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A fractional order generalized thermoelastic problem for a half-space by a Line-Focused Laser Irradiation

S. Esmail[1], S. E. Khader[2], Mona El. M. Khedr[3]

[1]Department of Mathematical and Theoretical Physics Atomic Energy Authority, EGYPT,
samia_hamed40@yahoo.com
[2]Department of Mathematical and Theoretical Physics Atomic Energy Authority, EGYPT,,
shabolaaa@gmail.com
[3]Department of Physics, Faculty of Science University of Alexandria, EGYPT
monakhedr25jan@yahoo.com

Abstract A two-dimensional model of the fractional order theory of thermoelasticity to a problem of a half-space under a line-focused laser irradiation is presented. The model is solved for the effects of thermal diffusion and optical penetration. Laplace and Fourier transforms techniques are applied to derive the solution. The inverse of Laplace transforms and Fourier transforms are obtained numerically by using the complex inversion formula of the transform together with Fourier expansion techniques. The predictions of the fractional order theory are discussed. Numerical results for the temperature, displacement and thermal stress, strain distributions are represented graphically. This research is useful in the way the laser affects live tissue.

Keywords Fourier Transform; Fractional order thermoelasticity; Laplace transform; line-source

1. Introduction

Wave motion can be generated by the irradiation of a solid surface by laser light, these waves called ultrasonic waves. There are many applications for laser-generated ultrasound in the industry, for example in non-destructive test areas (Independent Nondestructive Testing and Evaluation), grain size of materials and determine elastic constants. The characteristics of laser based ultrasonic waves depend on the thermal diffusion, the optical penetration, the elastic properties, and the geometry of materials, as well as the finite width and duration of the laser source. Many physical processes occur when a solid surface is illuminated by a laser beam, depending on the incident power. As a high powers produce damage to the material surface making them unsuitable for non-destructive tests. In the case of the low powers, the source of the laser generates heat waves through heat conduction, and elastic waves are generated in materials such as conductors. Some scientists have done some research. Irene Arias and Jan D. Achenbach [1] are solved a two-dimensional theoretical model for the field generated in the thermoelastic regime by line-focused laser illumination of a homogeneous. Jaegwon Yoo, C. H. Lim and S. H. Baik [2] are disused a numerical analysis formulation of thermoelastic surface waves in a homogeneous isotropic elastic half-space under a line-focused laser irradiation. I. A. Veres, T. Berer, P. Burgholzer [3] are solved numerical modeling of thermoelastic generation of ultrasound by laser irradiation in the coupled thermoelasticity. Yoo, Jae-Gwon; Baik, S.H. [4] are investigated a 2D finite-element numerical simulation of generation of ultrasonic waves in a homogeneous isotropic elastic slab under a line-focused laser irradiation. F. Reverdy and B. Audoin [5] are described A noncontact laser–ultrasonic technique that allows determination of material properties of anisotropic plate like samples. McDonald, F. A [6] is investigated the precursor in laser-generated ultrasound waveforms in metals.

Some scientists have done some research on the generalized thermoelasticity. Lord and Shulman [7] introduced the theory of generalized thermoelasticity with one relaxation time. Anwar and Sherief [8] studied A Problem in Generalized Thermoelasticity for an Infinitely Long Annular Cylinder Composed of Two Different Materials. Hany Sherief and Khader. S. E. [9] solved the problem Propagation of discontinuities in electromagneto generalized thermoelasticity in cylindrical regions. Some contribution works that use generalized thermoelasticity can be found in [10-18].

The fractional order theory of thermoelasticity was derived by Sherief. H, El-Sayed. A and Abd El-Latief. A.M. [19]. It is a generalization of both the coupled and the generalized theories of thermoelasticity. Sherief and Abd El Latief [20, 21] have solved a spherical cavity and 1D problems for a half space in this theory. Abd El-Latief. A. M. and Khader. S. E [22] they are applied the fractional order theory of thermoelasticity to a 1D problem for a half-space overlaid by a thick layer of a different material. S. Santra, N. C. Das, R. Kumar, and A. Lahiri [23] are solved Three-dimensional fractional order generalized thermoelastic problem under the effect of rotation in a half space. Z. Wang, D. Liu, Q. Wang, and J. Z. Zhou [24] are discussed the fractional order theory of thermoelasticity for elastic medium with variable material properties. . Some contribution works that use fractional calculus can be found in [25-30].

2. Formulation of the Problem

Consider a homogeneous isotropic thermoelastic solid occupying the half-space. The z-axis is taken perpendicular to the bounding plane pointing inward. From time; the medium is irradiated by a laser pulse depositing heat on its front surface.

The displacement vector u has the form

$$\mathbf{u}=(u_x,0,u_z)$$

The cubical dilatation e is given by

$$e=\operatorname{div}\mathbf{u}=\frac{\partial u_x}{\partial x}+\frac{\partial u_z}{\partial z} \tag{1}$$

The components of the thermoelastic stress tensor are given by

$$\sigma_{xx}=(\lambda+2\mu)e-2\mu\frac{\partial u_z}{\partial z}-\gamma(T-T_0) \tag{2a}$$

$$\sigma_{zz}=(\lambda+2\mu)e-2\mu\frac{\partial u_x}{\partial x}-\gamma(T-T_0) \tag{2b}$$

$$\sigma_{xz}=\mu\left(\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right) \tag{2c}$$

where λ and μ are Lamé's moduli, T is the absolute temperature of the medium, and γ is a material constant given by $\gamma = (3\lambda + 2\mu)\alpha t$ where αt is the coefficient of linear thermal expansion, T0 is a reference temperature assumed to be such that │(T-T0) / T0 │<<1.

The equations of motion have the vector form

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u}-\gamma\,\mathrm{grad}\,T=\rho\,\ddot{\mathbf{u}} \tag{3}$$

$$(\lambda+\mu)\frac{\partial e}{\partial x}+\mu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_z}{\partial z^2}\right)-\gamma\frac{\partial T}{\partial x}=\rho\frac{\partial^2 u_x}{\partial t^2} \tag{4}$$

$$(\lambda+\mu)\frac{\partial e}{\partial z}+\mu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial z^2}\right)-\gamma\frac{\partial T}{\partial z}=\rho\frac{\partial^2 u_z}{\partial t^2} \qquad \text{`(5)}$$

The equation of heat conduction has the form [16]

$$k\nabla^2 T=\frac{\partial}{\partial t}\left(1+\tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)(\rho c_E T+\gamma T_0\,e)-Q \tag{6}$$

where k is the thermal conductivity of the medium, cE is the specific heat at constant and ρ is the density. And ρ are two parameters of the theory. Q is the heat source due to the laser irradiation.

A suitable expression for the heat deposition over the irradiation zone of the laser pulse is

$$Q = E(1-R)h(\xi)f(x)g(t)$$

where , $f(x) = \frac{1}{\sqrt{2\pi}}\frac{2}{\varpi}e^{-2x^2/\varpi^2}$, $g(t) = \frac{8t^3}{\upsilon^4}e^{-2t^2/\upsilon^2}$, $h(\xi) = \xi e^{-\xi z}$,

Where E is the energy of the laser pulse per unit length, R is the surface reflectivity, R_G is the Gaussian beam radius, υ is the pulse duration time of the laser beam (full width at half maximum), and ξ is the extinction coefficient.

3. Solution of the Problem in the Laplace Transform Domain

Let us introduce the following non-dimension variables

$$(x^*, z^*) = c_0\eta(x, z) \quad , \quad u^* = c_0\eta u \quad , \quad \sigma^*_{ij} = \frac{\sigma_{ij}}{\mu} \quad , \quad \theta = \frac{\gamma(T - T_0)}{(\lambda+2\mu)},$$

$$t^* = c_0^2\eta t \quad , \quad \tau_0 = c_0^{2\alpha}\eta^\alpha\tau_0 \quad , Q^* = \frac{\gamma Q}{kc_0^2\eta^2(\lambda+2\mu)}$$

$$\eta = \frac{\rho c_E}{k} \quad , \quad c_1^2 = \frac{\lambda+2\mu}{\rho}.$$

The governing equations (2) - (6) in non-dimensional form become (dropping the asterisks for convenience)

$$\sigma_{xx} = \beta^2 e - 2\frac{\partial u_z}{\partial z} - \beta^2\theta, \tag{7a}$$

$$\sigma_{zz} = \beta^2 e - 2\frac{\partial u_x}{\partial x} - \beta^2\theta, \tag{7b}$$

$$\sigma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \tag{7c}$$

$$\nabla^2\mathbf{u} + (\beta^2 - 1)\,\text{grad div}\,\mathbf{u} - \beta^2\text{grad}\,\theta = \beta^2\ddot{\mathbf{u}}. \tag{8}$$

$$(\beta^2 - 1)\frac{\partial e}{\partial x} + \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial z^2}\right) - \beta^2\frac{\partial\theta}{\partial x} = \beta^2\frac{\partial^2 u_x}{\partial t^2} \tag{9}$$

$$(\beta^2 - 1)\frac{\partial e}{\partial z} + \left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial z^2}\right) - \beta^2\frac{\partial\theta}{\partial z} = \beta^2\frac{\partial^2 u_z}{\partial t^2} \tag{10}$$

$$\nabla^2\theta = \frac{\partial}{\partial t}\left(1 + \tau_0\frac{\partial^\alpha}{\partial t^\alpha}\right)(\theta + \varepsilon e) - Q. \tag{11}$$

where $\beta^2 = \frac{(\lambda+2\mu)}{\mu}$, $\varepsilon_1 = \frac{T_0\gamma^2}{c_E\rho^2 c_0^2}$

Applying Laplace transform with respect to variable t for equations (7)-(11), we obtain:

$$\bar{\sigma}_{xx} = \beta^2\bar{e} - 2\frac{\partial\bar{u}_z}{\partial z} - \beta^2\bar{\theta}, \tag{12a}$$

$$\bar{\sigma}_{zz} = \beta^2\bar{e} - 2\frac{\partial\bar{u}_x}{\partial x} - \beta^2\bar{\theta}, \tag{12b}$$

$$\bar{\sigma}_{xz} = \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \tag{12d}$$

$$\nabla^2\bar{\mathbf{u}} + (\beta^2 - 1)\,\text{grad div}\,\bar{\mathbf{u}} - \beta^2\,\text{grad}\,\bar{\theta} = \beta^2 s^2\bar{\mathbf{u}}. \tag{13}$$

$$(\beta^2-1)\frac{\partial \bar{e}}{\partial x}+\left(\frac{\partial^2 \bar{u}_x}{\partial x^2}+\frac{\partial^2 \bar{u}_x}{\partial z^2}\right)-\beta^2\frac{\partial \bar{\theta}}{\partial x}=\beta^2 s^2 \bar{u}_x \tag{14}$$

$$(\beta^2-1)\frac{\partial \bar{e}}{\partial z}+\left(\frac{\partial^2 \bar{u}_z}{\partial x^2}+\frac{\partial^2 \bar{u}_z}{\partial z^2}\right)-\beta^2\frac{\partial \bar{\theta}}{\partial z}=\beta^2 s^2 \bar{u}_z \tag{15}$$

$$\nabla^2\bar{\theta}=s\left(1+\tau_0 s^\alpha\right)\left(\bar{\theta}+\varepsilon\bar{e}\right)-\bar{Q}. \tag{16}$$

Taking the divergence for both side of equation (13), we obtain

$$\nabla^2\bar{e}-\nabla^2\bar{\theta}=s^2\bar{e}. \tag{17}$$

Eliminating $\bar{e}$ between equations (16) and (17), we get

$$\left\{\nabla^4-\left[s^2+s\left(1+\tau_0 s^\alpha\right)(1+\varepsilon)\right]\nabla^2+s^3\left(1+\tau_0 s^\alpha\right)\right\}\bar{\theta}=-\left(\nabla^2-s^2\right)\bar{Q} \tag{18}$$

In a similar manner we can show that $\bar{e}$ satisfy the equations

$$\left\{\nabla^4-\left[s^2+s\left(1+\tau_0 s^\alpha\right)(1+\varepsilon)\right]\nabla^2+s^3\left(1+\tau_0 s^\alpha\right)\right\}\bar{e}=-\nabla^2\bar{Q} \tag{19}$$

In order to solve equations (18) and (19), we shall use the exponential Fourier transform with respect to the variable *x* (denoted by an asterisk), the solutions of equations (18) and (19) are in the form:

$$\bar{\theta}^*=\sum_{i=1}^{2}A_i\left(k_i^2-s^2\right)e^{-m_i z}+\frac{\xi^2-\left(q^2+s^2\right)}{\left(\xi^2-m_1^2\right)\left(\xi^2-m_2^2\right)}\bar{Q}^* \tag{20}$$

$$\bar{e}^*=\sum_{i=1}^{2}A_i k_i^2 e^{-m_i z}+\frac{\xi^2-q^2}{\left(\xi^2-m_1^2\right)\left(\xi^2-m_2^2\right)}\bar{Q}^* \tag{21}$$

Where: $\bar{Q}^*=E(1-R)h(\xi)F^*(q)\bar{G}(s)$, $F^*(q)=\frac{1}{\sqrt{2\pi}}e^{-q^2\varpi^2/8}$.

$\bar{G}(s)=1+\frac{s^2\upsilon^2}{8}-\frac{\sqrt{2\pi}}{32}s\upsilon\left(12+s^2\upsilon^2\right)e^{s^2\upsilon^2/8}$ and $m_1^2=k_1^2+q^2$, $m_2^2=k_2^2+q^2$, A_1 and A_2 are constants. k_1^2 *and* k_2^2 are the roots of the characteristic equation:

$$k^4-\left[s^2+s\left(1+\tau_0 s^\alpha\right)(1+\varepsilon)\right]k^2+s^3\left(1+\tau_0 s^\alpha\right)=0$$

Appling the exponential Fourier transform to equation (15) to get the displacement components of u as follows:

$$\left(D^2-n^2\right)\bar{u}_z^*=\frac{\partial}{\partial z}\left[\left(1-\beta^2\right)\bar{e}^*+\beta^2\bar{\theta}^*\right] \tag{22}$$

where $n^2=\beta^2 s^2+q^2$

Substituting from equations (20) and (21) into equation (22), we obtain

$$\left(D^2-n^2\right)\bar{u}_z^*=\frac{\partial}{\partial z}\left[\sum_{i=1}^{2}\left(k_i^2-\beta^2 s^2\right)A_i e^{-m_i z}+\frac{\xi\left[\xi^2(\beta^2+1)-n^2\right]}{\left(\xi^2-m_1^2\right)\left(\xi^2-m_2^2\right)}\bar{Q}^*\right] \tag{23}$$

The solution of equation (23) is given by

$$\bar{u}_z^*=Ce^{-nz}-\sum_{i=1}^{2}A_i m_i e^{-m_i z}-\frac{\xi\left[\xi^2(\beta^2+1)-n^2\right]}{\left(\xi^2-m_1^2\right)\left(\xi^2-m_2^2\right)\left(\xi^2-n^2\right)}\bar{Q}^* \tag{24}$$

Taking the Laplace and exponential Fourier transforms of equation (1), and using equations (21), (24), we get

$$\bar{u}_x^*=\frac{1}{iq}\left[-nCe^{-nz}+q^2\sum_{i=1}^{2}A_i e^{-m_i z}+\frac{\xi^4\beta^2+q^2\left(\xi^2-n^2\right)}{\left(\xi^2-m_1^2\right)\left(\xi^2-m_2^2\right)\left(\xi^2-n^2\right)}\bar{Q}^*\right] \tag{25}$$

Substituting from equations (20), (21), (24) and (25) into equations (12), we obtain the stress components in the form

$$\bar{\sigma}_{zz}^{*}=-2nCe^{-nz}+\left(n^{2}+q^{2}\right)\sum_{i=1}^{2}A_{i}e^{-m_{i}z}+\frac{q^{2}+n^{2}}{\left(\xi^{2}-m_{1}^{2}\right)\left(\xi^{2}-m_{2}^{2}\right)}\bar{Q}^{*} \quad (28)$$

$$\bar{\sigma}_{xz}^{*}=\left(\frac{n^{2}+q^{2}}{iq}\right)Ce^{-nz}+\sum_{i=1}^{2}2iqm_{i}A_{i}e^{-m_{i}z}-\frac{\xi\left[\xi^{2}\beta^{2}(\xi^{2}+q^{2})+2q^{2}\left(\xi^{2}-n^{2}\right)\right]}{iq\left(\xi^{2}-m_{1}^{2}\right)\left(\xi^{2}-m_{2}^{2}\right)\left(\xi^{2}-n^{2}\right)}\bar{Q}^{*} \quad (29)$$

The constants A_1, A_2 and C will be obtained from the initial and boundary conditions.

4. Initial and boundary conditions

The initial conditions are that the half-space are rest and the boundary conditions include the mechanical and thermal conditions. We apply the initial and boundary conditions according to each case as follows:

First case

In this case, we assume the pulse energy is not completely absorbed at the surface and heat is flow into the half-space, the boundary conditions are:

$$\begin{cases}\bar{\theta}^{*}=\dfrac{\psi(q)}{s} \\ \bar{\sigma}_{zz}^{*}=\bar{\sigma}_{xz}^{*}=0\end{cases} \quad at\ z=0$$

Where $\psi(q)=\sqrt{\frac{2}{\pi}}\frac{\sin(qa)}{q}$

The solutions in this case are represented by equations (20), (28) and (29)

Second case

In this case, we assume the pulse energy is completely absorbed at the surface, no optical penetration $\xi\to 0$, we can set $\underset{\xi\to 0}{Lim}\,\xi e^{-\xi z}=1$, and heat is flow into the half-space, the boundary conditions are:

$$\begin{cases}\bar{\theta}^{*}=\dfrac{\psi(q)}{s} \\ \bar{\sigma}_{zz}^{*}=\bar{\sigma}_{xz}^{*}=0\end{cases} \quad at\ z=0$$

The solutions of this case are in the form:

$$\bar{\theta}^{*}=\sum_{i=1}^{2}A_{i}\left(k_{i}^{2}-s^{2}\right)e^{-m_{i}z}+\frac{q^{2}+s^{2}}{m_{1}^{2}m_{2}^{2}}\bar{Q}^{*} \quad (30)$$

$$\bar{u}_{z}^{*}=Ce^{-nz}-\sum_{i=1}^{2}A_{i}m_{i}e^{-m_{i}z} \quad (31)$$

$$\bar{u}_{x}^{*}=\frac{1}{iq}\left[-nCe^{-nz}+q^{2}\sum_{i=1}^{2}A_{i}e^{-m_{i}z}-\frac{q^{2}}{m_{1}^{2}m_{2}^{2}}\bar{Q}^{*}\right] \quad (32)$$

$$\bar{\sigma}_{zz}^{*}=-2nCe^{-nz}+\left(n^{2}+q^{2}\right)\sum_{i=1}^{2}A_{i}e^{-m_{i}z}-\left(\frac{2q^{2}+\beta^{2}s^{2}}{m_{1}^{2}m_{2}^{2}}\right)\bar{Q}^{*} \quad (33)$$

$$\bar{\sigma}_{xz}^{*}=\left(\frac{n^{2}+q^{2}}{iq}\right)Ce^{-nz}+\sum_{i=1}^{2}2iqm_{i}A_{i}e^{-m_{i}z} \quad (34)$$

Where $\bar{Q}^{*}=E(1-R)F^{*}(q)\bar{G}(s)$

Third case

In this case, we assume the pulse energy is completely absorbed at the surface. And the heat that is generated by the laser is deposited the half-space just under the surface, the boundary conditions are:

$$\begin{cases} \dfrac{d\bar{\theta}^*}{dz} = 0 \\ \bar{\sigma}^*_{zz} = \bar{\sigma}^*_{xz} = 0 \end{cases} \qquad at\ z = 0$$

The solutions of this case are given by equations (30-34)
The formula of inversion of the double transforms, who's presented in [13]

5. Numerical Results

For fixed values of *z*, *t* and α, the linear system are solved numerically for the unknowns A_1, A_2 and C and the results are substituted to obtain the values of functions θ, u_z and σ_{zz}. And we shall apply our results to the aluminum alloy. The material properties are

$$\lambda = 5.81\times10^{10}\,kg\,m^{-1}s^{-2},\ \mu = 2.61\times10^{10}\,kg\,m^{-1}s^{-2},\ \rho = 2700\,kg\,m^{-3},$$

$$k = 240\,kg\,m\,K^{-1}s^{-3}, C_E = 896\,m^2K^{-1}s^{-2}, T_0 = 293\,K,\ \alpha_t = 6.58\times10^{-5}\,K^{-1},$$

$$E = 1\,mJ,\ R = 1.06\,m,\ \varpi = 0.5\,mm,\ \upsilon = 10\,ns, \xi = 2\times10^8\,1/m, \tau = 0.02$$

The problem was solved for three values of fractional parameter $\alpha = 0.99$, $\alpha = 0.9$ and $\alpha = 0.01$ with fixed time t = 0.1. The graphs for the temperature, displacement and stress, for case 1 are shown in figure (1) – figure (3), for case 2 are shown in figure (4) – figure (6) and for case 3 are shown in figure (7) – figure (9). Black lines represent the solution for α = 0.01, red lines represent the solution for α = 0.9 and blue lines represent the solution when α = 1.

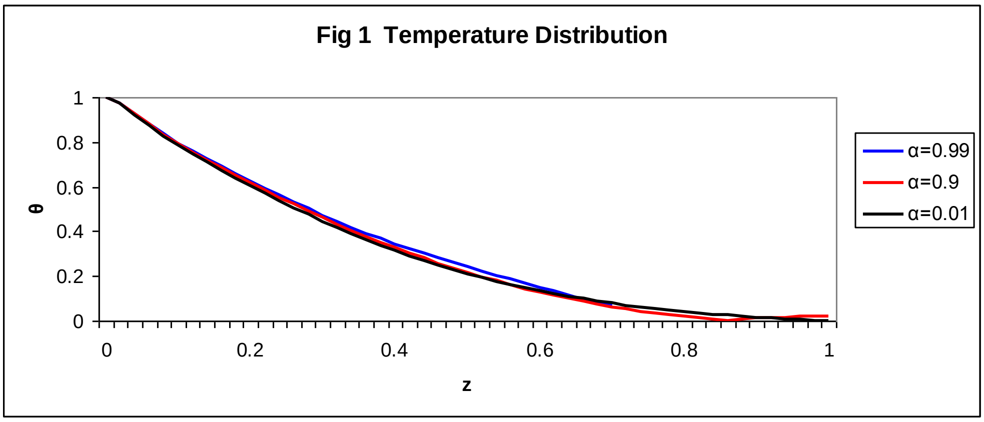

Figure 1: Temperature Distribution

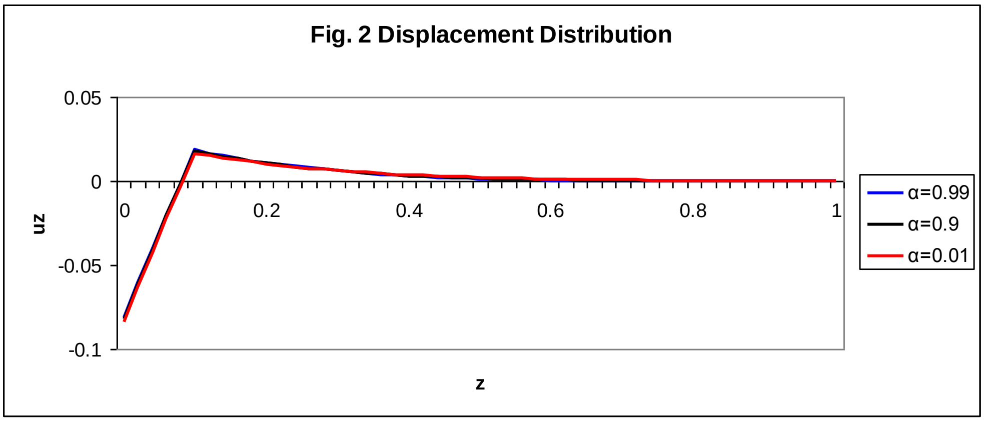

Figure 2: Displacement Distribution

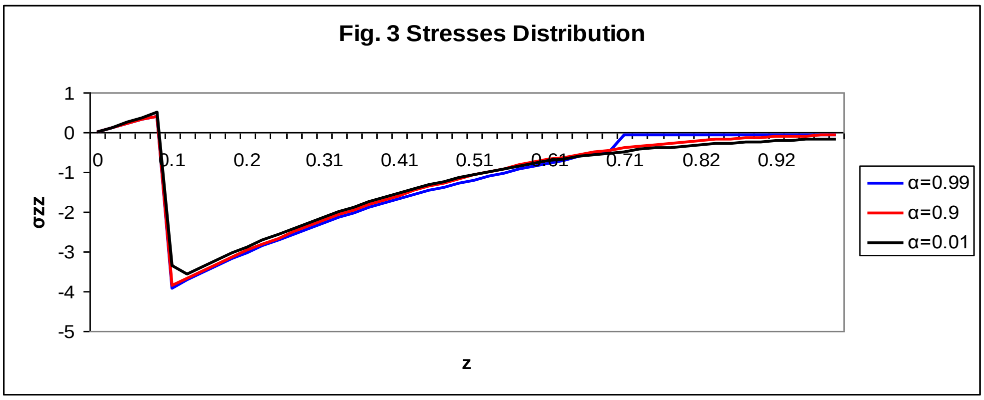

Figure 3: Stresses Distribution

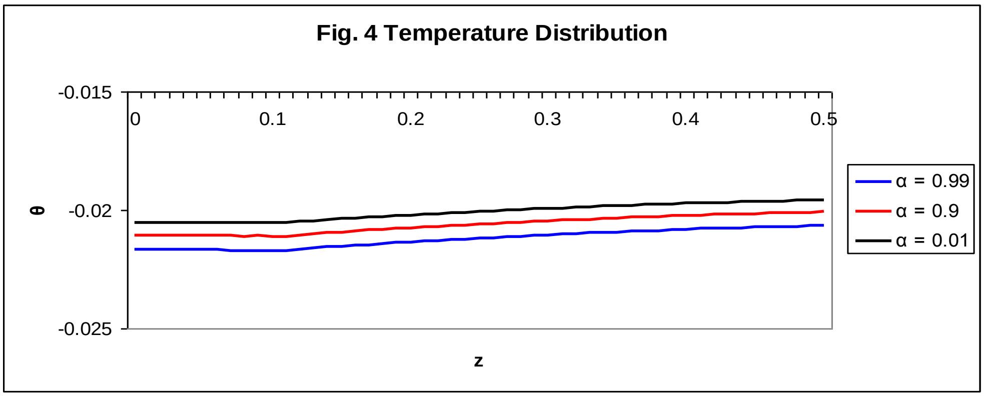

Figure 4: Temperature Distribution

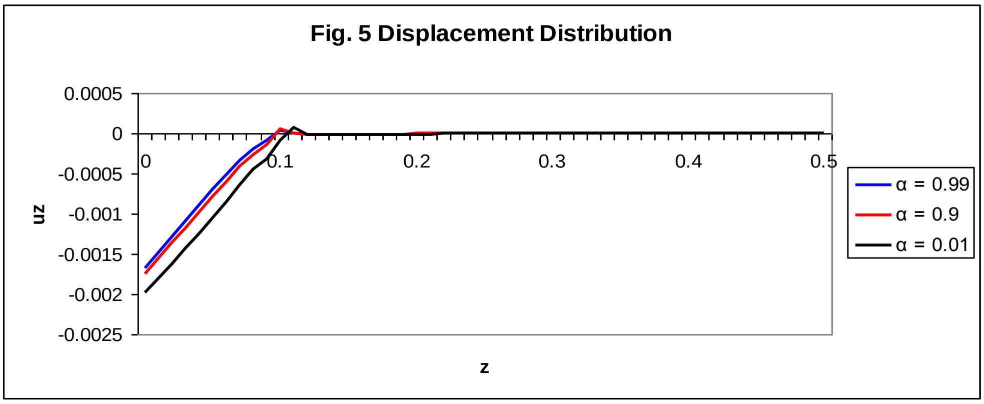

Figure 5: Displacement Distribution

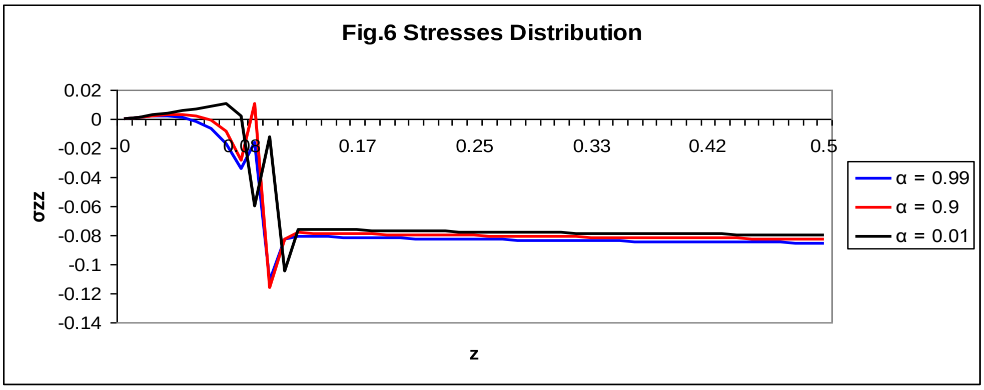

Figure 6: Stresses Distribution

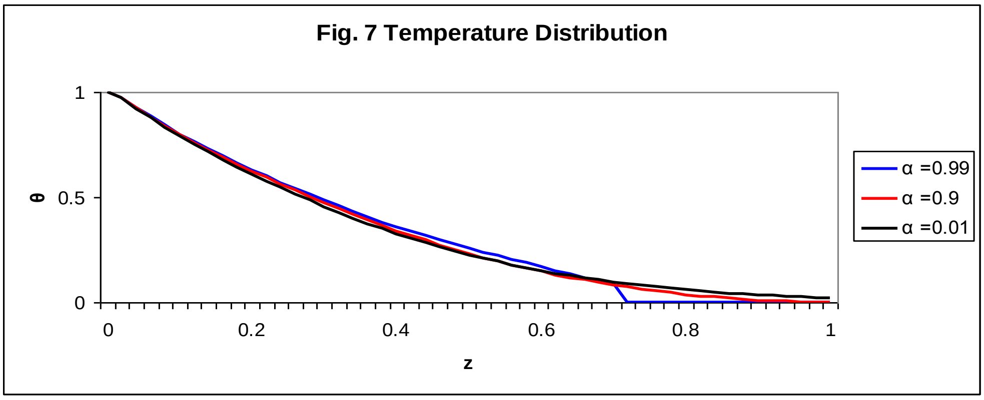

Figure 7: Temperature Distribution

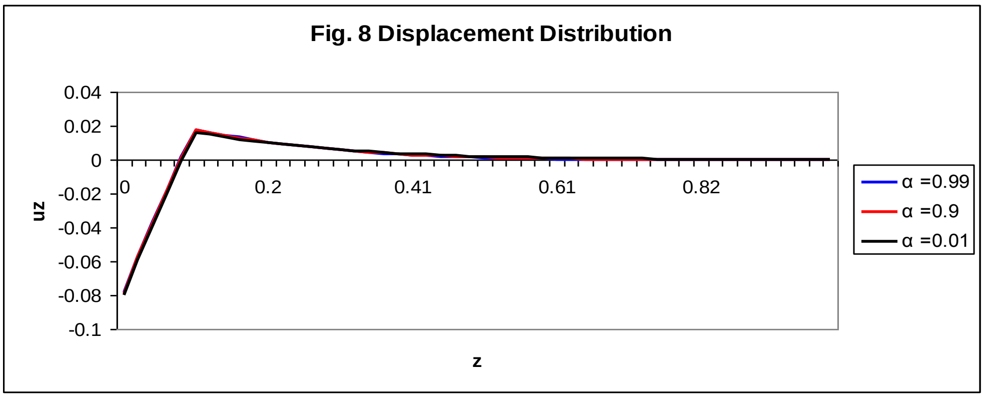

Figure 8: Displacement Distribution

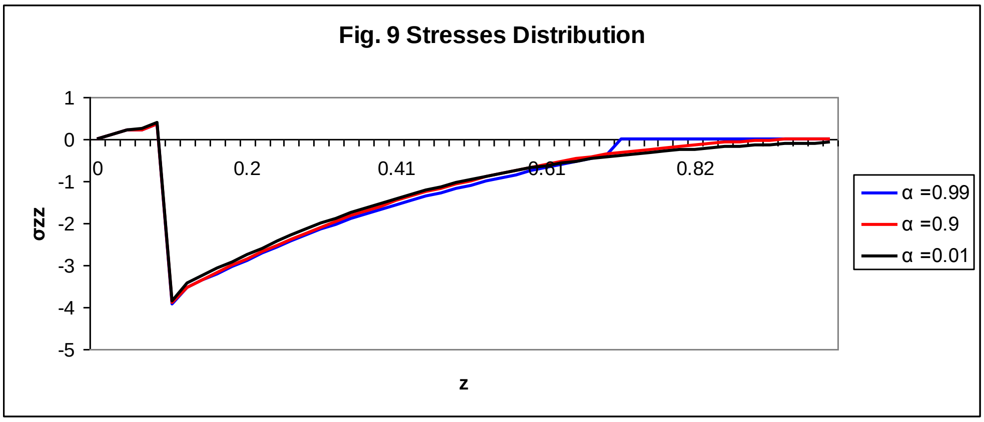

Figure 9: Stresses Distribution

In the first case, the pulse energy is not completely absorbed at the surface and heat is flow into the half-space From figures (1-3), we draw the figures with different values of fractional parameter, when, the equation of heat conduction becomes parabolic, it is predicted an infinite heat propagation speed, representing the dotted line. When, the equation of heat conduction becomes hyperbolic, it is predicted a finite heat propagation speed, representing the solid line.

In the second case, the pulse energy is completely absorbed at the surface and heat is flow into the half-space. From all the graphs we find that the behavior of all functions are the same the functions in the first case, we find that the function has a little effects, and the surface is completely absorbed the pulse energy.

In the third case, the pulse energy is completely absorbed at the surface. And the heat that is generated by the laser is deposited the half-space just under the surface. From all the graphs and compeer with [6], there is agreement in the results for all functions, where the solution has non-zero value only in the interval, i.e. the heat which is generated by the laser is deposited the half-space just under the surface.

6. Conclusions

A two-dimensional fractional order generalized thermoelastic problem for a half-space by a Line-Focused Laser Irradiation has been solved. The problem has been solved by using Laplace and Fourier transform techniques. The inversion process is carried out using a numerical method based on Fourier series expansions. The problem takes the effect of fractional parameter, the thermal diffusion and optical penetration.

The effect of fractional parameter has appeared when fractional parameter tends to zero, the curve has elongated, but when fractional parameter tends to one, the curve has shrunk. The effect of optical penetration is very small which Leeds to the pulse energy is completely absorbed at the surface. The effect of thermal diffusion has been studied on the surface of an aluminum half-space. This effect is noticeable near the hot area but in the remote area does not appeared as expected physically. The thermal diffusion is neglected in the case three.

This research is useful in laser surgery and laser teeth whitening. Surgical laser systems differ not only because of the wavelength, but also because of the light conduction system: the elastic fibers or the articulated arm, as well as other factors. It varies depending on how the laser affects live tissue and we recognize thermal, chemical, and electromechanical effects.

Author Contributions

The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

Conflict of Interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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