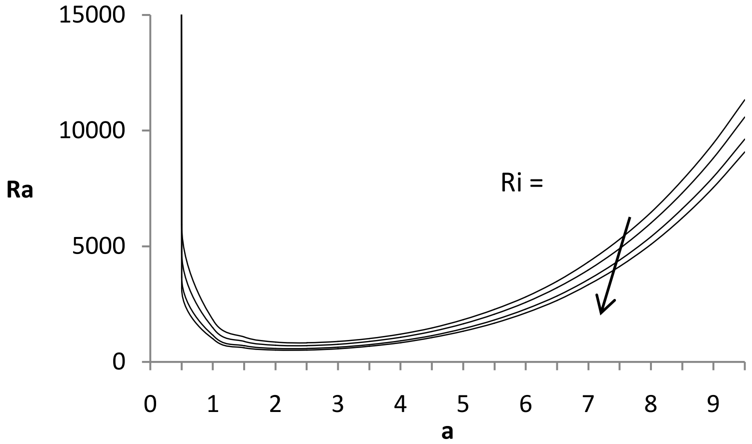

Figure 3: Variation of the thermal Rayleigh number, $\boldsymbol{Ra}$ *and the internal heat parameter,* $\boldsymbol{Ri}$ *for fixed values of* $\boldsymbol{Ha=2}, \boldsymbol{Rs=10}, \boldsymbol{Le=1}, \boldsymbol{Da=1}, \boldsymbol{\Lambda=1}$ *for stationary convection*

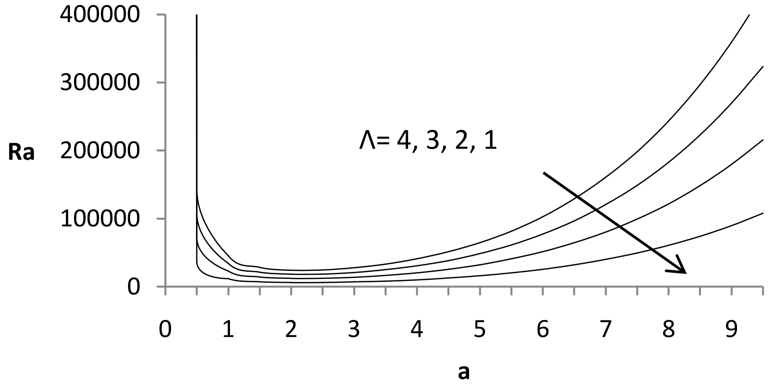

Figure 4: Variation of the thermal Rayleigh number, $\boldsymbol{Ra}$ *and coefficient of viscosities,* Λ *for fixed values of* $\boldsymbol{Ha=2}, \boldsymbol{Rs=10}, \boldsymbol{Ri=1}, \boldsymbol{Le=1}, \boldsymbol{Da=10}$ *for stationary convection*

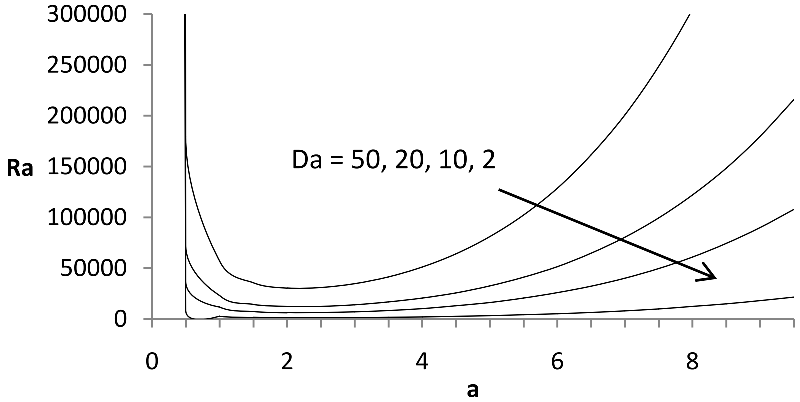

Figure 5: Influence of Darcy number, $\boldsymbol{Da}$ *on thermal Rayleigh number,* $\boldsymbol{Ra}$ *for fixed values of* $\boldsymbol{Ha=2}, \boldsymbol{Rs=10}, \boldsymbol{Ri=1}, \boldsymbol{Le=1}, \boldsymbol{\Lambda=1}$ *for stationary convection.*

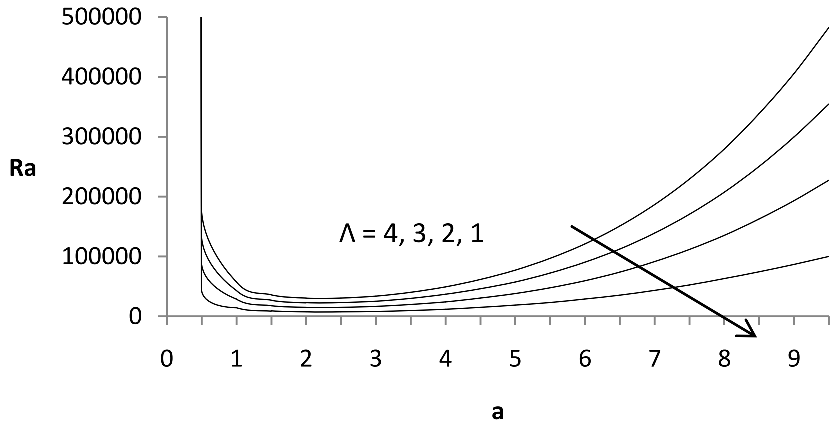

Figure 6: Variation of the thermal Rayleigh, $\boldsymbol{Ra}$ *and coefficient of viscosities,* $\boldsymbol{\Lambda}$ *for fixed values of* $\boldsymbol{Va} = \mathbf{200}, \boldsymbol{Ha} = \mathbf{2}, \boldsymbol{Ri} = \mathbf{2}, \boldsymbol{Da} = \mathbf{10}$ *for oscillatory convection.*

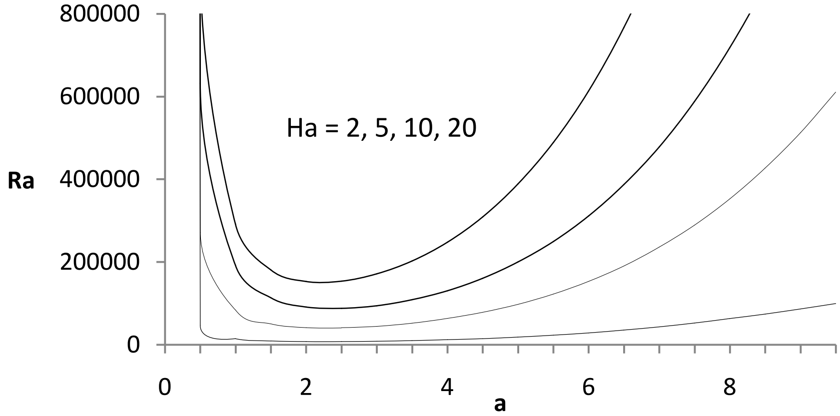

Figure 7: Influence of Magnetic field parameter, $\boldsymbol{Ha}$ *on thermal Rayleigh number,* $\boldsymbol{Ra}$ *for fixed values of* $\boldsymbol{Va} = \mathbf{200}, \boldsymbol{Ri} = \mathbf{2}, \boldsymbol{Da} = \mathbf{10}, \boldsymbol{\Lambda} = \mathbf{1}$ *for oscillatory convection*

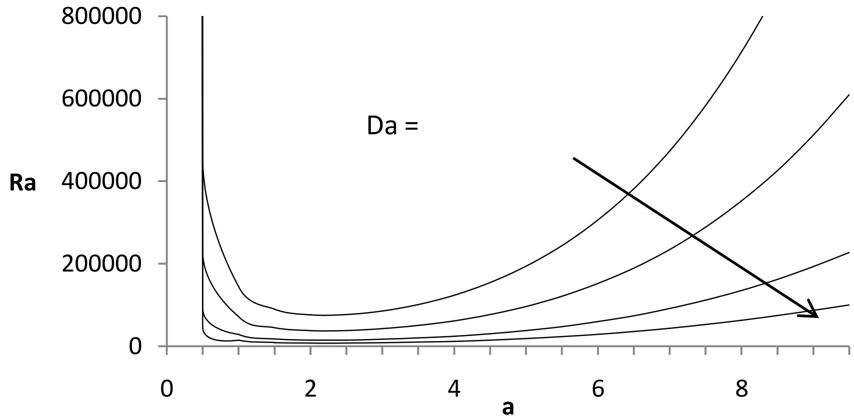

Figure 8: Variation of the thermal Rayleigh number, $\boldsymbol{Ra}$ *and Darcy number,* $\boldsymbol{Da}$ *for fixed values of* $\boldsymbol{Va} = \mathbf{200}, \boldsymbol{Ha} = \mathbf{2}, \boldsymbol{Ri} = \mathbf{2}, \boldsymbol{\Lambda} = \mathbf{1}$

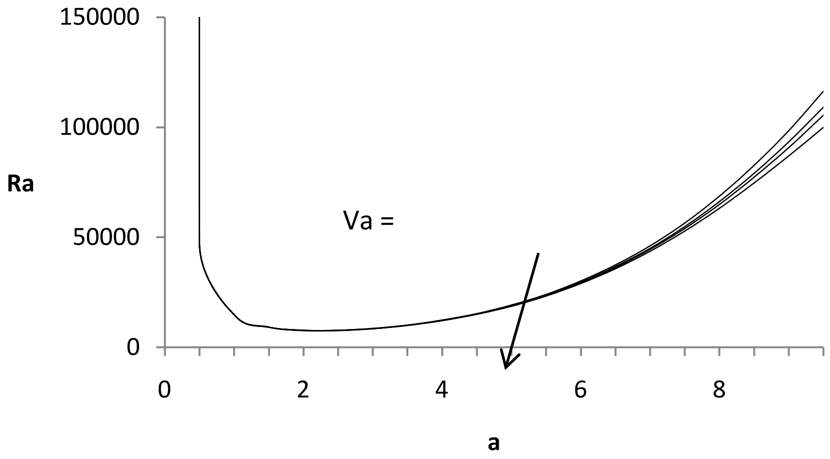

Figure 9: Variation of the thermal Rayleigh number, $\boldsymbol{Ra}$ and Vadasz number, $\boldsymbol{Va}$ for fixedvalues of $\boldsymbol{Ha} = \mathbf{2}, \boldsymbol{Ri} = \mathbf{2}, \boldsymbol{Da} = \mathbf{10}, \boldsymbol{\Lambda} = \mathbf{1}$ for oscillatory convection

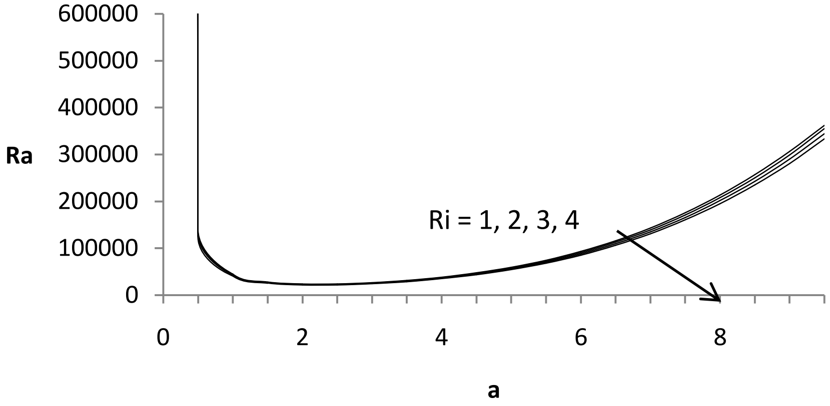

Figure 10: Influence of the internal heat parameter, $\boldsymbol{Ri}$ on thermal Rayleigh number, $\boldsymbol{Ra}$ for fixed values of $\boldsymbol{Va} = \mathbf{200}, \boldsymbol{Ha} = \mathbf{2}, \boldsymbol{Da} = \mathbf{10}, \boldsymbol{\Lambda} = \mathbf{3}$ for oscillatory convection

5. Discussion of Results

Recall the stationary Rayleigh number, Ra^{st} in Eq. (29)

$$Ra^{st} = \frac{4\pi^2 LeRsJ}{b_0} + \frac{4\pi^2}{a^2 b_0}[J^2(J + Ha^2\pi^2) + Da\Lambda J^4]$$

To validate Eq. (29) with some of the existing results in literature, we put

$Ri = 0, Da = 0$ or $\Lambda = 0$, we obtain

$$Ra^{st} = \frac{J(J + Ha^2\pi^2)}{a^2} + LeRs \quad (34)$$

The result in Eq. (34) coincides with the result of Israel-Cookey *et al*[11], Nield and Bejan [2]

Further, when $Ha = 0$, Eq. (34) reduces to

$$Ra^{st} = \frac{J^2}{a^2} + LeRs \quad (35)$$

Eq. (35) has critical value, $Ra_c^{st} = 4\pi^2$ with corresponding crtical wave number, $a_c = \pi$. This corresponds to the result of Nield and Bejan [2] for double diffusive convection in a porous medium with Darcy limit.

For a single fluid, $Rs \to 0$, the expression for the stationary Rayleigh number, Ra^{st} , in Eq. (35) reduces to

$$Ra^{st} = \frac{(\pi^2 + a^2)^2}{a^2} \quad (36)$$

The result in Eq. (36) corresponds to the results of Horton and Rogers [25], Lapwood [26].

Figure 1 shows the effect of magnetic field parameter, Ha on the thermal Rayleigh number, Ra for fixed $Rs = 10, Ri = 2, Le = 1, Da = 1, \Lambda = 1$. The result shows that, increase in magnetic field increases the thermal Rayleigh number. This is an indication that, magnetic field stabilize the system for the stationary mode.

Figure 2. We compute the influence of the solutal Rayleigh number, Rs on the thermal Rayleigh number, Ra for fixed $Ha = 2,\ Ri = 2, Le = 1, Da = 1, \Lambda = 1$. The result show that, increase in the solutal Rayleigh number, Rs increases the thermal Rayleigh number, Ra indicating that the effect of Rs is to enhance the stability of the system for the stationary mode.

Figure 3 shows the variation of the internal heat parameter, Ri on the thermal Rayleigh number, Ra for fixed values of $Ha = 2, Rs = 10, Le = 1, Da = 1, \Lambda = 1$. We found that the internal heat parameter, Ri increases with decrease in the thermal Rayleigh number, Ra, which implies that the internal heat parameter has a destabilizing effect on the system for stationary convection.

Figure 4 shows the variation of the ratio of viscosities, Λ on thermal Rayleigh number, Ra for fixed $Ha = 2, Rs = 10, Ri = 1, Le = 1, Da = 10$. The result indicates that, thermal Rayleigh number and ratio of viscosities increases proportionally which indicates that, the ratio of viscosity parameter enhances stability of the system for stationary convection.

Figure 5 depicts the influence of Darcy number, Da on thermal Rayleigh number, Ra for fixed values of $Ha = 2, Rs = 10, Ri = 1, Le = 1$. The result shows increase in Darcy number is accompanied by an increase in thermal Rayleigh number which is an indication that, the system is stabilized for stationary convection.

Figure 6 shows the effect of the coefficient of viscosity, Λ on the thermal Rayleigh number, Ra for fixed values of $Va = 200, Ha = 2, Ri = 2, Da = 10$. The result shows an increase in the coefficient of viscosity parameter lead to an increase in the thermal Rayleigh number which is an indication that the system is stabilized in the presence of the coefficient of viscosity for oscillatory convection.

Figure 7 shows the variation of magnetic field parameter, Ha and thermal Rayleigh number, Ra for fixed values of $Va = 200, Ri = 2, Da = 10, \Lambda = 1$. It is observed that increase in magnetic field lead to an increase in the thermal Rayleigh number. This indicates that magnetic field stabilize the system.

Figure 8 shows the effect of Darcy number, Da on thermal Rayleigh number, Ra for fixed values of $Va = 200, Ha = 2, Ri = 2, \Lambda = 1$. The result indicates that, increase in Darcy number lead to an increase in the thermal Rayleigh number, Ra. This indicates that, Darcy number advances the stability of the system.

Figure 9 shows the variation of Vadasz number, Va and thermal Rayleigh number, Ra for fixed values of $Ha = 2, Ri = 2, Da = 10, \Lambda = 1$. It is found that increase in Vadasz number increases the thermal Rayleigh number for oscillatory convection. This implies that, the presence of Vadasz number is to advance the stability of the system.

Figure 10 depicts the variation in internal heat parameter, Ri and thermal Rayleigh number, Ra for fixed values of $Va = 200, Ha = 2, Da = 10, \Lambda = 3$. The result shows increase in internal heat lead to a decrease in the thermal Rayleigh number. This means, internal heat hastens the onset of instability in the system.

6. Conclusion

The effect of magnetic field on the onset of Darcy-Brinkman convection in a thin porous layer induced by concentration based internal heating is investigated analytically. The criterion for the onset of stationary and oscillatory modes were obtained using the linear stability analysis method. The effects of the governing parameters on the stability of the flow is studied and the following conclusion was reached.

(i) The presence of Darcy and solutal Rayleigh numbers stabilized the system

(ii) Positive increase in the magnetic field parameter and Vadasz number enhances the stability of the system

(iii) Internal heat parameter triggers the onset of instability for both stationary and oscillatory modes, which leads to a destabilization of the system.

(iv) Coefficient of viscosity stabilizes the system.

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