Available online www.jsaer.com

***Journal of Scientific and Engineering Research*, 2020, 7(4):191-197**

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

Coefficient Bound Estimates for Certain Class of Analytic Functions

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Abstract In this paper, we introduce and investigate new subclasses of analytic functions on the open unit disk in complex plane. Several interesting geometric properties of the functions belonging to these classes are examined. Also, sharp inequalities for the coefficient bound estimates for the functions belonging to these classes are given. Some consequences of the results obtained here are also discussed.

Keywords Analytic function, Coefficient bound, Starlike function, Convex function

1. Introduction and Preliminaries

Let A be the class of analytic functions f in the open unit disk $U = \{z \in \mathbb{C}: |z| < 1\}$, normalized by $f(0) = 0 = f'(0) - 1$ of the form

$$f(z) = z + a_2 z^2 + a_3 z^3 + \ldots + a_n z^n + \ldots = z + \sum_{n=2}^{\infty} a_n z^n, a_n \in \mathbb{C}, \tag{1.1}$$

and S denote the class of all functions in A which are univalent in U.

Let T denote the subclass of all functions f in A of the form

$$f(z) = z - a_2 z^2 - a_3 z^3 - \ldots - a_n z^n - \ldots = z - \sum_{n=2}^{\infty} a_n z^n, a_n \geq 0. \tag{1.2}$$

We also denote by $S^*(\alpha)$, $C(\alpha)$ and $K(\alpha)$ the subclasses of S that are, respectively, starlike, convex and close-to-convex with respect to starlike function $g(z)$ (need not be normalized) of order $\alpha\left(\alpha \in [0,1)\right)$ in the open unit disk U. By definition, we have (see for details,[4, 5], also [8])

$$S^*(\alpha) = \left\{f \in A: Re\left(\frac{zf'(z)}{f(z)}\right) > \alpha, z \in U\right\}, \alpha \in [0,1), \tag{1.3}$$

$$C(\alpha) = \left\{f \in A: Re\left(1 + \frac{zf''(z)}{f'(z)}\right) > \alpha, z \in U\right\}, \alpha \in [0,1), \tag{1.4}$$

and

$$K(\alpha) = \left\{f \in A: Re\left(\frac{zf'(z)}{g(z)}\right) > \alpha, z \in U, g \in S^*\right\}, \alpha \in [0,1).$$

For convenience, $S^* = S^*(0), C = C(0)$ and $K = K(0)$ are, respectively, well-known starlike, convex and close-to-convex functions in U. It is well known that close-to-convex functions are univalent in U, but not necessarily the converse. It is easy to verify that $C \subset S^* \subset K \subset S$. For details on these classes one could refer to the monograph by Goodman [5].

An interesting generalization of the function class $K(\alpha)$ is provided by the class $K(\alpha, \beta; g)$ of functions $f \in A$, which satisfies the following condition

$$K(\alpha, \beta; g) = \left\{f \in A: Re\left(\frac{zf'(z) + \beta z^2 f''(z)}{g(z)}\right) > \alpha, z \in U, g \in S^*\right\},$$
$$\alpha \in [0,1), \beta \in [0,1]$$

with respect to function g (need not be normalized).

We will denote $K(\alpha, \beta; g) = K(\alpha, \beta)$.

Notation 1 *The class $K(\alpha, \beta), \alpha \in [0,1), \beta \in [0,1]$ is the first time introduced and examined in this paper.*

Note that, we will use $TS^*(\alpha), TK(\alpha)$ and $TC(\alpha)$ instead $S^*(\alpha), K(\alpha)$ and $C(\alpha)$, respectively, if $f \in T$. Also, we will denote $TK(\alpha,\beta)$ instead $K(\alpha,\beta)$ if $f \in T$.

An interesting unification of the functions $S^*(\alpha)$ and $C(\alpha)$ classes is provided by the class $A(\alpha,\beta)$ of functions $f \in A$, which also satisfies the following condition

$$Re\left\{\frac{zf'(z)+\beta z^2 f''(z)}{\beta z f'(z)+(1-\beta)f(z)}\right\} > \alpha, 0 \le \alpha < 1, 0 \le \beta \le 1, z \in U.$$

Thus,

$$A(\alpha,\beta) = \left\{f \in A: Re\left\{\frac{zf'(z)+\beta z^2 f''(z)}{\beta z f'(z)+(1-\beta)f(z)}\right\} > \alpha, z \in U\right\}, 0 \le \alpha < 1,$$
$$0 \le \beta \le 1.$$

In the special case, we have $A(\alpha,0) = S^*(\alpha)$ and $A(\alpha,1) = C(\alpha)$ in terms of the simpler classes $S^*(\alpha)$ and $C(\alpha)$, defined by (1.3) and (1.4), respectively.

We will denote $T(\alpha,\beta)$ if $f \in T$. Thus,

$$T(\alpha,\beta) = \left\{f \in A: Re\left\{\frac{zf'(z)+\beta z^2 f''(z)}{\beta z f'(z)+(1-\beta)f(z)}\right\} > \alpha, z \in U\right\}, 0 \le \alpha < 1,$$
$$0 \le \beta \le 1.$$

The class $T(\alpha,\beta)$ was investigated by Altintaş et al. [2] and [3] (in a more general way $T_n(p,\alpha,\beta)$) and (subsequently) by Irmak et al. [6]. In particular, the class $T_n(1,\alpha,\beta)$ was considered earlier by Altintaş [1].

Motivated by the aforementioned works, we define subclasses of analytic functions as follows.

Definition 1 *A function $f \in A$ given by (1.1) is said to be in the class $A(\alpha,\beta,\gamma)$, $\alpha \in [0,1), \beta, \gamma[0,1]$ if the following condition is satisfied*

$$Re\left\{\frac{zf'(z)+\beta z^2 f''(z)}{\gamma[\beta z f'(z)+(1-\beta)f(z)]+(1-\gamma)z}\right\} > \alpha,$$
$$z \in U$$

Thus

$$A(\alpha,\beta,\gamma) = \left\{f \in A: Re\left\{\frac{zf'(z)+\beta z^2 f''(z)}{\gamma[\beta z f'(z)+(1-\beta)f(z)]+(1-\gamma)z}\right\} > \alpha, z \in U\right\},$$
$$\alpha \in [0,1), \beta, \gamma[0,1].$$

Definition 2 *A function $f \in T$ given by (1.1) is said to be in the class $T(\alpha,\beta,\gamma)$, $\alpha \in [0,1), \beta, \gamma[0,1]$ if the following condition is satisfied*

$$Re\left\{\frac{zf'(z)+\beta z^2 f''(z)}{\gamma[\beta z f'(z)+(1-\beta)f(z)]+(1-\gamma)z}\right\} > \alpha,$$
$$z \in U.$$

Thus

$$T(\alpha,\beta,\gamma) = \left\{f \in T: Re\left\{\frac{zf'(z)+\beta z^2 f''(z)}{\gamma[\beta z f'(z)+(1-\beta)f(z)]+(1-\gamma)z}\right\} > \alpha, z \in U\right\},$$
$$\alpha \in [0,1), \beta, \gamma[0,1].$$

Remark 1 *Choose $\gamma = 1$ in Definition 1.1, we have function class $A(\alpha,\beta)$.*
Remark 2 *Choose $\gamma = 1$ and $\beta = 0$ in Definition 1.1, we have function class $S^*(\alpha)$.*
Remark 3 *Choose $\gamma = 1$ and $\beta = 1$ in Definition 1.1, we have function class $C(\alpha)$.*
Remark 4 *Choose $\gamma = 0$ in Definition 1.1, we have function class $K(\alpha,\beta)$.*
Remark 5 *Choose $\gamma = 0$ and $\beta = 0$ in Definition 1.1, we have function class $K(\alpha)$.*
Remark 6 *Choose $\gamma = 1$ in Definition 1.2, we have function class $T(\alpha,\beta)$.*
Remark 7 *Choose $\gamma = 1$ and $\beta = 0$ in Definition 1.2, we have function class $TS^*(\alpha)$.*
Remark 8 *Choose $\gamma = 1$ and $\beta = 1$ in Definition 1.2, we have function class $TC(\alpha)$.*
Remark 9 *Choose $\gamma = 0$ in Definition 1.2, we have function class $TK(\alpha,\beta)$.*
Remark 10 *Choose $\gamma = 0$ and $\beta = 0$ in Definition 1.2, we have function class $TK(\alpha)$.*

In this paper, two new subclasses $A(\alpha,\beta;\gamma)$ and $T(\alpha,\beta;\gamma)$ of the analytic functions in the open unit disk are introduced. The various geometric properties of the functions belonging to these classes are examined. Sharp inequalities for the coefficient bound estimates for the functions belonging to these classes are also given.

2. Coefficient bound estimates for the classes $A(\alpha,\beta;\gamma)$ and $T(\alpha,\beta;\gamma)$

In this section, we will examine some inclusion results of the subclasses $A(\alpha,\beta;\gamma)$ and $T(\alpha,\beta;\gamma)$ of analytic functions in the open unit disk. Furthermore, we give coefficient bound estimates for the functions belonging to these subclasses.

A sufficient condition for the functions in the class $A(\alpha,\beta;\gamma)$ is given by the following theorem.

Theorem 1 *Let $f\in A$. Then, the function f belongs to the class $A(\alpha,\beta;\gamma)$ if the following condition is satisfied*

$$\sum_{n=2}^{\infty}(n-\alpha\gamma)[1+(n-1)\beta]|a_n|\leq 1-\alpha \tag{2.1}$$

The result obtained here is sharp.

Proof. Assume that $A(\alpha,\beta;\gamma),\alpha\in[0,1),\beta,\gamma[0,1]$. From the Definition 1, we have

$$Re\left\{\frac{zf'(z)+\beta z^2f''(z)}{\gamma[\beta zf'(z)+(1-\beta)f(z)]+(1-\gamma)z}\right\}>\alpha. \tag{2.2}$$

It is clear that condition (2.2) is satisfied if provided following condition

$$\left|\frac{zf'(z)+\beta z^2f''(z)}{\gamma[\beta zf'(z)+(1-\beta)f(z)]+(1-\gamma)z}-1\right|\leq 1-\alpha. \tag{2.3}$$

In that case, it suffices to show that satisfied condition (2.3). From (1.1), by simple computation, we write

$$\left|\frac{zf'(z)+\beta z^2f''(z)}{\gamma[\beta zf'(z)+(1-\beta)f(z)]+(1-\gamma)z}-1\right|=\left|\frac{\sum_{n=2}^{\infty}(n-\gamma)[1+(n-1)\beta]a_nz^n}{z+\sum_{n=2}^{\infty}\gamma[1+(n-1)\beta]a_nz^n}\right|$$
$$\leq\frac{\sum_{n=2}^{\infty}(n-\gamma)[1+(n-1)\beta]|a_n|}{z-\sum_{n=2}^{\infty}\gamma[1+(n-1)\beta]|a_n|}.$$

The last expression in the last inequality is bounded above by $1-\alpha$ if and only if

$$\sum_{n=2}^{\infty}(n-\gamma)[1+(n-1)\beta]|a_n|\leq(1-\alpha)\{z-\sum_{n=2}^{\infty}\gamma[1+(n-1)\beta]|a_n|\},$$

which is equivalent to

$$\sum_{n=2}^{\infty}(n-\alpha\gamma)[1+(n-1)\beta]|a_n|\leq 1-\alpha. \tag{2.4}$$

Thus, the inequality (2.3) is true if condition (2.4) is satisfied. Hence, the inequality (2.1) is provided.

To see that inequality obtained in the theorem is sharp, it is sufficient to see that inequality is provided as equality for the function given below

$$f_n(z)=z+\frac{(1-\alpha)}{(n-\alpha\gamma)[1+(n-1)\beta]}z^n, z\in U$$

for every $n=2,3,\ldots$.

Thus the proof of Theorem 1 is completed.

From the Theorem 2.1, we can readily deduce the following results.

Corollary 1 *The function f definition by (1.1) belongs to the class $A(\alpha,\beta)$ if the following condition is satisfied*

$$\sum_{n=2}^{\infty}(n-\alpha)[1+(n-1)\beta]|a_n|\leq 1-\alpha.$$

The result is sharp for the function

$$f_n(z)=z+\frac{(1-\alpha)}{(n-\alpha)[1+(n-1)\beta]}z^n, z\in U$$

for every $n=2,3,\ldots$.

Corollary 2 *(see [7, p. 110, Theorem 1]) The function f definition by (1.1) belongs to the class $S^*(\alpha)$ if the following condition is satisfied*

$$\sum_{n=2}^{\infty}(n-\alpha)|a_n|\leq 1-\alpha.$$

The result is sharp for the function

$$f_n(z)=z+\frac{(1-\alpha)}{(n-\alpha)}z^n, z\in U$$

for every $n=2,3,\ldots$.

Corollary 3 *(see [7, p. 110,Corollay of Theorem 1]) The function f definition by (1.1) belongs to the class $C(\alpha)$ if the following condition is satisfied*

$$\sum_{n=2}^{\infty}n(n-\alpha)|a_n|\leq 1-\alpha.$$

The result is sharp for the function

$$f_n(z) = z + \frac{(1-\alpha)}{n(n-\alpha)} z^n, z \in U$$

for every $n = 2,3,\ldots$.

Corollary 4 *The function f definition by (1.1) belongs to the class $K(\alpha,\beta)$ if the following condition is satisfied*

$$\sum_{n=2}^{\infty} n\,[1+(n-1)\beta]|a_n| \le 1-\alpha.$$

The result is sharp for the function

$$f_n(z) = z + \frac{(1-\alpha)}{n[1+(n-1)\beta]} z^n, z \in U$$

for every $n = 2,3,\ldots$.

Corollary 5 *The function f definition by (1.1) belongs to the class $K(\alpha)$ if the following condition is satisfied*

$$\sum_{n=2}^{\infty} n\,|a_n| \le 1-\alpha.$$

The result is sharp for the function

$$f_n(z) = z + \frac{(1-\alpha)}{n} z^n, z \in U$$

for every $n = 2,3,\ldots$.

For the function in the class $T(\alpha,\beta;\gamma)$, the converse of Theorem 1 is also true. So, the following theorem gives the sufficient and necessary condition for the functions belonging to the class $T(\alpha,\beta;\gamma)$.

Theorem 2 *Let $f \in T$. Then, the function f belongs to the class $T(\alpha,\beta;\gamma)$ if and only if the condition (2.1) is satisfied. The result obtained here is sharp.*

Proof. In view of Theorem 1, we need only to prove the necessity of the Theorem 2. Assume that $f \in T(\alpha,\beta;\gamma)$, which is equivalent to $f \in T$ and

$$Re\left\{\frac{zf'(z)+\beta z^2 f''(z)}{\gamma[\beta z f'(z)+(1-\beta)f(z)]+(1-\gamma)z}\right\} > \alpha, z \in U. \quad (2.5)$$

From (1.2) and (2.5) by simple computation, we get

$$Re\left\{\frac{z-\sum_{n=2}^{\infty} n[1+(n-1)\beta]a_n z^n}{z-\sum_{n=2}^{\infty} \gamma[1+(n-1)\beta]a_n z^n}\right\} > \alpha.$$

The expression

$$\frac{z-\sum_{n=2}^{\infty} n[1+(n-1)\beta]a_n z^n}{z-\sum_{n=2}^{\infty} \gamma[1+(n-1)\beta]a_n z^n}$$

is real if choose z real. Thus, from the previous inequality letting $z \to 1^-$ through real values, we obtain

$$1-\sum_{n=2}^{\infty} n\,[1+(n-1)\beta]a_n \ge \alpha\{1-\sum_{n=2}^{\infty} n\,[1+(n-1)\beta]a_n\};$$

so that,

$$\sum_{n=2}^{\infty}(n-\alpha\gamma)[1+(n-1)\beta]a_n \le 1-\alpha.$$

This completed the proof of the necessity of theorem.

To see that inequality obtained in the theorem is sharp, it is sufficient to see that inequality is provided as equality for the function given below

$$f_n(z) = z - \frac{(1-\alpha)}{(n-\alpha\gamma)[1+(n-1)\beta]} z^n, z \in U$$

for every $n = 2,3,\ldots$.

Thus, the proof of Theorem2 is completed.

Special case of Theorem 2 has been proved by Altintaş et al [2], $\gamma = 1$ (here $p = n = 1$).

From the Theorem 2, we can readily deduce the following results.

Corollary 6 *The function f definition by (1.2) belongs to the class $T(\alpha,\beta)$ if and only if*

$$\sum_{n=2}^{\infty}(n-\alpha)[1+(n-1)\beta]a_n \le (1-\alpha).$$

The result is sharp for the function

$$f_n(z) = z - \frac{(1-\alpha)}{(n-\alpha)[1+(n-1)\beta]} z^n, z \in U$$

for every $n = 2,3,\ldots$.

Remark 11 *The result obtained in Corollary 6 verifies to the Theorem 1 in [2].*

Corollary 7 *(see [7, p. 110, Theorem 2]) The function f definition by (1.2) belongs to the class $TS^*(\alpha)$ if and only if*

$$\sum_{n=2}^{\infty}(n-\alpha)a_n \leq (1-\alpha).$$

The result is sharp for the function

$$f_n(z) = z - \frac{(1-\alpha)}{n-\alpha}z^n, z \in U$$

for every $n = 2,3,\ldots$.

Corollary 8 *(see [7, p. 110, Theorem 2]) The function f definition by (1.2) belongs to the class $TC(\alpha)$ if and only if*

$$\sum_{n=2}^{\infty} n(n-\alpha)a_n \leq (1-\alpha).$$

The result is sharp for the function

$$f_n(z) = z - \frac{(1-\alpha)}{n(n-\alpha)}z^n, z \in U$$

for every $n = 2,3,\ldots$.

Corollary 9 *The function f definition by (1.2) belongs to the class $TK(\alpha,\beta)$ if and only if*

$$\sum_{n=2}^{\infty} n[1+(n-1)\beta]a_n\, n[1+(n-1)\beta]a_n \leq (1-\alpha).$$

The result is sharp for the function

$$f_n(z) = z - \frac{(1-\alpha)}{n[1+(n-1)\beta]}z^n, z \in U$$

for every $n = 2,3,\ldots$.

Corollary 10 *The function f definition by (1.2) belongs to the class $TK(\alpha)$ if and only if*

$$\sum_{n=2}^{\infty} na_n \leq (1-\alpha).$$

The result is sharp for the function

$$f_n(z) = z - \frac{(1-\alpha)}{n}z^n, z \in U$$

for every $n = 2,3,\ldots$.

On the coefficient bounds of the functions belonging to the class $T(\alpha,\beta;\gamma)$, we give the following result.

Lemma 1 *Let the function f definition by (1.2) belongs to the class $T(\alpha,\beta;\gamma)$. Then,*

$$\sum_{n=2}^{\infty} a_n \leq \frac{1-\alpha}{(1+\beta)(2-\alpha\gamma)} \tag{2.6}$$

and

$$\sum_{n=2}^{\infty} na_n \leq \frac{2(1-\alpha)}{(1+\beta)(2-\alpha\gamma)}. \tag{2.7}$$

Proof. Assume that $f \in T(\alpha,\beta;\gamma), \alpha \in [0,1), \beta,\gamma[0,1]$.

In this case, according to Theorem 2, we get

$$(1+\beta)(2-\alpha\gamma)\sum_{n=2}^{\infty} a_n \leq \sum_{n=2}^{\infty}(n-\alpha\gamma)[1+(n-1)\beta]a_n \leq 1-\alpha,$$

which equivalent to the first inequality (2.6) of the lemma.

Similarly, we write

$$(1+\beta)\sum_{n=2}^{\infty}(n-\alpha\gamma)a_n \leq_{n=2}^{\infty} (n-\alpha\gamma)[1+(n-1)\beta]a_n \leq 1-\alpha;$$

that is,

$$(1+\beta)\sum_{n=2}^{\infty} na_n \leq 1-\alpha+(1+\beta)\alpha\gamma\sum_{n=2}^{\infty} a_n.$$

Since

$$\sum_{n=2}^{\infty} a_n \leq \frac{1-\alpha}{(1+\beta)(2-\alpha\gamma)},$$

from the previous inequality, we immediately obtain the following inequality

$$(1+\beta)\sum_{n=2}^{\infty} a_n \le \frac{2(1-\alpha)}{2-\alpha\gamma},$$

which immediately yields the second assertion of the lemma.
Thus, the proof of Lemma 1 is completed.
From the Lemma 1, we can easily obtain the following results.

Corollary 11 *Let the function f definition by (1.2) belongs to the class $T(\alpha,\beta)$. Then,*

$$\sum_{n=2}^{\infty} a_n \le \frac{1-\alpha}{(1+\beta)(2-\alpha)} \text{ and } \sum_{n=2}^{\infty} na_n \le \frac{2(1-\alpha)}{(1+\beta)(2-\alpha)}$$

Remark 12 *The result obtained in the Corollary 11 verifies to the Lemma 2 (with $n = p = 1$) of [2].*
Corollary 12 *Let the function $f(z)$ definition by (1.2) belongs to the class $TS^*(\alpha)$. Then,*

$$\sum_{n=2}^{\infty} a_n \le \frac{1-\alpha}{2-\alpha} \text{ and } \sum_{n=2}^{\infty} na_n \le \frac{2(1-\alpha)}{2-\alpha}.$$

Corollary 13 *Let the function f definition by (1.2) belongs to the class $TC(\alpha)$. Then,*

$$\sum_{n=2}^{\infty} a_n \le \frac{1-\alpha}{2(2-\alpha)} \text{ and } \sum_{n=2}^{\infty} na_n \le \frac{1-\alpha}{2-\alpha}.$$

Corollary 14 *Let the function f definition by (1.2) belongs to the class $TK(\alpha,\beta)$. Then,*

$$\sum_{n=2}^{\infty} a_n \le \frac{1-\alpha}{2(1+\beta)} \text{ and } \sum_{n=2}^{\infty} na_n \le \frac{1-\alpha}{(1+\beta)}.$$

Corollary 15 *Let the function f definition by (1.2) belongs to the class $TK(\alpha)$. Then,*

$$\sum_{n=2}^{\infty} a_n \le \frac{1-\alpha}{2} \text{ and } \sum_{n=2}^{\infty} na_n \le 1-\alpha.$$

From Theorem 2, we have the following result on the coefficient bound estimates.
Corollary 16 *If $f \in T(\alpha,\beta;\gamma)$, then*

$$a_n \le \frac{1-\alpha}{(n-\alpha\gamma)[1+(n-1)\beta]}, \quad n = 2,3,\ldots.$$

Numerous consequences of the Corollary 16 can indeed be deduced by specializing the various parameters involved. Many of these consequences were proved by earlier workers on the subject (cf., e.g., [1, 7, 9]).
Now, we give following properties of the class $T(\alpha,\beta;\gamma)$.
Lemma 2 *The subclass $T(\alpha,\beta;\gamma)$ of the analytic functions in the open unit disk is convex set.*
Proof. Assume that each of the functions $f,g \in T(\alpha,\beta;\gamma), \alpha \in [0,1), \beta,\gamma \in [0,1]$, with $g(z) = z - \sum_{n=2}^{\infty} b_n z^n$. Then, for $\lambda \in [0,1]$, we write

$$\varphi(z) = \lambda f(z) + (1-\lambda)g(z) = z - \sum_{n=2}^{\infty} c_n z^n,$$

where

$$c_n = \lambda a_n + (1-\lambda)b_n, n = 2,3,\ldots .$$

Using necessary part of the Theorem2, we write

$$\begin{aligned}\sum_{n=2}^{\infty} (n-\alpha\gamma)[1+(n-1)\beta]c_n &= \lambda\sum_{n=2}^{\infty} (n-\alpha\gamma)[1+(n-1)\beta]a_n + \\ &(1-\lambda)\sum_{n=2}^{\infty} (n-\alpha\gamma)[1+(n-1)\beta]b_n \\ &\le \lambda(1-\alpha) + (1-\lambda)(1-\alpha) = 1-\alpha;\end{aligned}$$

That is,

$$\sum_{n=2}^{\infty} (n-\alpha\gamma)[1+(n-1)\beta]c_n \le 1-\alpha.$$

Next, using the sufficiently part of the Theorem 2, we have $\varphi \in T(\alpha,\beta;\gamma)$.
Thus, the proof of Lemma 2 is completed.

3. Results and Discussion

In this paper, was introduced two new subclasses $A(\alpha,\beta;\gamma)$ and $T(\alpha,\beta;\gamma)$ of the analytic functions on the open unit disk in the complex plane. The various geometric properties of the functions belonging to these classes have examined. Also, sharp inequalities for the coefficient bound estimates for the functions belonging to these classes are given.
Taking advantage of the results obtained in this study, can be given the distortion and growth theorems for the class $T(\alpha,\beta;\gamma)$ defined in the study. In addition, the radii of starlikeness and convexity can be examined for this class.

References

[1]. Altintaş, O. (1991). On a subclass of certain starlike functions with negative coefficient. Math. Japon., 36: 489-495.

[2]. Altintaş, O., Irmak, H. and Srivastava, H. M. (1995). Fractional calculus and certain starlike functions with negative coefficients. Comput. Math. Appl., 30(2): 9-16.

[3]. Altintaş, O., Özkan, Ö. and Srivastava, H. M. (2004). Neighborhoods of a Certain Family of Multivalent Functions with Negative Coefficients. Comput. Math. Appl., 47: 1667-1672.

[4]. Duren, P. L. (1983). Univalent Functions. Grundlehren der Mathematischen Wissenshaften, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo.

[5]. Goodman, A. W. (1983). Univalent Functions. Volume I, Polygonal, Washington.

[6]. Irmak, H., Lee, S. H. and Cho, N. E. (1997). Some multivalently starlike functions with negative coefficients and their subclasses defined by using a differential operator. Kyungpook Math. J., 37: 43-51.

[7]. Silverman, H. (1975). Univalent Functions with Negative Coefficients. Proc. Amer. Math. Soc., 51(1): 106-116.

[8]. Srivastava, H. M. and Owa, S. (1992). Editors, Current Topics in Analytic Funtion Theory. World Scientific, Singapore.

[9]. Srivastava, H. M., Owa, S. and Chatterjea, S. K. (1987). A note on certain classes of starlike functions. Rend. Sem. Mat. Univ. Padova, 77: 115-124.