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## Study of the Heat Exchange Coefficients in Frequency Modulation for the Determination of Minimum Thickness of Thermal Insulation with Rice Husk Panel

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**Abstract** In this article, we propose a method for determining the optimal thickness of a rice husk panel with thermal insulation from the exchange coefficients on the rear face. Analytical expressions for temperature and heat flux density are established from the heat modulation equation in frequency modulation. The influence of the stress period on the optimal thickness has been shown.

**Keywords** rice husk – thermal insulation - thermal exchange coefficient - minimum thickness - frequency dynamic regime

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### 1. Introduction

Global climate change warming of the earth is mainly caused increase energy consumption, producing greenhouse gases. His reduction is a major issue.

Work has proposed the development of green energies: solar energy [1, 2, 3], wind energy [4,5], hydraulic energy [6, 7], biomass [8, 9]. To better manage these energies produced, the researchers propose the use of materials allowing thermal insulation for energy saving in buildings.

The materials are synthetic [10,11], or vegetable origin [12, 13, 14, 15] or animal [16,17]. They are generally designed in the form of panels.

With the National program for rice self-sufficiency in Senegal, the ball of this crop which is almost not exploited. Thus it could be used in the manufacture of panels for thermal insulation.

The objective of his work is to propose a method for determining the minimum thickness of the rice husk panel from the surface heat exchange coefficients.

#### 1.1. Theory

The device is a material made of rice husk (figure 1), it's in the form of a panel:



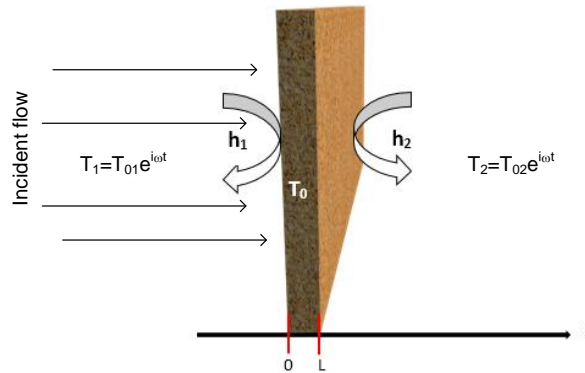


Figure 1: Study model composed of rice husk subjected to external climatic stresses  
( $L = 0,05\text{m}$ ,  $T_{01} = 35^\circ\text{C}$ ,  $T_{02} = 20^\circ\text{C}$  et  $T_0 = 25^\circ\text{C}$ ).

Where :

- $T_1$  ( $^\circ\text{C}$ ) and  $T_2$  ( $^\circ\text{C}$ ): temperature in frequency dynamic mode of external and indoor environment respectively,
- $T_{01}$  and  $T_{02}$  ( $^\circ\text{C}$ ): maximum amplitude of  $T_1$  and  $T_2$  respectively,
- $T_0$  ( $^\circ\text{C}$ ): initial temperature of insulating material,
- $h_1$ : heat exchange coefficient front face
- $h_2$ : heat exchange coefficient rear face
- $\omega$  is excitation pulse (rad/s) and  $t$  represents times (s)

When the material is excited by its external face, there is then a phenomenon of thermal diffusion through the material, governed by the following heat equation [18]:

$$\lambda \cdot \frac{\partial^2 T(x, h_1, h_2, \omega, t)}{\partial x^2} + P_p = \rho \cdot c \cdot \frac{\partial T(x, h_1, h_2, \omega, t)}{\partial t} \quad (1)$$

Where:

- $\rho$  ( $\text{Kg} \cdot \text{m}^{-3}$ ): density of material,
- $c$  ( $\text{J} \cdot \text{Kg}^{-1} \cdot \text{K}^{-1}$ ): mass thermal capacity,
- $\lambda$  ( $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ ): thermal conductivity of material,
- $P_p$  ( $\text{W} \cdot \text{m}^{-3}$ ): internal heat supply (heat sink) of material.
- $x$  (m): depth position

Simplified form of this equation, in absence of internal heat sinks and for constant thermal conductivity (assumed isotropic material) is given by:

$$\frac{\partial^2 T(x, h_1, h_2, \omega, t)}{\partial x^2} = \frac{1}{\alpha} \cdot \frac{\partial T(x, h_1, h_2, \omega, t)}{\partial t} \quad (2)$$

$$\text{With } \alpha = \frac{\lambda}{\rho \cdot c} \quad (3)$$

$\alpha$  is thermal diffusivity coefficient of the material ( $\text{m}^2 \cdot \text{s}^{-1}$ ).

$T(x, h_1, h_2, \omega, t)$  is temperature in material with rice husk panel

The study system is subject to the following boundary conditions:

i) At the front of the panel ( $x=0$  m) :

$$-\lambda \cdot \left. \frac{\partial T(x, h_1, h_2, \omega, t)}{\partial x} \right|_{x=0} = h_1 [T_1 - T(0, h_1, h_2, \omega, t)] \quad (4)$$

ii) at the back of the panel ( $x=L$ )

$$-\lambda \cdot \left. \frac{\partial T(x, h_1, h_2, \omega, t)}{\partial x} \right|_{x=L} = h_2 [T(L, h_1, h_2, \omega, t) - T_2] \quad (5)$$

iii) The initial temperature condition's written :

$$T(x, h_1, h_2, \omega, 0) = T_i \quad (6)$$

To introduce the initial temperature, we set the variable change:

$$\tilde{T}(x, h_1, h_2, \omega, t) = T(x, h_1, h_2, \omega, t) - T_i \quad (7)$$



So the heat equation and the boundary conditions of the panel become:

$$\frac{\partial^2 \tilde{T}(x, h1, h2, \omega, t)}{\partial x^2} = \frac{1}{\alpha} \cdot \frac{\partial \tilde{T}(x, h1, h2, \omega, t)}{\partial t} \quad (8)$$

iv) At the front (x=0) :

$$\lambda \cdot \frac{\partial \tilde{T}(x, h1, h2, \omega, t)}{\partial x} \Big|_{x=0} = h1[\tilde{T}(0, h1, h2, \omega, t) + T_i - T_1] \quad (9)$$

v) On the back side(x=L)

$$-\lambda \cdot \frac{\partial \tilde{T}(x, h1, h2, \omega, t)}{\partial x} \Big|_{x=L} = h1[\tilde{T}(L, h1, h2, \omega, t) + T_i - T_2] \quad (10)$$

$$\tilde{T}(x, h1, h2, \omega, 0) = T_i \text{ is initial temperature condition} \quad (11)$$

The general solution of equation (8) is given by the expression below:

$$\tilde{T}(x, h1, h2, \omega, t) = [A(h1, h2, \omega, t) \cdot \sinh(\beta(\omega) \cdot x) + B(h1, h2, \omega, t) \cdot \cosh(\beta(\omega) \cdot x)] \cdot \exp(i\omega t) \quad (12)$$

Components A(h1, h2, ω, t) and B(h1, h2, ω, t) are obtained from the boundary conditions (9) and (10).

The density of heat flux characterizing the heat exchange between the interface of a solid and an ambient medium is given from FOURIER's law:

$$\vec{\varphi}(x, h1, h2, \omega, t) = -\lambda \overrightarrow{\text{grad}} T \quad (13)$$

## 2. Results and Discussion

### 2.1. Modules of temperature, variation of the temperature and heat flux density as a function of decimal logarithm of the excitation pulse

Figures 2, 3 and 4 show evolution of the temperature, its variation and the density of heat flux as a function of decimal logarithm excitation pulse under influence of material thickness.

For pulses lower than  $10^{-4.7}$  rad/s, the temperature in the material is roughly the same as that external environment. In this zone, the material does not exchange a thermal wave with the external environment, thus resulting in a small variation in temperature (Figure 3) and the amount of heat per unit area.

For pulses between  $10^{-4.7}$  rad/s and  $10^{-3}$  rad/s, we note a drastic decrease in temperature compared to that of the external environment corresponding to a thermal response of the material. However, the pulsations in the magnitude of  $10^{-3}$  rad/s lead to maximum thermal variation, resulting in an optimal heat flow.

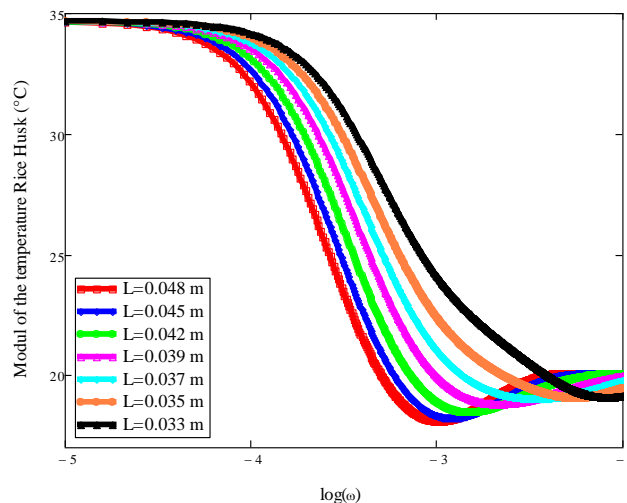


Figure 2:: Temperature as function of the decimal logarithm excitation pulse ( $h1=150W.m^{-2}.K$ ).



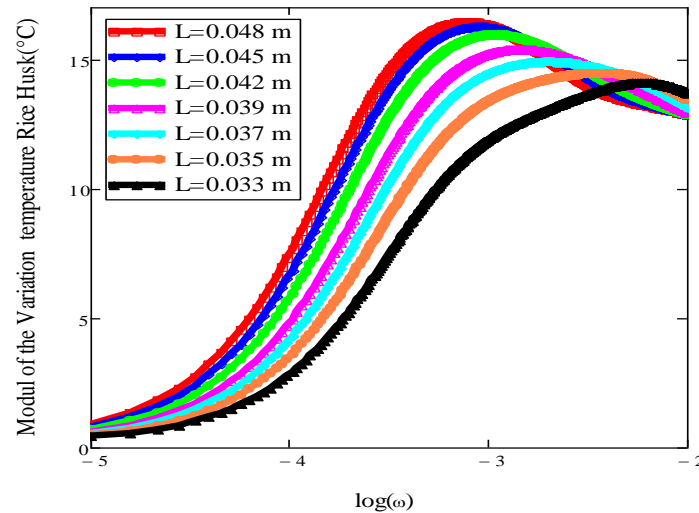


Figure 3: Variation temperature as function of the decimal logarithm excitation pulse ( $h1=150W.m^{-2}.K$ )

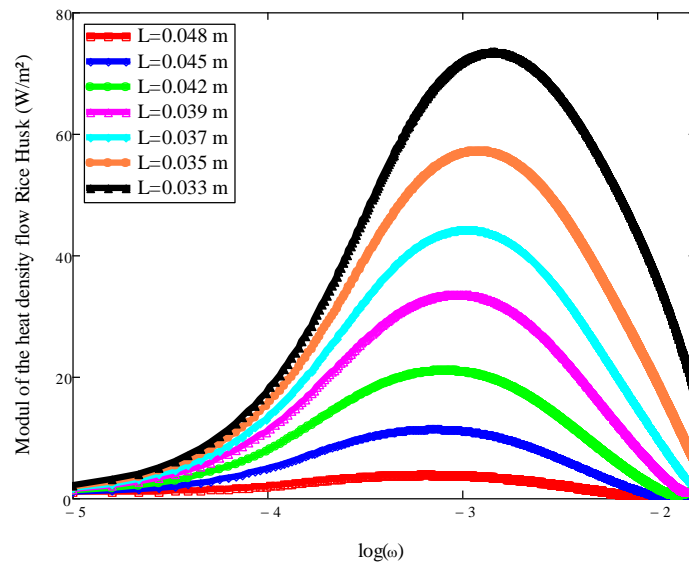


Figure 4: Density of heat flow as function of the decimal logarithm excitation pulse ( $h1=150W.m^{-2}.K$ ).

Figure 4, the optimum pulse is deduced from maximum temperature and presented in the table 1, corresponding to each thickness L of the panel.

**Table 1:** Values optimal pulse the variation in temperature and heat flow for differents thicknesses of panel.

Thickness (m)	0.048	0.045	0.042	0.039	0.037	0.035	0.033
Excitation Pulse opt (rad/s)	$10^{-2.82}$	$10^{-2.89}$	$10^{-2.99}$	$10^{-3.01}$	$10^{-3.07}$	$10^{-3.12}$	$10^{-3.14}$
$\Delta T_{max}(^{\circ}C)$	16.418	16.25	15.921	15.381	14.901	43.414	49.932
Density of heat flow( $W/m^2$ )	73.444	57.118	44.171	33.586	21.022	11.356	3.857

**2.2. Modules of temperature, variation of the temperature and heat flux density as function the thermal exchange coefficientat the front face for different values excitation pulse**

The Figures 4, 6 and 7, we note an increase in temperature, its variation and the density of heat flow as a function of the heat exchange coefficient on the front face ( $h1$ ) for different values excitation pulse .

For a long period of stress, the temperature tends towards that of outside environment under its conditions. Thus the material stores a large amount of energy corresponding to thermal saturation.

We note an increase in temperature, its variation and the density of flux of as a function of the coefficient of heat exchange. This characterizes the heat exchange between the interface of the wall and the outside environment.

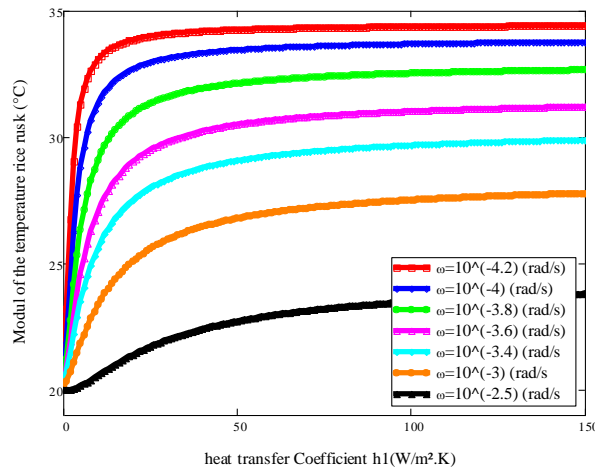


Figure 5: Temperature rice husk as function of the heat exchange coefficient on the front face for different values excitation pulse,  $x=0.033m$

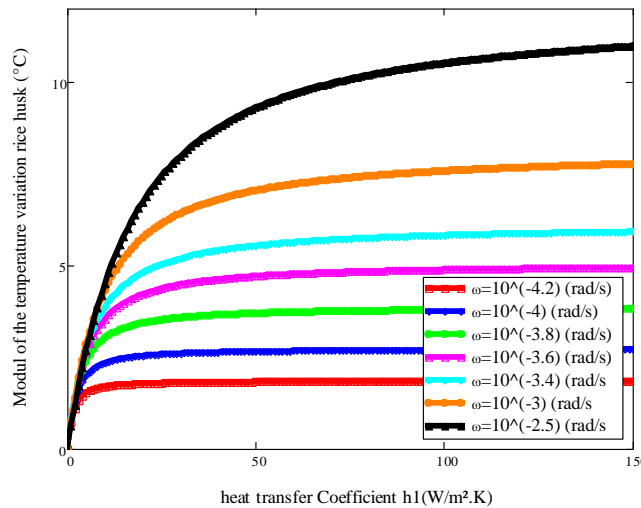


Figure 6: Temperature variation rice husk as function of the heat exchange coefficient on the front face for different values excitation pulse. ( $L= 0.033m$ )

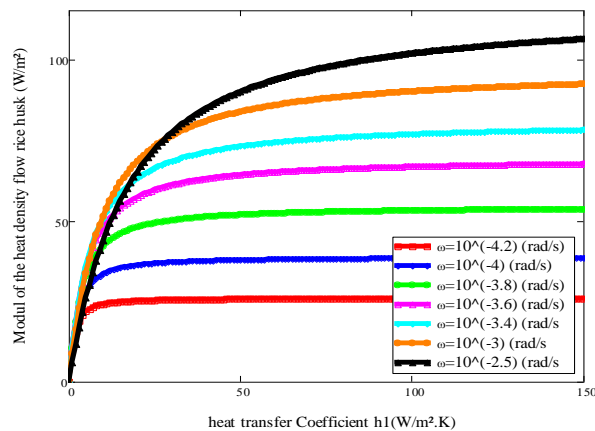


Figure 7: heat density flow rice husk as function of the heat exchange coefficient on the front face for different excitation pulse values ( $L=0.033m$ )

**2.3. Determination of the heat exchange coefficient on the back side h2**

Figure 7 shows a plateau for different stress periods, so the derivative of the expression of the heat flux density with respect to h1 is zero [19], [20] [21], by:

$$\frac{\partial \varphi(x, h_1, h_2, \omega, t)}{\partial h_1} = 0 \tag{14}$$

The resolution of this equation gives the following expressions h2 and h'2 of the heat exchange coefficient on the rear face:

$$h_2(\omega, l) = \frac{\lambda}{L(\omega)} \cdot \frac{T_{01}(\sinh(\beta(\omega).l))}{T_{02} - T_{01} \cosh(\beta(\omega).l)} \tag{15}$$

$$h'_2(\omega, l) = -\frac{\lambda}{L(\omega)} \cdot \frac{(\sinh(\beta(\omega).(l-x)))}{\cosh(\beta(\omega).(l-x))} \tag{16}$$

FIG. 8 gives the profile of the two heat exchange coefficients on the rear face as a function of the thickness of the panel for different excitations pulses.

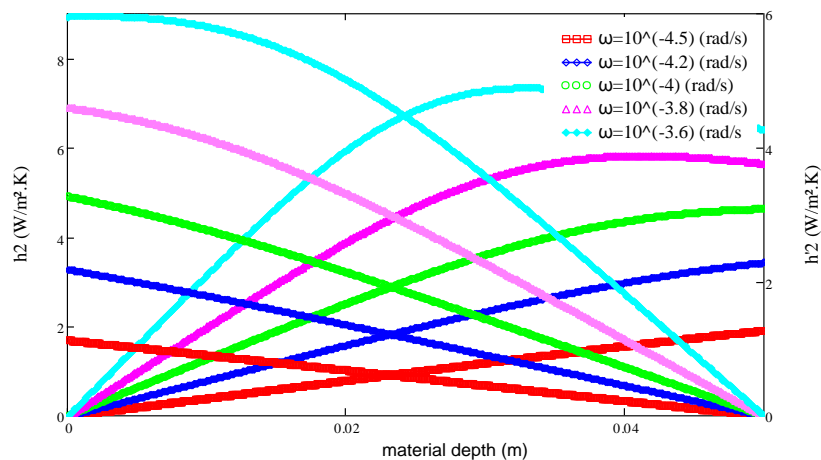


Figure 8: heat exchange coefficients as a function of the depth rice husk for different excitations pulses

The abscissa of intercept point of the two curves h2 and h'2 as a function of the depth of the panel, makes it possible to obtain a minimum thickness leading to good thermal insulation. This thickness called the effective thermal insulation layer is established by other authors [22, 23, 24] while considering these coefficients as constant. Table 2 gives the variation of the minimum panel thickness for different pulses, as well as the heat exchange coefficients and the density corresponding heat flow.

**Table 2:** Values of the minimum thickness and the heat exchange coefficients (h2 and h'2) for different excitation pulses

Excitation Pulses (rad/s)	10 <sup>-4.5</sup>	10 <sup>-4.2</sup>	10 <sup>-4</sup>	10 <sup>-3.8</sup>	10 <sup>-3.6</sup>
H <sub>min</sub> (cm)	0.0231	0.0232	0.0233	0.0235	0.0242
h2(W.m <sup>-2</sup> K <sup>-1</sup> )	0.9237	1.8413	2.8948	4.4661	6.7136
h'2(W.m <sup>-2</sup> K <sup>-1</sup> )	0.6158	1.2275	1.9298	2.9774	4.4757

Figure 9 shows the representation of the minimum thickness of the panel as a function of the excitation frequency. The best fit of H<sub>min</sub> curve as a function of excitation pulse is given by equation 17.

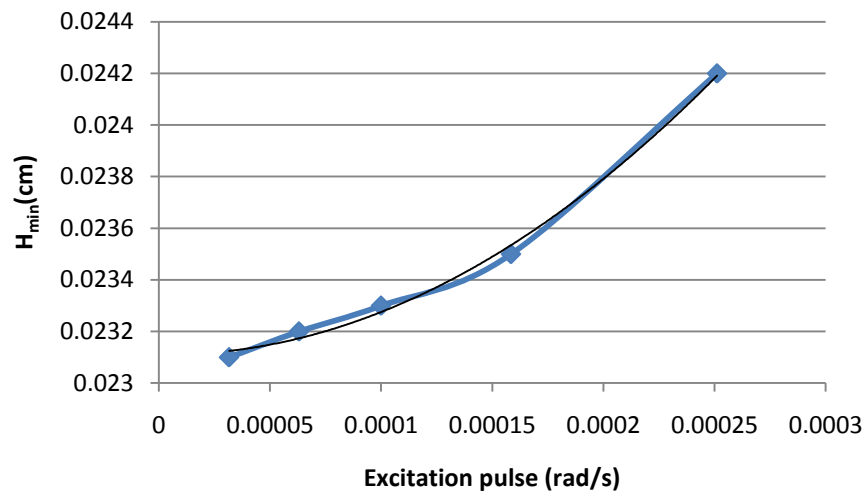


Figure 9: Minimum thickness as a function of the excitation pulse

$$H(\text{cm}) = a\omega^2 + b\omega + c \quad (17)$$

$$a = 17539 \text{ cms}^2/\text{rad}^2$$

$$b = -0.1023 \text{ cm.s/rad}$$

$$c = 0.0231 \text{ cm}$$

The use of the equation makes it possible to determine the required thickness of material for the development of a panel, for good thermal insulation. This study thus allows a saving of material which certainly leads to a reduction in the costs of producing thermal insulation devices, while taking into account the external solicitation frequency.

### 3. Conclusion

In this work, a method for determining the minimum thickness of the panel as a function of excitation pulse of external stresses is proposed from the study of theoretical expressions of the heat exchange coefficients on the rear face. The minimum thickness of the panel for good thermal insulation is modeled by a mathematical relationship highlighting the impact of the excitation frequency.

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