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Method of Performance Evaluation of Accurate Poverty Alleviation Based on Picture Fuzzy Cosine Similarity Measure

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Abstract In the paper, the problem of performance evaluation of accurate poverty alleviation in the environment of picture fuzzy information is studied. A picture fuzzy cosine similarity measure model considered five terms like degree of positive membership, degree of neutral membership, degree of negative membership, degree of refusal membership and the score function is proposed to overcome the limitations of the extant picture fuzzy cosine similarity measure models. The multi-attribute decision making method is constructed based on the picture fuzzy cosine similarity measure model. Finally, the effectiveness and flexibility of the proposed method are further verified through a case study of performance evaluation of accurate poverty alleviation.

Keywords Precise poverty alleviation, Picture fuzzy set, Cosine similarity measure

1. Introduction

Poverty is an important problem that China must face to build a well-off society in an all-round way. In order to fully implement the tasks set out in the 13th Five-Year Plan, let the people share the fruits of reform and development, eliminate the gap between the rich and the poor, and realize the political commitment of common prosperity, the CPC Central Committee with Comrade Xi Jinping as the core has successively issued a series of policy documents, which has set off a nationwide battle for precision poverty alleviation striving to lift all the poor out of poverty by 2020. Precision poverty alleviation is a huge system engineering, it should not only require "hard work", but also "harvest". "Harvest" means to evaluate whether the benefit of precision poverty alleviation is really implemented to the precise object. The performance evaluation of precision poverty alleviation projects is a very important link in the process of precision poverty alleviation, and a scientific and reasonable performance evaluation index system and evaluation methods are the top priority of this link. Many experts and scholars have conducted in-depth research on the performance evaluation methods of precision poverty alleviation. Combing with the 4E evaluation theory and the key performance evaluation index evaluation methods, Yang and Li [1] proposed a system for the government's accurate poverty alleviation performance management evaluation. Index system for evaluating poverty alleviation was constructed from the three dimensionality of precision recognition, precision assistance, precision management, meanwhile, the indexes' operability was verified in poverty alleviation evaluating of Hebei in 2016 [2]. When the attribute value of the accurate poverty alleviation evaluation index is fuzzy number intuitionistic fuzzy number, Dai et al. [3] defined the fuzzy number intuitionistic fuzzy number score function, and ranked the performance score of each poverty alleviation area according to the fuzzy comprehensive evaluation value.

It is very difficult to take real attribute values, because of complexity presented in serious level in the fifield of decision environment. Cuong [4] proposed picture fuzzy set (PFS) and investigated the some basic operations and properties of PFS. The picture fuzzy set is characterized by three functions expressing the degree of membership, the degree of neutral membership and the degree of negative membership. Basically, PFS based models can be applied to situations requiring human opinions involving more answers of types: yes, abstain, no,

refusal, which can't be accurately expressed in the traditional intuitionistic fuzzy set [5]. Singh gave a geometrical interpretation of picture fuzzy sets and proposed correlation coefficients for picture fuzzy sets, and he also applied the correlation coefficient to clustering analysis under picture fuzzy environments [6]. Wei presented another form of eight similarity measures between PFSs based on the cosine function between PFSs by considering the degree of positive membership, degree of neutral membership, degree of negative membership and degree of refusal membership in PFSs. Then, he applied these weighted cosine function similarity measures between PFSs to strategic decision making [7]. In the paper [8], Wei et al. defined ten similarity measures between spherical fuzzy sets based on the cosine function, and they designed two illustrative examples are to show the efficiency of the similarity measures for pattern recognition and medical diagnosis.
Picture fuzzy numbers (PFNs) can describe fuzzy and uncertain information involved in practical decision-making problems, and PFNs can mitigate information loss. However, studies of accurate poverty alleviation with picture fuzzy information are relatively fewer. Cosine similarity measure is a more comprehensive measure to describe the difference or similarity between two evaluations. Nevertheless, the extant cosine similarity measure model of PFSs has some limitations. In the paper, we propose a picture fuzzy cosine similarity measure model to overcome the limitations of the extant cosine similarity measure model of PFSs. The picture fuzzy cosine similarity measure model for PFSs is applied to strategic decision making problem for performance evaluation of accurate poverty alleviation.

2. Preliminaries

In this section, we present some basic concepts related to IFSs, PFSs and correlation coefficients.

Definition 2.1 [5] An intuitionistic fuzzy set A on a universe of discourse X is of the form

$$A=\left\{(x,\mu_A(x),\nu_A(x))\,|\,x\in X\right\},$$

Where $\mu_A(x)\in[0,1]$ is called the "degree of membership of x in A", $\nu_A(x)\in[0,1]$ is called the "degree of non-membership of x in A", and where $\mu_A(x)+\nu_A(x)\le 1,\ \forall x\in X$.

Cuong generalized the concept of intuitionistic fuzzy sets to the concept of picture fuzzy sets as follows.

Definition 2.2 [4] Let X be a universe of discourse. A picture fuzzy set (PFS) A on the universe X is an object of the form

$$A=\left\{(x,\mu_A(x),\eta_A(x),\gamma_A(x))\,|\,x\in X\right\},$$

Where $\mu_A(x)\ (\in[0,1])$ is called the "degree of positive membership of A", $\eta_A(x)\ (\in[0,1])$ is called the "degree of neutral membership of A" and $\gamma_A(x)\ (\in[0,1])$ is called the "degree of negative membership", and $\mu_{\mathrm{A}},\eta_{\mathrm{A}},\gamma_A$ satisfy: $\mu_A(x)+\eta_A(x)+\gamma_A(x)\le 1,\ \forall x\in X$. For any $x\in X$, $\nu_A(x)=1-(\mu_A(x)+\eta_A(x)+\gamma_A(x))$ could be called the degree of refusal membership of x in *A*.In particularly, if X only has one element, then $A(x)=(\mu_A(x),\eta_A(x),\gamma_A(x))$ is called a picture fuzzy number (PFN). For convenience, a PFN is denoted as (μ_A,η_A,γ_A).

Definition 2.3 [4,9] Given two PFSs represented by A and B on universe X, the inclusion, union, intersection and complement operations are defined as follows:

(1) $A\cap B=\{(x,\min\{\mu_A(x),\mu_B(x)\},\min\{\eta_A(x),\eta_B(x)\},\max\{\gamma_A(x),\gamma_B(x)\})\,|\,x\in X\}$;

(2) $A\cup B=\{(x,\max\{\mu_A(x),\mu_B(x)\},\min\{\eta_A(x),\eta_B(x)\},\min\{\gamma_A(x),\gamma_B(x)\})\,|\,x\in X\}$;

(3) $A\subseteq B\Leftrightarrow\forall x\in X,\ \mu_A(x)\le\mu_B(x),\eta_A(x)\le\eta_B(x),\ \gamma_A(x)\ge\gamma_B(x)$;

(4) $A = B \Leftrightarrow A \subseteq B$ and $B \subseteq A$.

Definition 2.4 [10] Let $\alpha = (\mu_\alpha, \eta_\alpha, \gamma_\alpha)$ a PFN, then a score function s of a PFN can be represented as follows:

$$s(\alpha) = \frac{1 + \mu_\alpha - \gamma_\alpha}{2}, \quad s(\alpha) \in [0,1].$$

3. Improved picture fuzzy cosine similarity measures

In an analogous manner to the cosine similarity measure for intuitionistic fuzzy set [11], Wei proposed cosine similarity measures for picture fuzzy sets as follows.

Definition 3.1[7] For any two PFSs A and B on a discrete universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$, a cosine similarity measure between A and B is defined by

$$cos_1(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_A(x_i)\mu_B(x_i) + \eta_A(x_i)\eta_B(x_i) + \gamma_A(x_i)\gamma_B(x_i)}{\sqrt{\mu_A^2(x_i) + \eta_A^2(x_i) + \gamma_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \eta_B^2(x_i) + \gamma_B^2(x_i)}}$$

When $cos_1(A,B)$ is large, B is close to A. And, if $cos_1(A,B) \geq cos_1(A,C)$, then B is closer to A than C.

When the four terms like degree of positive-membership, degree of neutral-membership, degree of negative membership and degree of refusal-membership are considered in PFSs, Singh proposed a cosine similarity measure (which were also called a correlation coefficient) model for picture fuzzy sets.

Definition 3.2 [6] For any two PFSs A and B on a discrete universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$, a cosine similarity measure between A and B is defined by

$$cos_2(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\mu_A(x_i)\mu_B(x_i) + \eta_A(x_i)\eta_B(x_i) + \gamma_A(x_i)\gamma_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i) + \eta_A^2(x_i) + \gamma_A^2(x_i) + \nu_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \eta_B^2(x_i) + \gamma_B^2(x_i) + \nu_B^2(x_i)}}.$$

Example 3.3 Let $A = \{(x,0,0,0.9)\}$, $B = \{(x,0,0,0.3)\}$ and $C = \{(x,0.1,0,0.2)\}$ be three PFSs on a discrete universe of discourse $X = \{x\}$. It is easy to see that $A \subseteq B \subseteq C$, according to our intuition, B is much closer to C than A. However, the result calculated according to the formula of Definition 3.1 shows that, $cos_1(A,C) = cos_1(B,C) = 0.8944$, that means A and B are similar close to C, which is not conform to our intuition.

Example 3.4 Let $D = \{(x,0.2,0.8,0.0)\}$, $E = \{(x,0.8,0.2,0.0)\}$ and $F = \{(x,0.5,0.5,0)\}$ be three PFSs on $X = \{x\}$. The result calculated according to the formulas of Definition 3.1 and Definition 3.2 shows that, $cos_1(D,F) = cos_1(E,F) = cos_2(D,F) = cos_2(E,F) = 0.8575$, it means that D and E are similar close to F, which is not conform to our intuition.

The results obtained by the cosine similarity measure model of Definition 3.1 and Definition 3.2 are occasionally counter-intuitive, in order to solve the problem, we propose a cosine similarity measure measure model for picture fuzzy sets which considers the degree of positive membership, degree of neutral membership, degree of negative membership, the degree of refusal membership and the score function.

Definition 3.5 For any two PFSs A and B on a discrete universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$, an improved cosine similarity measure between A and B is defined by

$$k(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A(x_i)\mu_B(x_i)+\eta_A(x_i)\eta_B(x_i)+\gamma_A(x_i)\gamma_B(x_i)+\nu_A(x_i)\nu_B(x_i)+s_A(x_i)s_B(x_i)}{\sqrt{\mu_A^2(x_i)+\eta_A^2(x_i)+\gamma_A^2(x_i)+\nu_A^2(x_i)+s_A^2(x_i)}\sqrt{\mu_B^2(x_i)+\eta_B^2(x_i)+\gamma_B^2(x_i)+\nu_B^2(x_i)+s_B^2(x_i)}}.$$

Example 3.6 Consistent with Example 3.3 and Example 3.4, let $A=\{(x,0,0,0.9)\}$, $B=\{(x,0,0,0.3)\}$, $C=\{(x,0.1,0,0.2)\}$, $D=\{(x,0.2,0.8,0.0)\}$, $E=\{(x,0.8,0.2,0.0)\}$ and $F=\{(x,0.5,0.5,0)\}$ be PFSs on $X=\{x\}$. Using the improved cosine similarity measure model , we can get

$$k(A,C)=0.3558\le k(B,C)=0.9840,$$

$$k(D,F)=0.9037\le k(E,F)=0.9339.$$

Thereby, the results indicates that B is considerably closer to C than A, and E is considerably closer to F than D, conforming to our intuition. Therefore, the proposed cosine similarity measure model is correct and superior.

In many cases, the weight of the elements $x_i\in X$ should be taken into account. For example, in MADM, the considered attributes usually have different importance, and thus need to be assigned different weights. As a result, an improved weighted cosine similarity measure between PFSs A and B is also proposed as follows:

$$k_w(A,B)=\sum_{i=1}^{n}w_i\frac{\mu_A(x_i)\mu_B(x_i)+\eta_A(x_i)\eta_B(x_i)+\gamma_A(x_i)\gamma_B(x_i)+\nu_A(x_i)\nu_B(x_i)+s_A(x_i)s_B(x_i)}{\sqrt{\mu_A^2(x_i)+\eta_A^2(x_i)+\gamma_A^2(x_i)+\nu_A^2(x_i)+s_A^2(x_i)}\sqrt{\mu_B^2(x_i)+\eta_B^2(x_i)+\gamma_B^2(x_i)+\nu_B^2(x_i)s_B^2(x_i)}}.$$

4. A picture fuzzy decision-making approach based improved picture fuzzy cosine similarity measures

Based the improved weighted cosine similarity measure model, in this section, we shall propose the model for multiple attribute decision making with picture fuzzy information.

Let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives, and $C=\{C_1,C_2,\cdots,C_n\}$ be the set of attributes, $w=\{w_1,w_2,\cdots,w_n\}$ is the weighting vector of the attribute C_j $(j=1,2,\cdots,n)$, where $w_j\in[0,1]$, $\sum_{j=1}^{n}w_j=1$.

The specific procedures are shown as following:

Step 1: Obtain the picture fuzzy decision matrix $D=(d_{ij})_{mn}=(\mu_{ij},\eta_{ij},\nu_{ij})_{mn}$, where μ_{ij} indicates the degree of positive membership that the alternative A_i satisfies the attribute C_j given by the decision maker, η_{ij} indicates the degree of neutral membership, ν_{ij} indicates the degree of negative membership.

Step 2: Normalize the decision matrix. The decision matrix has two types of criteria, namely benefit criteria and cost criteria. The evaluation need to be unified by transforming the cost criteria into benefit criteria by utilizing the complementary set of PFSs in Definition 2.3. Therefore, a normalized decision matrix $R=(r_{ij})_{mn}$ can be obtained using Eq. (M1).

$$r_{ij}=\begin{cases}d_{ij}, & \text{for benefit attribute } C_j,\\ d_{ij}^{c}, & \text{for cost attribute } C_j.\end{cases}\quad\text{(M1)}$$

Step 3: Determine the positive ideal solution $A^+ = (r_1^+, r_2^+, \cdots, r_n^+)$ and the negative ideal solution $A^- = (r_1^-, r_2^-, \cdots, r_n^-)$, where

$$r_j^+ = (\mu_j^+, \eta_j^+, \nu_j^+) = (\max_i \mu_{ij}, \min_i \eta_{ij}, \min_i \nu_{ij}),$$

$$r_j^- = (\mu_j^-, \eta_j^-, \nu_j^-) = (\min_i \mu_{ij}, \min_i \eta_{ij}, \max_i \nu_{ij}).$$

Step 4: Calculate the weight cosine similarity measure $k_w(A_i, A^+)$ between each alternative A_i and the positive ideal solution A^+, and the weight cosine similarity measure $k_w(A_i, A^-)$ between each alternative A_i and the negative ideal solution A^-, where

$$k_w(A_i, A^+) = \sum_{i=1}^{n} w_i \frac{\mu_{ij}\mu_j^+ + \eta_A(x_i)\eta_j^+ + \gamma_{ij}\gamma_j^+ + \nu_{ij}\nu_j^+ + s_{ij}s_j^+}{\sqrt{\mu_{ij}^2 + \eta_{ij}^2 + \gamma_{ij}^2 + \nu_{ij}^2 + s_{ij}^2}\sqrt{\mu_j^+ + \eta_j^+ + \gamma_j^+ + \nu_j^+ + s_j^+}} \quad (1)$$

$$k_w(A_i, A^-) = \sum_{i=1}^{n} w_i \frac{\mu_{ij}\mu_j^- + \eta_A(x_i)\eta_j^- + \gamma_{ij}\gamma_j^- + \nu_{ij}\nu_j^- + s_{ij}s_j^-}{\sqrt{\mu_{ij}^2 + \eta_{ij}^2 + \gamma_{ij}^2 + \nu_{ij}^2 + s_{ij}^2}\sqrt{\mu_j^- + \eta_j^- + \gamma_j^- + \nu_j^- + s_j^-}} \quad (2)$$

Step 5: Calculate the closeness degree $\rho(A_i)$ of each scheme A_i $(i = 1, 2, \cdots, m)$, where

$$\rho(A_i) = \frac{k_w(A_i, A^+)}{k_w(A_i, A^+) + k_w(A_i, A^-)} \qquad (i = 1, 2, \cdots, m) \quad (3)$$

Step 6: Rank all the alternatives and select the best one(s).

5. Numerical example

In this section, we utilize a practical multiple attribute decision making problems to illustrate the application of the developed approaches.

Suppose that there are five regions A_1, A_2, A_3, A_4, A_5 be considered for the performance evaluation of precision poverty alleviation, and evaluation experts conduct comprehensive research around precision poverty alleviation in the five regions. The index system of six evaluation indexes is assumed to be established, namely: poverty alleviation projects C_1, poverty alleviation funds C_2, poverty alleviation objects C_3, poverty alleviation management C_4, poverty alleviation process C_5, poverty alleviation response C_6. The weight vector of the six attributes is given by the decision maker as follows:

$$w = \{0.1924, 0.1694, 0.1664, 0.1686, 0.1653, 0.1379\}.$$

Table 1: The Picture Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.53,0.33,0.09)	(0.89,0.08,0.03)	(0.42,0.35,0.18)	(0.08,0.89,0.02)	(0.33,0.51,0.12)	(0.17,0.53,0.13)
A_2	(0.73,0.12,0.08)	(0.13,0.64,0.21)	(0.03,0.82,0.13)	(0.73,0.15,0.08)	(0.52,0.31,0.16)	(0.51,0.24,0.21)
A_3	(0.91,0.03,0.02)	(0.07,0.09,0.05)	(0.04,0.85,0.10)	(0.68,0.26,0.06)	(0.15,0.76,0.07)	(0.31,0.39,0.25)
A_4	(0.85,0.09,0.05)	(0.74,0.16,0.10)	(0.02,0.89,0.05)	(0.08,0.84,0.06)	(0.16,0.71,0.05)	(1.00,0.00,0.00)
A_5	(0.90,0.05,0.02)	(0.68,0.08,0.21)	(0.05,0.87,0.06)	(0.13,0.75,0.09)	(0.15,0.73,0.08)	(0.91,0.03,0.05)

The five regions A_1, A_2, A_3, A_4, A_5 are evaluated by the decision maker under the six attributes according to picture fuzzy concept, and the decision matrix $D=(d_{ij})_{5\times 6}$ is presented in Table 1, where are in the form of PFNs.

Step1: the picture fuzzy decision matrix $D=(d_{ij})_{mn}$ is shown as Table 1.

Step2: Since all the attributes are benefit criteria, so we get the normalized matrix $R=(r_{ij})_{mn}=(d_{ij})_{mn}$.

Step3: Determine the positive ideal solution A^+ and the negative ideal solution A^-, where

$$A^+=\begin{Bmatrix}(0.91, 0.03, 0.02),(0.89, 0.08, 0.03),(0.42, 0.35, 0.05),\\(0.73, 0.15, 0.02),(0.52, 0.31, 0.05),(1.00, 0.00, 0.00)\end{Bmatrix},$$

$$A^-=\begin{Bmatrix}(0.53, 0.03, 0.09),(0.07, 0.08, 0.21),(0.02, 0.35, 0.18),\\(0.08, 0.15, 0.09),(0.15, 0.31, 0.16),(0.17, 0.00, 0.25)\end{Bmatrix}.$$

Step4: Calculate the weight cosine similarity measure $k_w(A_i,A^+)$ and $k_w(A_i,A^-)$, then

$k_w(A_1,A^+)=0.8447$, $k_w(A_2,A^+)=0.8666$, $k_w(A_3,A^+)=0.7933$, $k_w(A_4,A^+)=0.8480$, $k_w(A_5,A^+)=0.8574$,
$k_w(A_1,A^-)=0.6766$, $k_w(A_2,A^-)=0.6985$, $k_w(A_3,A^-)=0.7718$, $k_w(A_4,A^-)=0.6756$, $k_w(A_5,A^-)=0.6870$.

Step5: Calculate the closeness degree $C(A_i)$ by Eq. (3), then

$$\rho(A_1)=0.5553,\ \rho(A_2)=0.5537,\ \rho(A_3)=0.5069,\ \rho(A_4)=0.5566,\ \rho(A_5)=0.5552.$$

Step6:Rank all the alternatives A_1-A_5in accordance with the values of $\rho(A_i)$, since

$$\rho(A_4)>\rho(A_1)>\rho(A_5)>\rho(A_2)>\rho(A_3),$$

then $A_4 \succ A_1 \succ A_5 \succ A_2 \succ A_3$. Note that "$\succ$" means "preferred to". Thus, the best region is A_4.

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