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Frequencies of Gantries Free Oscillation at Internal Ball-Joints

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Abstract This paper deals with the determination of free oscillation frequencies of internal ball-joints systems having a broken configuration and supporting the action of several construction masses. At first, a gantry with an internal ball-joint was examined by the force method, and the results were compared to those obtained by stiffness matrix method proposed in this article. This method has therefore been extended to the analysis of fairly complex gantries. It has been found that for rigid structures, frequencies of the free oscillations increase and the natural period of vibration decreases. On the other hand, for the more flexible structures: the rigidity decreases, which decreases the frequencies of the free oscillations and thus increases the natural period of vibration of the structures. Also these frequencies of free oscillations depend neither on the time, nor the amplitude of the oscillations, nor the phase angle but rather on the stiffness and the mass of the structures. Application of method purposed in this article for determination of the frequencies of free oscillations of systems is very advantageous because of the reliability of the results obtained, its ease of execution and above all the time saved during the study phase.

Keywords Frequency of free oscillations, Gantries, Internal ball-joints, Stiffness Matrix Method

1. Introduction

In the modern world, engineers are trying to take up a new challenge, which is to design slender structures, thus tall buildings are in vogue, every year more and more large buildings are built. One of the most common building designs for high-rise buildings is the gantry system, which is formed by a combination of vertical (columns) and horizontal (beams) load-bearing elements. However, as a building increase in height, it must have sufficient strength and stiffness to withstand side loads imposed by wind or earthquakes. Faced with this revolution in the design of structures, dynamic loads due to wind are becoming an inevitable and significant design criterion. Conceptions based in the meantime on the static effects of loads and the neglect of the dynamic effects of wind on structures must today be understood differently. Over the last ten decades, there has therefore been a significant renewed interest in the problem of the stability of structures subject to time-dependent loads. It should therefore be noted that buildings dimensioned in the form of gantry especially when they are a little slender, behave in a rather fragile manner under the effect of strong earthquakes or a strong wind. This observed fragility is often due to a lack of resistance to gantry nodes and a poor dissipation of energy during these seismic loads [1]. According to [2], the most sensitive areas are the nodal points of the porticoes where seismic performance is reduced. These nodal areas therefore become critical areas that act as force transfer cores from the beams to the columns and are the ultimate failure site that can compromise the behavior of the entire structure.

To avoid this fragile fracture of the structure, the recommended technique is to orient the appearance of the ball-joints to promote their positioning at the level of the beams and not the posts and as far away from the nodal zones [3]. When designing structures, it may be necessary to join two members. In doing so, the connection between these members is established using a process called splicing. Splicing can be a moment connection or a

shear connection. The introduction of a shear connection limits the development of moments at the connection and only the shear forces are present. While moment connections are like fixed connections giving rise to both moment and shear. A shear or splice connection is usually called an internal ball-joint [4]. The introduction of an internal ball-joint reduces the indeterminacy of the structures of a unit. A beam embedded at one end and simply supported at the other, without internal ball joint is hyperstatic of degree1 and cannot be solved using the 3 equilibrium equations. However, if an internal ball joint is introduced, the structure becomes isostatic and therefore determined as it provides an additional equation to solve the system. In the short term, hyperstatic structures are stable compared to the isostatic structures, but can lead to stress concentration and a very rigid structure [5]. The introduction of an internal ball-joint therefore makes the structures much more flexible and sometimes isostatic. They also used to isolate the member transferring its moments on supports or on an adjacent chord and vice versa. Similarly, internal ball-joints are used to remove any redundant or incompatible moment in a structure [6].
The bending moment at an internal ball-joint is always zero. The internal ball joint makes the structure more flexible. This allows the structure to move, which reduces the reactive stresses. Internal ball joints are sometimes necessary to solve problems created by additional stresses such as settlement or temperature constraints [7]. In some cases, they are intentionally introduced so that the excess load breaks the weak zone rather than damaging other structural elements [8]. The flexibility of the structures in which the internal ball joints are introduced makes them very popular in structures subject to dynamic loads [9]. It is often difficult to use structural analysis techniques, such as the force method, the method of displacements and so one to determine the internal forces of structures under static loads. Dynamic analysis is all the more complex because inertial forces come from displacements of the structure which are in turn dependent on the frequency of the free oscillations of the structures. The coincidence of this free oscillation frequency with that of forced oscillations caused by the wind or seismic loads, or other dynamic loads, leads the phenomenon of resonance, which is very dangerous for the structures. The dynamic analysis of structures with internal ball joints that necessarily require the resolution of a static problem is very complex, even tedious when performing the structural analysis manually, especially for gantries.
The consideration of dynamic problems in the field of civil engineering is necessary to ensure the reliability of structures in many applications. The increasing need to solve complex structural dynamics problems like stepped gantries with internal ball-joints requires the development of new ideas, innovative methods and digital tools to provide accurate numerical solutions. It is in this context that in this paper, the stiffness matrix method has been used to obtain the frequencies of the free oscillations which constitute the starting point of any dynamic analysis.

2. Material and Method:

2.1. Governing equation of the dynamic movement

The systems to be examined being subjected to the action of construction masses and abandoned to themselves after excitation by one or of the force (s) external (s) are thus the seat of free oscillations by one or the external force (s) are thus the seat of free oscillations [7].The equation governing their dynamic movement is as follows:

$$[M]\ddot{X}+[K]X=0 \tag{1}$$

$[M]$: Mass Matrix; X : Displacement vector; $[K]$: Stiffness Matrix; $\ddot{X}$: Acceleration vector;

The solution of this differential equation (1) is in the form:

$$X=A\sin(\omega t+\varphi) \tag{2}$$

A: Amplitude of oscillations; ω: Oscillation frequencies; t : Time in second; φ: Phase angle.

By deferring (2) to (1) the following equation is obtained:

$$\{A\}[K]-\omega^2[M]=0 \tag{3}$$

Equation (3) represents a system of n equations with n unknowns which are the components of the vector. A non-trivial solution ($A \neq 0$) is only possible if the determinant (D) of the matrix below is zero.

$$\left[[K]-\omega^2[M]\right]=0 \tag{4}$$

Since the global stiffness matrices $[K]$ and the mass matrix $[M]$ are positive, it follows that Eq. (4) possesses N real roots ω_i^2 and by developing this determinant Eq. (4), we obtain an equation of degree N in ω_i^2, where N is the dimension of the matrix [K] and [M]: that is to say the degree of freedom of the system. Eq (4) is in the form:

$$\left[[A]-\lambda[I]\right]=0 . \quad (5)$$

In mathematics, it is a problem of eigenvalue to which an eigenvector is associated. λ is an eigenvalue assimilable to $1/\omega^2$ with ω, which is nothing other than the frequency of the oscillations. The lowest of these frequency values is called the fundamental frequency. The determination of these frequency values constitutes the object of this article. There are several different mathematical methods for the numerical solution of the eigenvalue problem. They all have advantages for certain types of problems. The movement of the building masses being harmonic, and the sine function being periodic of period 2π the period of the oscillations is easily determined by adding to time (t), time (T), in the following way:

$$\omega(t+T)+\varphi-(\omega t-\varphi)=\omega T$$
$$\omega T=2\pi \quad \Rightarrow T=\frac{2\pi}{\omega} \quad (6)$$

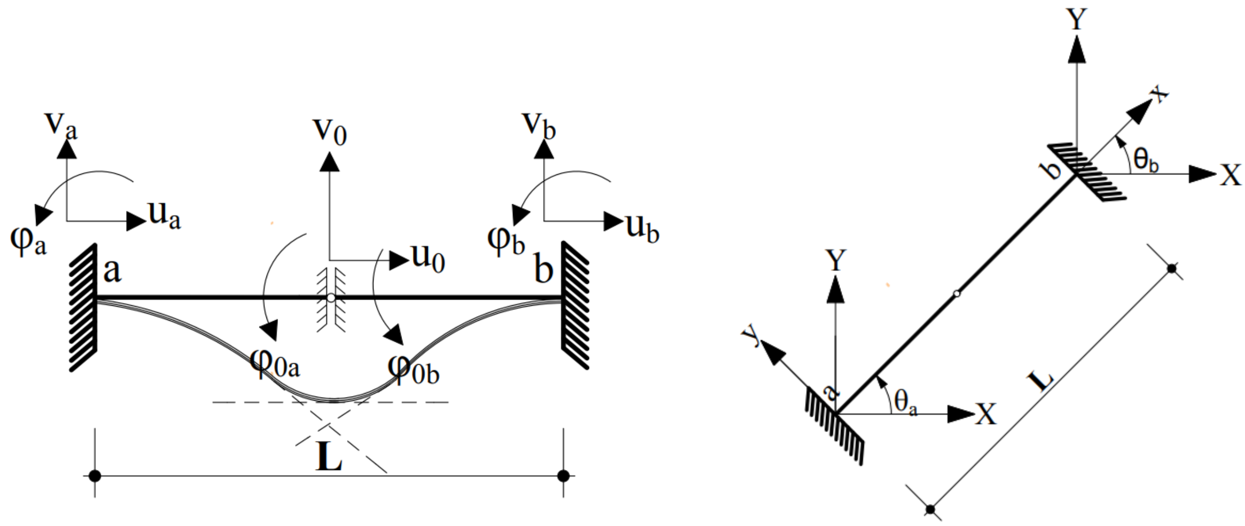


Figure 2.1: Deformed position and degree of freedom of a gantry frame

2.2. Description of the method

The stiffness matrix method is mainly used for linear static analysis. The development of this method was born in the 1940s and is generally considered as the fundamental method of finite element analysis. Linear static analysis is appropriate if deflections are small and vary only slowly. Linear static analysis omits time as a variable. This also excludes the plastic action and deformations that change the way loads are applied. The stiffness matrix method for linear static analysis follows the laws of statics and the laws of the resistance of materials. The method uses matrices and matrix algebra to organize and solve the equations of the governance system. Matrices, which are arrays of ordered numbers, are subject to specific rules, and can be used to help the solution process in a compact and elegant way. Arrays, which are arrays of ordered numbers, are subject to specific rules, and can be used to help the solution process in a compact and elegant way. The stiffness matrix method is an analytical method where the main unknowns are the displacements of the joints. This method of analysis can be extended to the dynamic analysis of systems by first calculating the frequencies of the free oscillations of the gantries; what is the purpose of this article.

The stiffness matrix method is the basis of almost all commercial structural analysis programs [10]. This is a specific case of the more general finite element method, which has been partly responsible for the latter's development. It is therefore hoped that understanding the basics presented in this article should lead to a more successful use of available computational software. The stiffness matrix method has two approaches: the direct and indirect approach [11]. The direct approach is the one that will be used in this article because it requires visual recognition of the relationship between forces and structural displacements, forces and displacements of elements induced by the charging system applied. The indirect approach is mainly used in the development of computer programs to allow automatic correlation between displacements. In other words the direct approach allows the users of computational software to better understand the concepts involved and the procedure to follow during a computer analysis. In the use of both approaches, it is necessary to develop element stiffness matrices, linked to a (local) coordinate system of the elements and a structural stiffness matrix linked to a global coordinate system.

The organization of the solution requires for each member: to identify, to number the nodes, to configure the coordinate system and to select a starting node (node 1) and an end node (node 2). An arrow will be used along each member of the system to indicate the direction of the starting node at the end node. This will establish the local coordinate system for each element. The three global degrees of freedom (dof) will be tagged at each node starting from node A and proceeding sequentially [11]. It should be noted that in flat structures, a node has three degrees of freedom because it has two translations and one rotation. Therefore, there are six possible degrees of freedom for a gantry frame and the resulting stiffness matrix is of the same order. With this method, the counterclockwise moments and counterclockwise rotations are considered positive. The positive direction of translation and rotation is also represented at each node and in the presence of a building mass. The stiffness matrix of a member is symmetrical and is in the form:

$$[k_i] = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \quad (7)$$

E: is the longitudinal elastic modulus or YOUNG modulus of the member material;
A: is the area of the cross section of a member;
I: is the moment of inertia of the section;
L: is the length of a member.

The axes that are suitable for relations with the members are called local axes, but the axes that are convenient for treating the structure as a whole are called global axes (Bishop et al., 1965). The displacement and force components can be expressed using one of the two previous systems. Stiffness matrices are often derived and defined for a local axis system. The derivation of the stiffness matrix for different types of members is probably the most delicate part of the stiffness matrix method (Argyris et al., 1964). However, there is a local or global axis system for the structure as a whole. We must therefore transform the matrices of the local coordinate system into global coordinate system matrices. The relationship between the components in the two axis systems is expressed as a matrix called transformation matrix $[T_i]$.

$$[T_i] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

The stiffness matrix$[k_i]$of a member of a gantry in the global coordinate system is in the following form:

$$[K_i] = [T_i]^T.[k_i].[T_i] \quad (9)$$

The stiffness matrices are assembled taking into account the interconnection between the different members of the system. Boundary conditions are then applied to the overall stiffness matrix to obtain a reduced stiffness matrix. These boundary conditions are necessary because without them the system will be insoluble [10] and technically the determining factor of the stiffness matrix is zero. This mathematically represents the fact that until we apply the boundary conditions, the structure moves in space. To impose known displacements in the equations of the structure, we modify the global stiffness matrix in order to recover the result of the zero displacement we know.

2.3. Resolution steps

The resolution steps by the stiffness matrix method are as follows:

- **Step 1:** Model the structure with a number of members, nodes and choose the global axis system;
- **Step 2:** Numbering nodes (A, B, C, D, ...) and degrees of freedom (dof) of the structure (0 for the prevented ddl, and 1, 2, 3, 4, ..., for the others) ;
- **Step 3:** Numbering the members and assign a direction of mileage (an arrow identifies the nodes i and j of a member);
- **Step 4:** Introduction into the Excel spreadsheet, geometric characteristics (area, inertia, length) of each member and its rotation angle θ, as well as the Young's modulus of the material constituting the member;
- **Step 5:** Formation of nodal transformation matrix $[T_i]$ of each member and calculate its stiffness matrix $[K_i]$ in the global axis system using Eq.(8);
- **Step 6:** Assembly of the overall stiffness matrix of the system (assembly of formed matrices taking into account the interconnection between the members);
- **Step 7:** Application of boundary conditions (according to step 2);
- **Step 8:** Formation of reduced stiffness matrix [kr] (by removing the rows and columns of worthless displacements), and the matrix of the masses [M] (diagonal matrix);
- **Step 9:** Formation of determinant of oscillations using equation (4);
- **Step 10:**Introduction of frequency value $\omega = 0.01$ in (4) and iteration;
  - ✓ If the determinant (D) changes sign; go to step 11;
  - ✓ Otherwise, continue the iteration until a sign change of the determinant.
- **Step 11:** Stop the iteration as soon as the determinant sign changes and retain the value of the frequency whose determinant D> 0;
- **Step 12:** Take into account the positive value of the determinant from which the sign change is observed and proceed to a frequency value increment ω =0.0001 in order to improve the accuracy in determining the frequencies.

A flowchart for calculating the frequency of free oscillation of the systems has therefore been proposed:

2.4. Flowchart for calculating the frequency of free oscillations

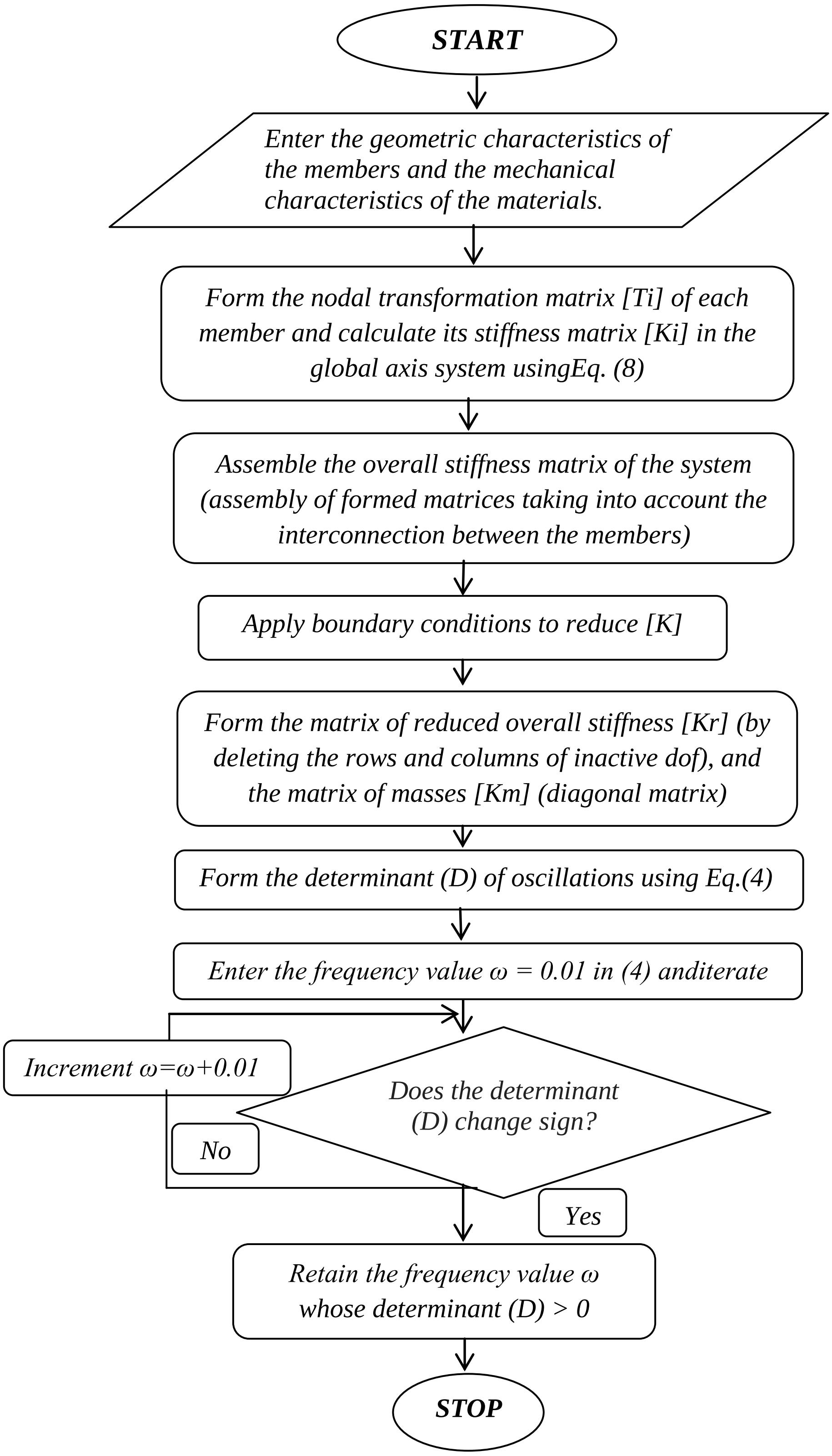


Figure 2.2: Flowchart for calculating the frequency of free oscillations (ω)

The algorithm required for calculating the frequencies of the free oscillations of the porticoes (by the stiffness matrix method) is represented by the above flowchart. This flowchart shows the important tasks to be performed in order to determine the frequency of free oscillations of the systems with as input parameters the geometrical characteristics (area; inertia, length) and the mechanical characteristics of the materials constituting the members and as output parameter the oscillation frequencies from which the free oscillation frequencies of the system are derived.

2.5. Numerical examples

The first step will be to consider a fairly simple gantry whose frequency of free oscillations will be determined by the force method and the stiffness matrix method and compare the results in order to verify the accuracy of stiffness matrix method proposed in this article. Then we will extend the proposed method to more complex gantries for which we will determine the frequencies of free oscillations and the corresponding periods.

2.5.1. Assumptions

The following assumptions are those to be considered in this study:

- Materials constituting the different members of the porticoes are assumed to be elastic;
- Gantries examined are loaded in the plane of their greatest inertia;
- Building masses are concentrated in the floors (beam);
- Floors have a high stiffness in their plan;
- Posts are embedded in the foundation;
- Displacements are considered small compared to the dimensions of the elements of the gantries;
- Rotation of masses in space is neglected (2D study);
- Frequency values obtained in this study are taken to the nearest 10000th;
- Studied porticoes are assumed to be unamortized;
- Gantries are made of reinforced concrete.

2.5.2. Exercise Test

Consider the simple isostatic gantry below supporting through the internal ball joint, a building mass as shown in the figure below (Figure 2). This gantry having undergone, through its mass, a punctual excitation by an external force which disappeared after excitation. It is desired to determine the frequency of the free oscillations of this gantry.

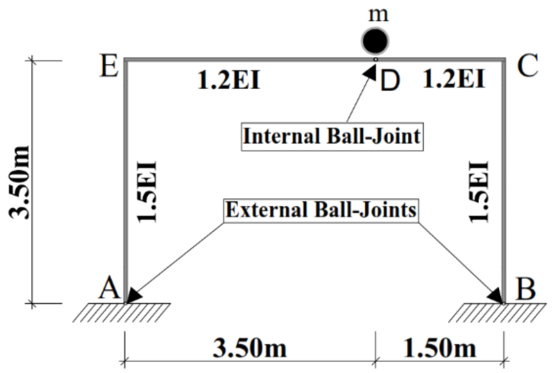


Figure 2.3: Gantry test

To do this we will determine the frequencies by the force method and the stiffness matrix method. Stiffness (EI) is assumed to be constant. The mass can move vertically and horizontally, giving it a degree of freedom equal to 2 (ddl=2).

2.5.2.1. Resolution by the Forces method

- **<u>Step</u> 1**: Determination of the degree of freedom of the system (dof) of the mass: dof=2 (possibility of movement along the axis x and along the axis y).

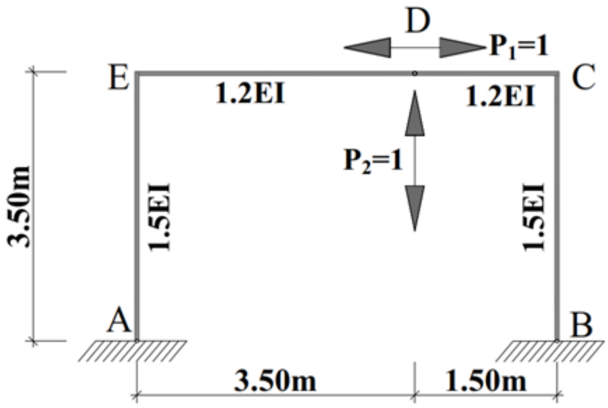


Figure 2.4: Degree of freedom of the test gantry

We are in the presence of an isostatic system, it is therefore possible for us to draw the unitary drawings directly in order to calculate the frequencies of the free oscillations.

- ❖ **Step 2**: Construction of the unit diagrams with respect to the degree of freedom (the mass is transformed into arbitrary and directional unit forces in accordance with the dof).

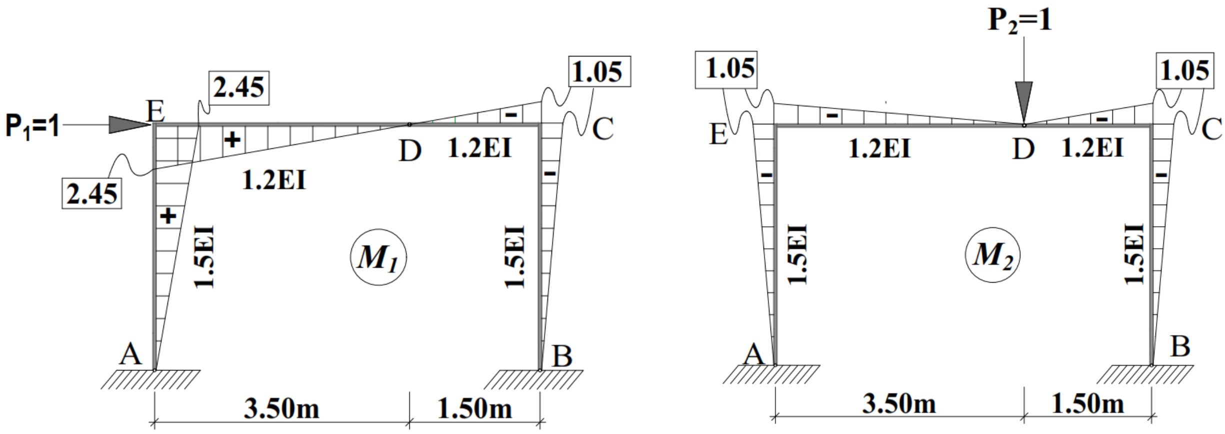


Figure 2.5: Diagrams of bending moments M_1 and M_2 under unit load.

- ❖ **Step 3**: Calculation of canonical coefficients and formation of the determinant of oscillations.

The different coefficients being calculated using the integrals of Möhr, this calculation will be facilitated by the VERECHAGUINE method.

$$\delta_{11} = \int \frac{M_1 M_1}{EI}\, dz = \frac{1}{1.5EI}\left[\left(\frac{2.45*3.5}{2}*\frac{2}{3}*2.45\right)\right] +$$

$$+\frac{1}{1.2EI}\left[\left(\frac{2.45*3.5}{2}*\frac{2}{3}*2.45\right)\right] + \frac{1}{1.2EI}\left[\left(\frac{1.05*1.5}{2}*\frac{2}{3}*1.05\right)\right] + \frac{1}{1.5EI}\left[\left(\frac{1.05*3.5}{2}*\frac{2}{3}*1.05\right)\right]$$

$$\delta_{11} = \frac{\mathbf{11.8213}}{\mathbf{EI}}$$

By analogy we will have:

$$\delta_{12} = \delta_{21} = \int \frac{M_1 M_2}{EI}\, dz = -\frac{3.185}{EI} \;;*\delta_{22} = \int \frac{M_2 M_2}{EI}\, dz = \frac{11.8213}{EI}$$

General expression of the determinant of free oscillations according to [15] is:

$$\begin{vmatrix} \delta_{11}m-\lambda & \delta_{12}m \\ \delta_{21}m & \delta_{22}m-\lambda \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} \frac{11.8213}{EI}m-\frac{1}{\omega^2} & -\frac{3.185}{EI}m \\ -\frac{3.185}{EI}m & \frac{3.2463}{EI}m-\frac{1}{\omega^2} \end{vmatrix} = 0$$

$$(\delta_{11}m-\lambda)(\delta_{22}m-\lambda)-(\delta_{12}m)^2 = 0 \quad \text{avec}\, \lambda = \frac{1}{\omega^2}$$

Finally, we obtain the following values: $\boldsymbol{\omega_1 = 0.2472\sqrt{\frac{EI}{m}}\,rad.s^{-1}}$; $\boldsymbol{\omega_2 = 0.4439\sqrt{\frac{EI}{m}}\,rad.s^{-1}}$

2.7. Resolution by matrix stiffness method

- **Steps 1-2 et 3 :**Numbering of the nodes (A, B, C, D, …) and the degrees of freedom (dof) of the structure (0 for the prevented dof, and 1, 2, 3, 4, …, for the others) ;

In the solution, we will use the method without constant number of unknowns in the node. The internal ball-joint connecting the ED and DC members is located at point D and has four unknown displacements, two distinct rotation angles, one for the ED member and the other for the DC member and a vertical and horizontal displacement at the point D. The directions of movement of each of the members constituting the study gantry are indicated in local and global coordinates and the rotation angle of each of the members is specified in **step 4**

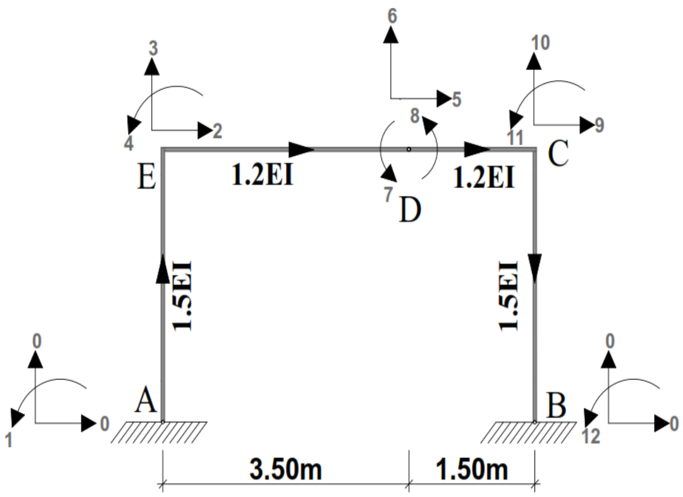


Figure 2.6: Nodes numbering and degree of freedom of the test gantry

- **Step 4 :**Introduction into the Excel spreadsheet, geometric characteristics (area, inertia, length) of each member and its rotation angle θ, as well as the Young's of the material constituting the member:

Table 2.1: Geometric characteristics of the members

Member N°1	**AE**	Unity	Member N°2	**ED**	Unity	Member N°3	**DC**	Unity	Member N°4	**CB**	Unity
L=	3.50	m	L=	3.50	m	L=	3.50	m	L=	3.50	m
A=	1.00	m^2	A=	1.0	m^2	A=	1.00	m^2	A=	1.0	m^2
βI=	1.00	m^4	βI=	1.0	m^4	βI=	1.00	m^4	βI=	1.0	m^4
E=	1.00	MN/m^2	E=	1.0	MN/m^2	E=	1.00	MN/m^2	E=	1.0	MN/m^2
θ =	90	°	θ =	0	°	θ =	90	°	θ =	0	°

Stiffness Matrices in the Local Coordinate System

- **Step 5 :**Formation of the nodal transformation matrix $[T_i]$ of each member and calculate its stiffness matrix [Ki] in the global axis system using Eq. (8);

Local $[k]_{AE}$Local $[k]_{ED}$

$$\begin{bmatrix} 0.29 & 0 & 0 & -0.29 & 0 & 0 \\ 0 & 0.42 & 0.73 & 0 & -0.42 & 0.73 \\ 0 & 0.73 & 1.71 & 0 & -0.73 & 0.86 \\ -0.29 & 0 & 0 & 0.29 & 0 & 0 \\ 0 & -0.42 & -0.73 & 0 & 0.42 & -0.73 \\ 0 & 0.73 & 0.86 & 0 & -0.73 & 1.71 \end{bmatrix} \begin{bmatrix} 0.29 & 0 & 0 & -0.29 & 0 & 0 \\ 0 & 0.34 & 0.59 & 0 & -0.34 & 0.59 \\ 0 & 0.59 & 1.37 & 0 & -0.59 & 0.69 \\ -0.29 & 0 & 0 & 0.29 & 0 & 0 \\ 0 & -0.34 & -0.59 & 0 & 0.34 & -0.59 \\ 0 & 0.59 & 0.69 & 0 & -0.59 & 1.37 \end{bmatrix}$$

Local $[k]_{DC}$ Local $[k]_{CB}$

$$\begin{bmatrix} 0.67 & 0 & 0 & -0.67 & 0 & 0 \\ 0 & 4.27 & 3.20 & 0 & -4.27 & 3.20 \\ 0 & 3.20 & 3.20 & 0 & -3.20 & 1.60 \\ -0.67 & 0 & 0 & 0.67 & 0 & 0 \\ 0 & -4.27 & -3.20 & 0 & 4.27 & -3.20 \\ 0 & 3.20 & 1.60 & 0 & -3.20 & 3.20 \end{bmatrix} \begin{bmatrix} 0.29 & 0 & 0 & -0.29 & 0 & 0 \\ 0 & 0.42 & 0.73 & 0 & -0.42 & 0.73 \\ 0 & 0.73 & 1.71 & 0 & -0.73 & 0.86 \\ -0.29 & 0 & 0 & 0.29 & 0 & 0 \\ 0 & -0.42 & -0.73 & 0 & 0.42 & -0.73 \\ 0 & 0.73 & 0.86 & 0 & -0.73 & 1.71 \end{bmatrix}$$

Members stiffness matrices in the overall system

Global $[k]_{AE}$Global $[k]_{ED}$

	0	0	1	2	3	4
0	0.42	0	–0.73	–0.42	0	0.73
0	0	0.29	0	0	–0.29	0
1	–0.73	0	1.71	0.73	0	0.86
2	–0.42	0	0.73	0.42	0	0.73
3	0	–0.29	0	0	0.29	0
4	–0.73	0	0.86	0.73	0	1.71

	2	3	4	5	6	7
2	0.29	0	0	–0.29	0	0
3	0	0.34	0.59	0	–0.34	0.59
4	0	0.59	1.37	0	–0.59	0.69
5	–0.29	0	0	0.29	0	0
6	0	–0.34	–0.59	0	0.34	–0.59
7	0	0.59	0.69	0	–0.59	1.37

Global $[k]_{DC}$ Global $[k]_{CB}$

	5	6	8	9	10	11
5	0.67	0	0	–0.67	0	0
6	0	4.27	3.20	0	–4.27	3.20
8	0	3.20	3.20	0	–3.20	1.60
9	–0.67	0	0	0.67	0	0
10	0	–4.27	–3.20	0	4.27	–3.20
11	0	3.20	1.60	0	–3.20	3.20

	9	10	11	0	0	12
9	0.29	0	0	–0.29	0	0
10	0	0.42	0.73	0	–0.42	0.73
11	0	0.73	1.71	0	–0.73	0.86
0	–0.29	0	0	0.29	0	0
0	0	–0.42	–0.73	0	0.42	–0.73
12	0	0.73	0.86	0	–0.73	1.71

- **Step 6 :** Assembly of the global system stiffness matrix (assembly of the formed matrices taking into account the interconnection between the members);
- **Step 7 :** Application of boundary conditions (according to step 2);
- **Step 8 :** Formation of the reduced stiffness matrix $[k_r]$ (by removing the rows and columns of the zero displacements), and the mass matrix [M] (diagonal matrix);

Formation of the Reduced Stiffness Matrix

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.71	0.73	0	0.86	0	0	0	0	0	0	0	0
2	0.73	0.71	0	0.73	−0.29	0	0	0	0	0	0	0
3	0	0	0.62	0.59	0	−0.34	0.59	0	0	0	0	0
4	0.86	0.73	0.59	3.09	0	−0.59	0.69	0	0	0	0	0
5	0	−0.29	0	0	0.95	0	0	0	−0.67	0	0	0
6	0	0	−0.34	−0.59	0	4.6	−0.59	3.20	0	−4.27	3.20	0
7	0	0	0.59	0.69	0	−0.59	1.37	0	0	0	0	0
8	0	0	0	0	0	3.20	0	3.20	0	−3.20	1.60	0
9	0	0	0	0	−0.67	0	0	0	1.09	0	0.73	0.73
10	0	0	0	0	0	−4.27	0	−3.20	0	4.55	−3.20	0
11	0	0	0	0	0	3.20	0	1.60	0.73	−3.20	4.91	0.86
12	0	0	0	0	0	0	0	0	0.73	0	0.86	1.71

- **Step 9 :** Formation of the oscillation determinant using Eq (4) ;
- **Step 10 :** Introduction of the frequency value ω=0.01 in (4) and iteration;

Table 2.2 below present the calculation of the determinant (D) and the value of the frequency leading to the change in sign of the determinant (D).

Table 2.2: Frequencies values and corresponding determinants

N°	Frequency (ω)	Determinant (D)	ω
1	0.01	6.045E-02	
2	0.02	6.004E-02	
3	0.03	5.935E-02	
-	-	-	
22	0.22	6.430E-03	
23	**0.23**	**2.693E-03**	**ω_1**
24	**0.24**	**-9.739E-04**	
25	0.25	-4.540E-03	
-	-	-	
43	0.43	-8.598E-03	
44	**0.44**	**-2.663E-03**	**ω_2**
45	**0.45**	**4.264E-03**	
46	0.46	1.224E-02	

$\omega_1 \in [0.23 - 0.24]$		$\omega_2 \in [0.44 - 0.45]$	
Frequency	**determinant**	**Frequency**	**determinant**
0.2301	2.656E-03	0.4401	-2.598E-03
0.2302	2.619E-03	0.4402	-2.534E-03
-	-	-	-
-	-	-	-
0.2372	4.402E-05	0.4439	-8.232E-05
0.2373	7.543E-06	0.444	-1.415E-05
0.2374	-2.893E-05	**0.4441**	5.413E-05
0.2375	-6.539E-05	0.4442	1.225E-04
-	-	-	-
-	-	-	-
0.24	-9.739E-04	0.45	4.264E-03

2.8. Comparison of the results obtained for the test exercise

From the application of the two methods for the determination of the frequencies of the free oscillations of the proposed test exercise, the following comparative table appears.

Table 2.3: Comparative frequencies values of the free oscillations (ω) of the test gantry

Forces Method		Stiffness Matrix Method	
Frequency	**Values (rad.S^{-1})**	**Frequency**	**Values (rad.S^{-1})**
ω_1	$0.2372\sqrt{\frac{EI}{m}}$	ω_1	$0.2374\sqrt{\frac{EI}{m}}$
ω_2	$0.4439\sqrt{\frac{EI}{m}}$	ω_2	$0.4441\sqrt{\frac{EI}{m}}$

The comparative table above shows us that the determination of the free oscillation frequency values gives the same results by the force method and by the stiffness matrix method proposed in this article. However, according to several authors for a tall building, a tall tower or a chimney, the first response mode generally represents the essence of the answer and the second mode is only considered for possible problems of discomfort due to accelerations. This test exercise therefore testifies to the accuracy of the matrix method proposed in this paper and makes this method reliable for determining the free oscillation frequencies of vibrating systems.The projection of these points on the x-axis allows us to obtain the two oscillation frequencies which are here the free oscillation frequencies of the studied portico. After the second passage of the curve on the abscissa axis, it is easy to notice that the latter grows indefinitely, which shows that the free oscillation frequencies of the system have already been reached.

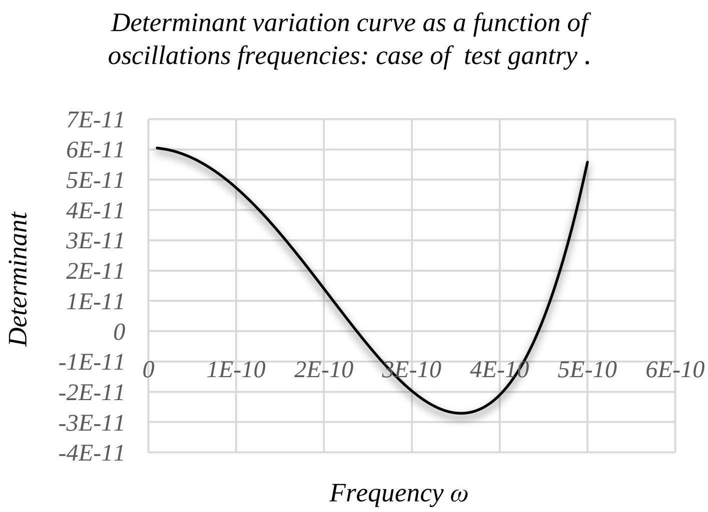


Figure 2.7: Determinant curve-frequency of the test gantry

Given the accuracy of the proposed method, its reliability and, above all, its ease of execution in a computer program, this method will therefore be extended to very complex gantries in order to determine on them the values of the frequencies of the free oscillations and consequently the periods of the associated oscillations.

2.9. Numerical Examples: (Case of gantries at internal ball-joints and several masses)

- **Resolution by Stiffness Matrix Method**

Consider the following gantries with internal ball-joints and also undergoing the action of several masses as shown in *Figure 2.8.* These gantries having undergone, through their construction masses, excitations caused by external forces that have disappeared after excitation. For the purpose of a dynamic study of these gantries, it is now desired to determine the frequencies of the free oscillations of these gantries and consequently to determine the periods corresponding to these free oscillation frequencies using the stiffness matrix method.

The numerical values to be considered are as follows: $g=9.81 m.s^{-2}$, Q= 8.5kN, E= 2.10^5MPa, A=600cm², I_z = 45000cm^4, b=20cm and h=30cm.

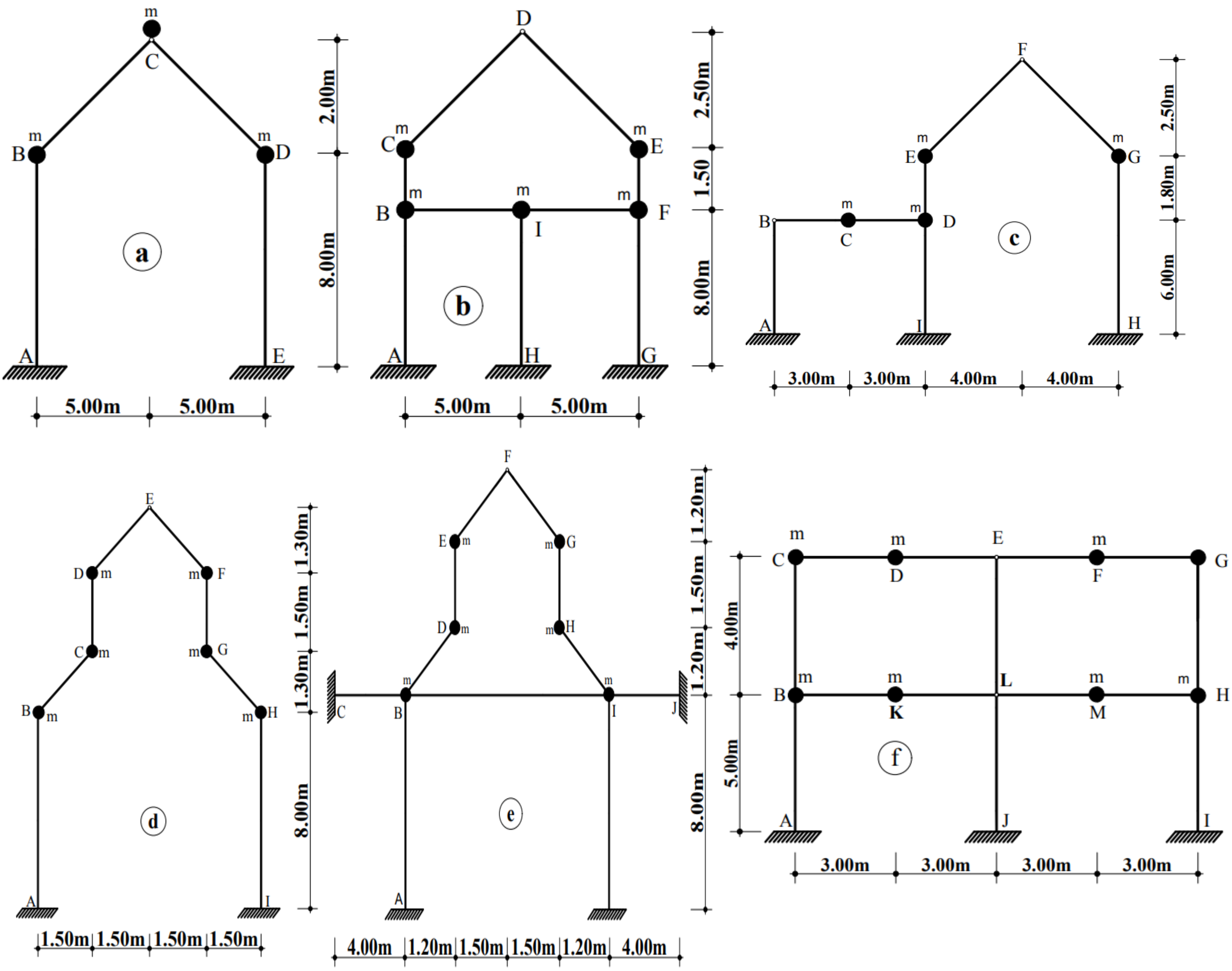
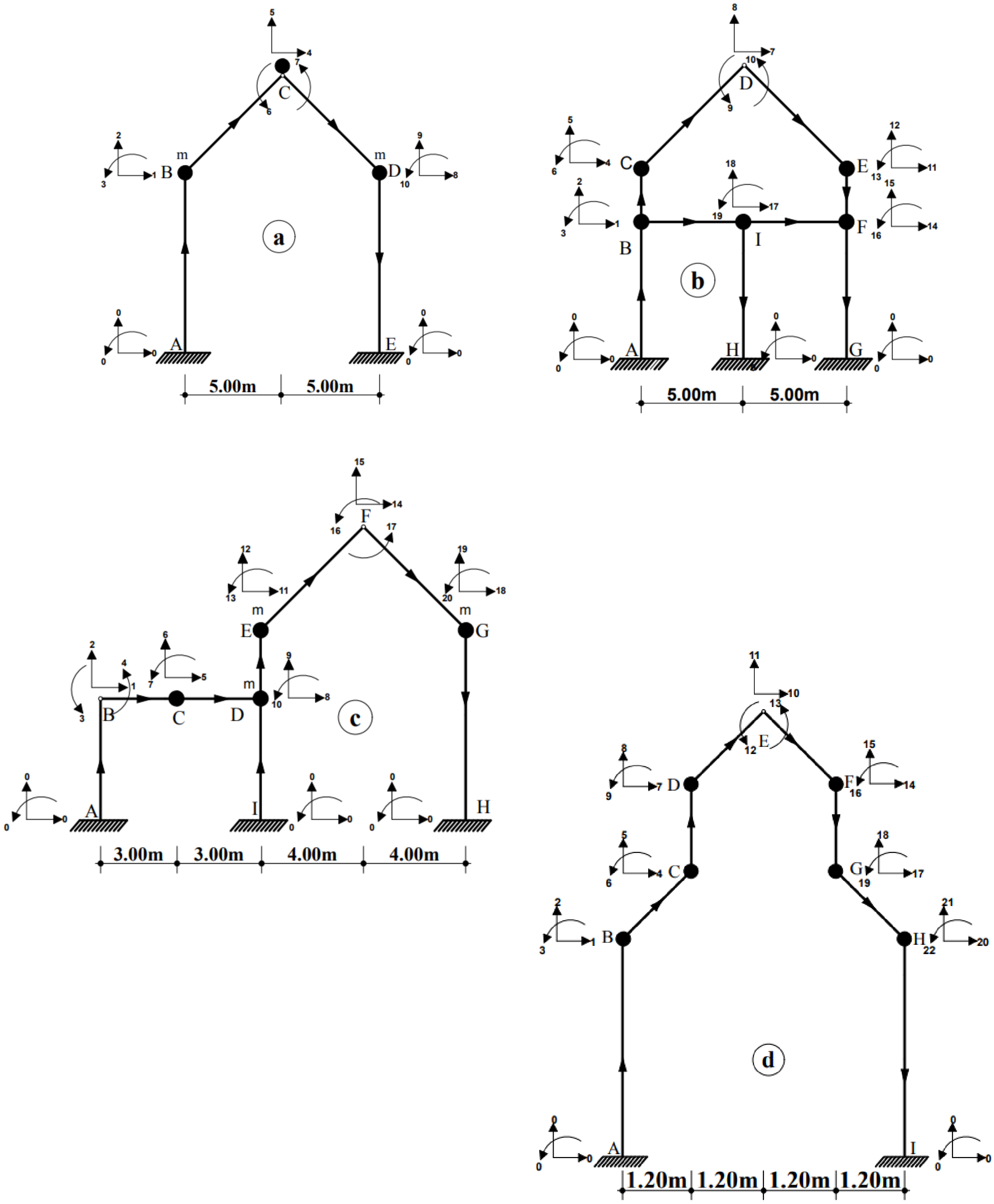


Figure 2.8: Categories of gantries to be studied

To do this, it is necessary to proceed to step 2 above specified as follows:



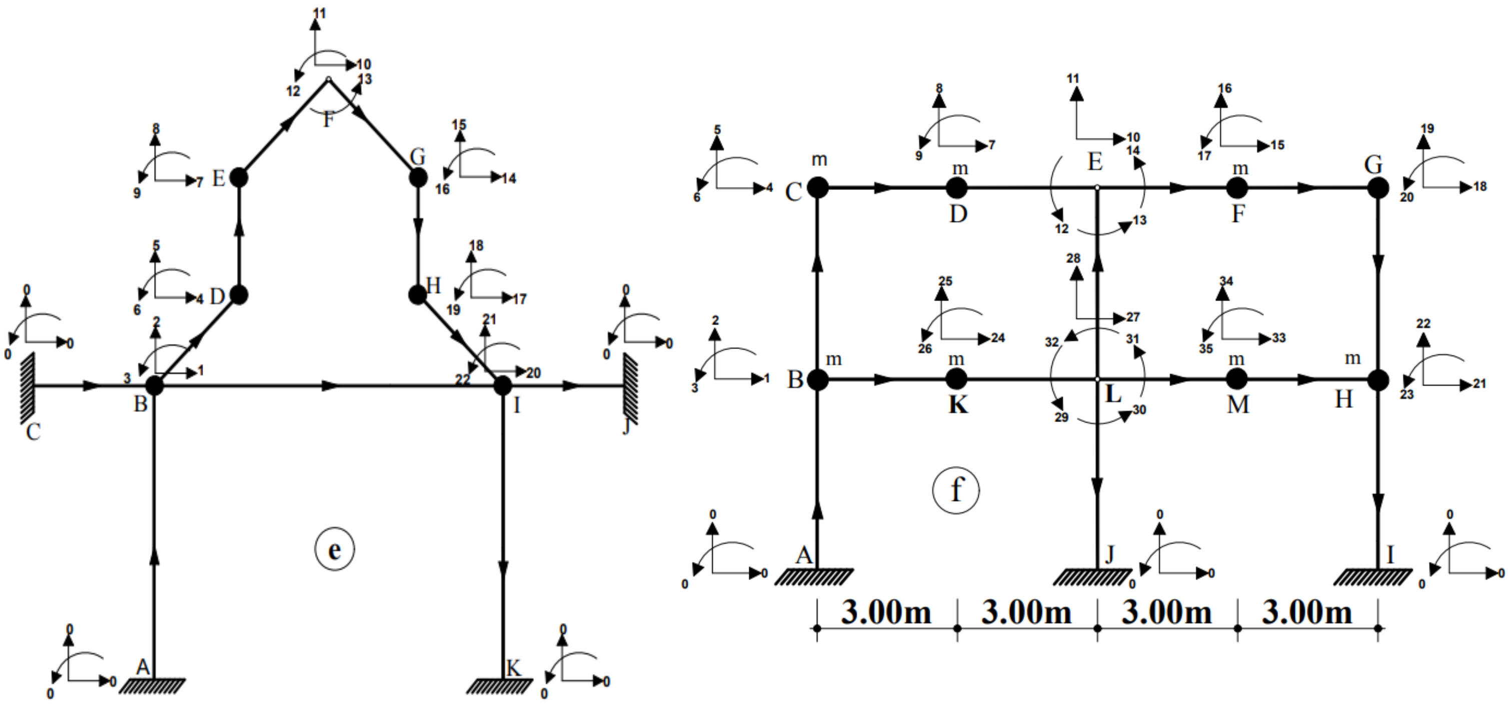


Figure 2.9: Numbering of nodes and d.o.f of the gantries to be studied

3. Results

For the calculation of the frequencies of the free oscillations of the gantries in *Figure 2.8*, based on the flowchart for calculating the frequencies of the free oscillations using the stiffness matrix method proposed above and *Figure 2.9*, it is easy to reach **step 12**, which gives the following results:

Table 3.1: Frequencies values of oscillations (ω) of gantry **a**

Frequency ($rad.S^{-1}$)		***ω* ($rad.S^{-1}$)**	***T*(S)**	***f* (H_Z)**
ω_1	$0.0926\sqrt{\frac{EI}{m}}$	30.581	0.2055	4.8671
ω_2	$0.1779\sqrt{\frac{EI}{m}}$	58.7512	0.1069	9.3505
ω_3	$0.4848\sqrt{\frac{EI}{m}}$	160.105	0.0392	25.481

Table 3.2: Frequencies values of oscillations (ω) of gantry **b**

Frequency ($rad.S^{-1}$)		***ω* ($rad.S^{-1}$)**	***T*(S)**	***f* (H_Z)**
ω_1	$0.0926\sqrt{\frac{EI}{m}}$	97.0601	0.0647	15.448
ω_2	$0.1779\sqrt{\frac{EI}{m}}$	123.48	0.0509	19.652

Table 3.3: Frequencies values of oscillations (ω) of gantry **c**

	Frequency (rad.S^{-1})	ω (rad.S^{-1})	T(S)	f (H$_Z$)
ω_1	$0.112\sqrt{\frac{EI}{m}}$	36.9878	0.170	5.8868
ω_2	$0.3785\sqrt{\frac{EI}{m}}$	125.00	0.0503	19.8942
ω_3	$0.4744\sqrt{\frac{EI}{m}}$	156.670	0.0401	24.9348

Table 3.4: Frequencies values of oscillations (ω) of gantry **d**

	Frequency (rad.S^{-1})	ω (rad.S^{-1})	T(S)	f (H$_Z$)
ω_1	$0.0557\sqrt{\frac{EI}{m}}$	18.3948	0.3416	2.9276
ω_2	$0.1759\sqrt{\frac{EI}{m}}$	58.0907	0.1082	9.2454
ω_3	$0.3460\sqrt{\frac{EI}{m}}$	114.266	0.0550	18.186
ω_4	$0.8332\sqrt{\frac{EI}{m}}$	275.163	0.0228	43.794
ω_5	$1.0328\sqrt{\frac{EI}{m}}$	341.081	0.0184	54.2847

Table 3.5: Frequencies values of oscillations (ω) of gantry **e**

	Frequency (rad.S^{-1})	ω (rad.S^{-1})	T(S)	f (H$_Z$)
ω_1	$0.2431\sqrt{\frac{EI}{m}}$	80.2834	0.0783	12.7775
ω_2	$0.2666\sqrt{\frac{EI}{m}}$	88.0443	0.0714	14.0127
ω_3	$0.4522\sqrt{\frac{EI}{m}}$	149.338	0.0421	23.7679
ω_4	$0.5126\sqrt{\frac{EI}{m}}$	170.276	0.0369	27.100

Table 3.6: Frequencies values of oscillations (ω) of gantry **f**

	Frequency (rad.S^{-1})	ω (rad.S^{-1})	T(S)	f (H$_Z$)
ω_1	$0.0868\sqrt{\frac{EI}{m}}$	28.6656	0.2192	4.5623
ω_2	$0.2652\sqrt{\frac{EI}{m}}$	87.5819	0.0717	13.939
ω_3	$0.3046\sqrt{\frac{EI}{m}}$	100.594	0.0625	16.010
ω_4	$0.3762\sqrt{\frac{EI}{m}}$	124.234	0.0506	19.773
ω_5	$0.5017\sqrt{\frac{EI}{m}}$	165.6857	0.0379	26.37
ω_6	$0.0557\sqrt{\frac{EI}{m}}$	177.1784	0.0355	28.199

4. Results Analysis

The curves below show the variation of the determinant of the reduced stiffness matrix and the mass matrix as a function of the argument (ω) for the six gantries systems previously studied.

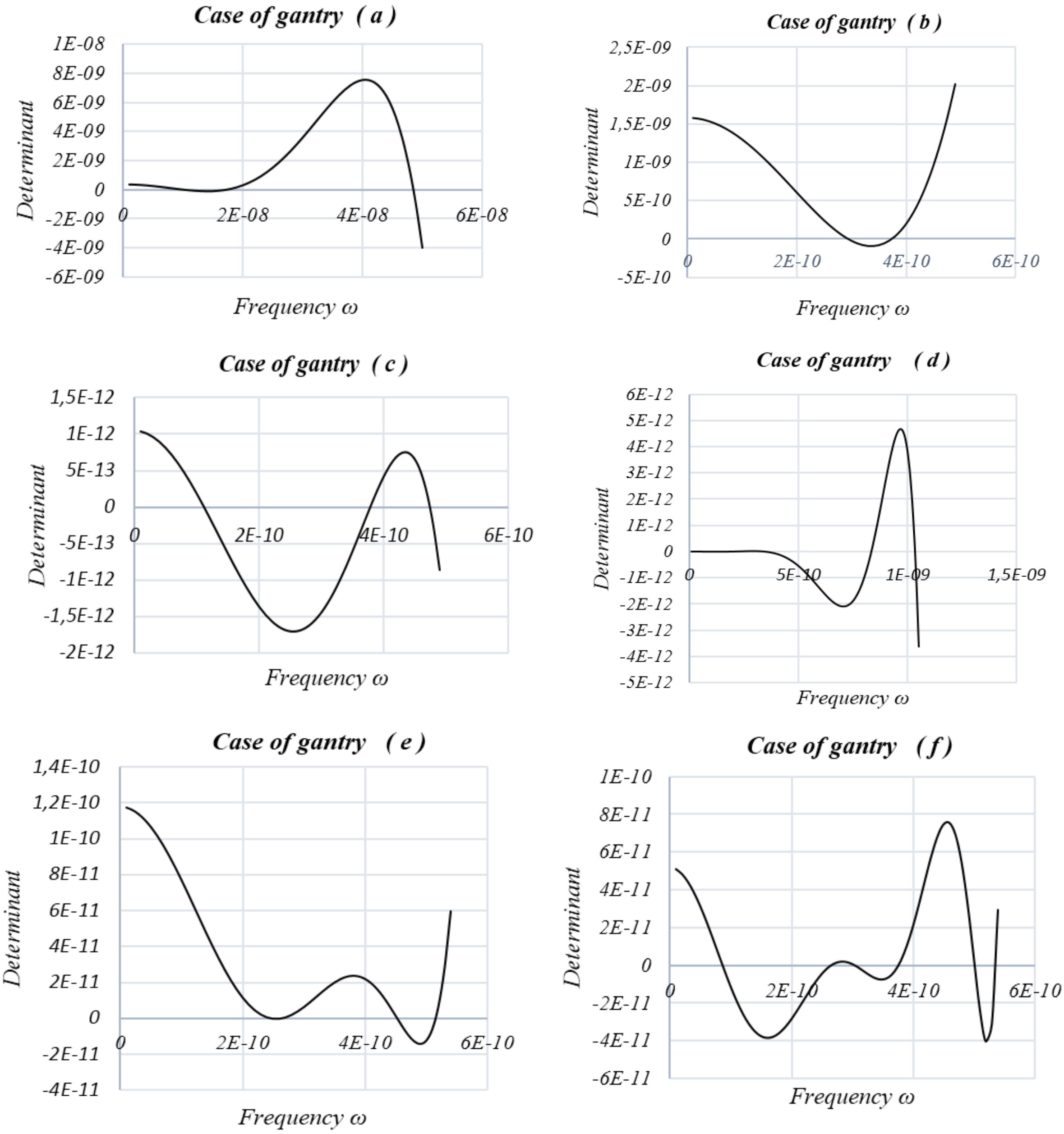


Figure 4.1: Determinant-Frequency curves of the gantries at internal ball-joints

The analysis of the curves in *Figure 4.1* show the variation of the frequency of the free oscillations as a function of the oscillations determinant. We easily notice that these curves coincide with the x-axis at different points. The projection of these points on the abscissa axis allows us to obtain the oscillations frequencies which are nothing other than the free frequencies oscillations of each of the studied gantries. It is also noted that the curves increase indefinitely after their last passes through the abscissa, which shows that the free oscillation frequencies of the system have already been reached. Table 3.8 summarizes the value of the smaller frequencies of the systems examined and the values of the associated periods. These are the fundamental frequencies with which a

dynamic study will have to be carried out. If these values are different from the frequencies of the forced oscillations, it can be stated that the systems studied above do not risk the phenomenon of resonance.
To overcome the difficulty related to the analytical resolution, the stiffness matrix method was used, based on the d'Alembert principle. Thus, as we can see, the stiffness matrix method completely dispenses with the drawing of bending moment diagrams under unit loads and the calculation of canonical coefficients because, here, it is simply a matter of a formal operation of matrices. From the results of the Test treated exercise, it is clear that the determination of the frequencies of free oscillations by the analytical method (force method) and by the method developed in this article called the stiffness matrix method gives exactly the same fundamental frequency values of free oscillations. This confirms the statements of [4-5] and many authors.
Ultimately, free oscillation frequencies that are nothing more than the parameters of the vibrating system are necessary to carry out a dynamic study. At the end of this study, it appears that the free oscillation frequencies do not depend on time, oscillation amplitude or phase angle, but rather on the mass of the system and its stiffness. The same applies to the periods associated with these frequencies. Since tall buildings have higher periods. The dynamic response is much more important on flexible structures than on stiff structures and the explanation of this phenomenon is directly related to the natural frequency of structures. Softer structures have lower frequencies. By varying the transverse dimensions of the proposed gantries it is noted that for more rigid structures, the circular natural frequency increases, the frequency increases and the vibration period decreases. On the other hand for the more flexible structures where the rigidity decreases, which decreases the eigenfrequency and the natural frequency and increases the period of vibration of these last ones.

5. Conclusion

To perform a dynamic analysis, many structures can be modeled entirely or partially in the form of constant or variable section gantries. In this study, the frequencies of free oscillations of internal ball-joints gantries with several construction masses were determined by the stiffness matrix method. This calculation method makes it possible to find the natural frequencies in linear analysis of mechanical engineering systems, which can be compared with the frequencies of forced oscillations in order to conclude whether or not the structure is subject to resonance. The natural frequency values acquired with this method give the same results as for analytical calculation proposed by [15]. These values of free oscillation frequencies can be obtained by analytical solutions of the problem, but this takes more time and may be tedious or even ineffective due to the risk of errors in their resolution.
The application of the Stiffness Matrix Method in the determination of the frequencies of free oscillations of systems is very advantageous because of the reliability of the results obtained, its ease of execution and above all the time saved during the study phase. The dynamic response is much more important on flexible structures than on stiff structures and the explanation for this phenomenon is directly related to the natural frequency of the structure. Softer structures have much lower frequencies Eigenvalues, i. e. the frequencies of free oscillations, play a fundamental role in determining the dynamic behavior of vibrating systems and in particular in civil engineering structures and therefore deserve particular attention, which requires the application of a fast and easy to implement method.

Consent for Publication
Not applicable

Conflict of Interest
The authors declare no conflict of interest, financial or otherwise.

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Declared none

Abbreviations List

m	*Mass of the structure*
C	*Amplitude of oscillations*
$[k_i]$	*Stiffness matrix of each member (local system)*
$[M]$	*Matrix of the masses of the structure*
(D)	*Determining oscillations*
dof	*Degree of freedom*
$\ddot{x}$	*Acceleration*
x	*displacement*
$[K_r]$	*Reduced Stiffness matrix*
$[K]$	*Matrix of global rigidity*
2D	*Dimension 2*
E	*Longitudinal elasticity module*
A	*Area of the cross section of the member*
I	*Moment of inertia of the section*
L	*Length of the member*

Subscripts and superscripts	
Matrix Transposed	
T	*Matrix Transposed*
i	*Node number*
r	*Reduce*
z	*Inertia axis*

$[T_i]$	*Transformation Matrix*
g	*Gravity acceleration*
b	*Base*
EI	*Stiffness*
Cste	*Constant*
u	*Axial displacement*
v	*Lateral displacement*
X	*Horizontal axis (global coordinate)*
Y	*Vertical axis (global coordinate)*
x	*Horizontal axis (local coordinate)*
y	*Vertical axis (local coordinate)*
h	*Height*
S	*Second*
T	*Period*
Q	*Load of the structure*
I_z	*Moment of inertia*
Hz	*Hertz*
rad	*Radian*
S	*Second*

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