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Research Article

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Hybrid Five-Dimensional Spacetime and Introducing Gravitational Effects into Particle Physics

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Abstract We propose five-dimensional spacetime with usual four-dimensions and plus one-discrete dimension constructed by using the Planck length, where $x_5 = L_{Pl} \cdot a \cdot \overleftarrow{n}$. Here a is dimensionless parameter or number of lattice and $\overleftarrow{n}$ is a unit vector. It turns out that motion equations of free particles remain unchanged, while those causal-Green functions or propagators are modified and carried gravitational effects in particle physics.

Keywords Five-dimensional spacetime, Planck length, metric tensor, interval of events, propagators, formfactor, Newtonian, Coulomb and Yukawa potentials. Klein-Gordon equation, D'Alembertian.

1. Metric Tensor, Interval of Events in Five-Dimensional Spacetime

In five-dimensional spacetime, co-ordinate points of events are denoted by

$$x^{\mu} = (x_0 = ct, x_1, x_2, x_3, x_5 = L_{Pl} a \overleftarrow{n}), \tag{1}$$

where a is dimensionless parameter or number of lattice and $\overleftarrow{n}$-is an arbitrary unit vector. Then the metric tensor is

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix} \tag{2}$$

and an interval reads

$$S^2 = g^{\mu\nu} x_{\mu} x_{\gamma} = x_0^2 - \overleftarrow{x}^2 - L_{Pl}^2 a^2 \tag{3}$$

Without generality, we assume in (1) $a = 1$, then such type of five-dimensional spacetime was considered by Markov [1]. D'Alembertian in five-dimensional spacetime takes the form

$$\Delta_5 = -\frac{1}{c^2}\frac{\partial^2}{\partial^2} + \frac{\partial^2}{\partial \overleftarrow{x}^2} + \frac{\partial^2}{L_{Pl}^2 \partial a^2} \tag{4}$$

2. Klein-Gordon Type Equation in Five-Dimensional Spacetime

We propose following type of motion equation for a scalar particle

$$(\Delta_5 - m^2)\Phi(x^{\mu}) = 0 \tag{5}$$

Then plane-wave solution of this equation is given by

$$\Phi(x^{\mu}) = A\cdot e^{ix_i\cdot p^i - ip^5x_5} \quad (6)$$

where

$$p^5 = \sqrt{m^2 - p^2} = \sqrt{m^2 - p_0^2 + \overleftarrow{p}^2}, \quad \overleftarrow{n}\cdot\overleftarrow{n} = 1,$$

$$ix_{\mu}\cdot p^{\mu} = ix_0p^0 - i\overleftarrow{x}\overleftarrow{p} - i\sqrt{m^2 - p^2}L_{Pl}a\overleftarrow{n}, \quad (7)$$

Then direct calculation for equation (5) gives

$$p_0^2 - \overleftarrow{p}^2 - (m^2 - p_0^2 + \overleftarrow{p}^2) - m^2 = 0$$

or

$$2(p_0^2 - \overleftarrow{p}^2 - m^2) = 0. \quad (8)$$

It means that motion of equation for free particles does not charge in five-dimensional spacetime case, where

$$E = \pm\sqrt{m^2 + \overleftarrow{p}^2}$$

is valid as for Klein-Gordon case.

3. Effect of Gravity Affects on Causal-Green and Propagator of Interacting Particles

Direct and easy way to introduce influence of gravity to the particle motion is construction of particle propagators in five-dimensional spacetime by using the Fourier transform of Newtonian, Coulomb and Yukawa interaction potentials.

It is well known that in the static limit the Coulomb, the Newtonian and the Yukawa potentials are related with photon, graviton and scalar particle propagators by using the Fourier transforms:

$$U_c(r) = \frac{e}{(2\pi)^3}\int\frac{d^3p}{\overleftarrow{p}^2}e^{i\overleftarrow{p}\overleftarrow{r}} = \frac{e}{4\pi}\frac{1}{r}, \quad (9)$$

$$U_N(r) = \frac{G\cdot M}{2\pi^2}\int\frac{d^3p}{\overleftarrow{p}^2}e^{i\overleftarrow{p}\overleftarrow{r}} = \frac{G\cdot M}{r}, \quad (10)$$

and

$$U_Y(r) = \frac{g}{(2\pi)^3}\int\frac{d^3p}{\overleftarrow{p}^2 + m^2}e^{i\overleftarrow{p}\overleftarrow{r}} = \frac{g}{4\pi}\frac{1}{r}e^{-mr}. \quad (11)$$

Then, inverse Fourier transforms for photon, graviton and scalar particle propagators are given by following formulas in five-dimensional spacetime:

$$\frac{1}{\overleftarrow{p}^2} \Rightarrow D_g^{\gamma}(\overleftarrow{p}^2) = c\int_0^{\infty}\frac{dr\cdot r^3}{r^2}\int_0^{2\pi}d\varphi\times$$

$$\int_0^{\pi}d\theta\cdot sin\theta\cdot e^{iprcos\theta}\int_0^{\pi}d\theta_1\sin^2\theta_1 e^{iL_{Pl}\sqrt{\overleftarrow{p}^2}cos\theta_1}, \quad (12)$$

where c is normalization constant and we have used the formula (7) with a=1, $\overleftarrow{n} = cos\theta_1$ is the directed cosine. Last integral reads

$$Z_3 = \int_0^{\pi}d\theta_1 sin^2\theta_1 e^{ipL_{Pl}cos\theta_1} = \int_{-1}^{1}d\lambda\sqrt{1-\lambda^2}e^{ipL_{Pl}\lambda} =$$

$$2\int_0^1 d\lambda\sqrt{1-\lambda^2}\cos(pL_{Pl}\lambda)=\frac{\pi}{pL_{Pl}}J_1(pL_{Pl}). \tag{13}$$

Here for a scalar particle case $p\to\sqrt{m^2+\overleftarrow{p}^2}$, $J_1(x)$ is the Bessel function of the order 1 and we have used quantum potential form [2]

$$U_5(r)\sim\frac{1}{r^2} \tag{14}$$

This potential form leads to the quantum forces

$$F_5^N(r_5)=GL_{Pl}\frac{M_1\cdot M_2}{r_5^3},\quad F_5^C(r_5)=k_C L_{Pl}\cdot\frac{q_1\cdot q_2}{r_5^3}, \tag{15}$$

$$r_5=\sqrt{x_1^2+x_2^2+x_3^2+x_4^2}$$

in five-dimensional spacetime for two bodies with masses M_1, M_2 and electric charges q_1, q_2. Here

$$G=6.6743\times10^{-11}\frac{m^3}{kg\cdot sec^2},$$

$$k_C=\frac{1}{4}c^2\cdot10^7\frac{m^3}{kg}=2.2469\times10^{23}\frac{m^3}{kg} \tag{16}$$

are the Newtonian and the Coulomb constants.

Finally, photon and graviton propagators in usual four-dimensional space take the forms

$$D_{\mu\nu}^{\gamma}(x)=\frac{g_{\mu\nu}}{(2\pi)^4 i}\int d^4p\frac{V(-p^2L_{Pl}^2)}{-p^2-i\varepsilon}e^{ipx} \tag{17}$$

$$D_{\mu\nu,\rho\delta}^{g}(x)=\frac{1}{(2\pi)^4 i}[g_{\mu\rho}g_{\nu\delta}+g_{\nu\rho}g_{\mu\delta}-\frac{2}{D-2}g_{\mu\nu}g_{\rho\delta}]\times\int d^4p\frac{V(-p^2L_{Pl}^2)}{-p^2-i\varepsilon}e^{ipx} \tag{18}$$

where $p^2=p_0^2-\overleftarrow{p}^2$.

It is obviously that in formula (12) we have used the following integrals:

$$Z_1=\int_0^{\pi}d\theta\cdot sin\theta e^{iprcos\theta}=\frac{2sinpr}{pr}, \tag{19}$$

$$Z_2=\int_0^{\infty}drsinpr=\frac{1}{p}. \tag{20}$$

For the Yukawa potential last integral takes the form

$$Z_2^Y=\frac{1}{p}\int_0^{\infty}dre^{-mr}\cdot sinpr=$$

$$\frac{1}{p}\frac{1}{\sqrt{m^2+\overleftarrow{p}^2}}sin(arctg\frac{p}{m})=\frac{1}{m^2+\overleftarrow{p}^2}, \tag{21}$$

Where $\mathrm{arctg}\,x=\arcsin\frac{x}{\sqrt{1+x^2}}$,

and therefore for scalar particle propagator one gets

$$D_M^Y(r)=\frac{1}{(2\pi)^4 i}\int d^4p\frac{V_m(-p_D^2L_{Pl}^2)}{m^2-p^2 i\varepsilon}\cdot e^{ipx} \tag{22}$$

For form factors $V(-p^2L_{Pl}^2)$ and $V_m(-p^2L_{Pl}^2)$ the following Mellin representations are valid

$$V(-p^2L_{Pl}^2)=\frac{1}{2i}\int_{-\beta+i\propto}^{-\beta-i\propto}d\xi\frac{(-\frac{p^2L_{Pl}^2}{4})^{\xi}}{sin\pi\xi\cdot\Gamma(1+\xi)\cdot\Gamma(2+\xi)},\qquad(23)$$

$$V_m(-p^2L_{Pl}^2)=\frac{1}{2i}\int_{-\beta+i\propto}^{-\beta-i\propto}d\xi\frac{\left[\frac{(m^2-p^2)}{4}\right]^{\xi}L_{Pl}^{2\xi}}{\sin\pi\xi\cdot\Gamma(1+\xi)\Gamma(2+\xi)},\qquad(24)$$

$$0<\beta<1.$$

Finally, notice that due to form factors (22) and (23) which are analytic functions on the left hand complex plane and there are no pales there, all Feynman diagrams [except vacuum polarization ones] for electromagnetic, electro-weak and gravitational interactions between elementary particles, and for self-interactions of scalar particles are finite (see [3] for construction of finite-nonlocal quantum electrodynamics).

Spin one massive vector fields carrying electro-weak interactions are $W^{\pm}$ and Z -bosons and those causal-Green function or propagator have also nonlocal form.

$$D_{\mu\nu}^{m}(x)=\frac{1}{(2\pi)^4 i}\int d^4pe^{ipx}(g_{\mu\nu}-\frac{p_\nu p_\mu}{m^2})\frac{V_m(-p^2L_{Pl}^2)}{m^2-p^2-i\xi}\qquad(25)$$

Appendix 1

For the Yukawa potential case

$$U_4^Y(r)=\frac{g_4}{r}e^{-mr}=g_5'\left(\frac{m}{r}\right)^{1/2}K_{1/2}(mr)$$

and therefore

$$D_S^Y(p)=const\cdot\int_0^{\infty}drr^{1+\frac{1}{2}}K_{1/2}(mr)\frac{sinpr}{p}\times$$

$$\int_0^{\pi}d\theta_1 sin^2\theta_1 e^{i\sqrt{m^2+\overleftarrow{p}^2}L_{Pl}cos\theta_1}=\frac{const}{p}\sqrt{\pi}(2m)^{1/2}\Gamma\left(\frac{3}{2}+\frac{1}{2}\right)p(m^2+\overleftarrow{p}^2)^{-1}\times$$

$$\pi\frac{J_1(\sqrt{m^2+\overleftarrow{p}^2}L_{Pl})}{\sqrt{m^2+\overleftarrow{p}^2}L_{Pl}}$$

Thus, after normalization, we have

$$D_S^Y(p)=\frac{1}{m^2+\overleftarrow{p}^2}\cdot\frac{J_1(\sqrt{m^2+\overleftarrow{p}^2}L_{Pl})}{L_{Pl}\cdot\sqrt{m^2+\overleftarrow{p}^2}},$$

as it should be.

It is important to notice that the form of the Yukawa propagator in the momentum space has universal character independing on dimensions of spacetime in the static limit:

$$\frac{1}{m^2-p^2-i\xi}\Rightarrow\frac{1}{m+\overleftarrow{p}^2}\cong\int dr\cdot r^{D-2}\left(\frac{m}{r}\right)^{\frac{D-3}{2}}K_{\frac{D-3}{2}}(mr)\times$$

$$\frac{J_{\frac{D-3}{2}}(pr)}{(pr)^{\frac{D-3}{2}}}$$

$$D = 5,6,\ldots,10,11. \quad \overleftarrow{p}^2 = p_1^2 + p_2^2 + \ldots + p_{D-1}^2$$

where we have used the following integral

$$\int_0^\infty dx \cdot x \cdot K_\nu(mx) J_\nu(px) = \frac{p^\nu}{m^\nu (m^2 + \overleftarrow{p}^2)}$$

Free and interaction Lagrangians are constructed by means of fields $\Phi^i(x_D^\nu), \Psi(x_D^\nu)$ and those differentials $\partial\Phi^i(x_D^\mu)/\partial x_D^\nu), \partial\Psi(x_D^\nu)/\partial x_D^\mu$ in the general form for any D-dimensions of spacetime:

$$L_{free}(\Phi^i(x_D^\mu), \partial\Phi^i/\partial x_D^\rho) + L_{free}(\Psi(x_D^\mu), \partial\Psi/\partial x_D^\rho) +$$
$$+ L_{in}(\Psi(x_D^\mu), \Phi^i(x_D^\nu)) \quad .$$

Then S-matrix for interaction between $\Phi^i(x_D^\mu)$ and $\Psi(x_D^\nu)$-fields is given by the form

$$S = Texp\{i\int d^D x L_{in}()\} \quad .$$

Matrix elements of this S-matrix are determined by causal-Green functions or propagators of these fields by using T-product, for example, for scalar field:

$$\langle 0 | \{\Phi^i(x_D^\mu), \Phi^j(x_D^\nu)\} | 0 \rangle = \delta_{ij} D_D(x-y) \quad .$$

Regularization of S matrix is carried out by means of gravity with fundamental length or the Planck length $L_{Pl} = \sqrt{G\hbar/c^3}$. In a such approach force carrying photon, graviton and scalar particles propagators in any D-dimensions of spacetime are chosen as follows:

$$D_{\mu\nu}^\gamma(x_D) = \frac{g^{\mu\nu}}{(2\pi)^D \cdot i} \int d^D x e^{ip\cdot x} \frac{V(-p_D^2 L_{Pl}^2)}{-p_D^2 - i\varepsilon},$$

$$D_{\mu\nu,\rho\delta}^g(x_D) = \frac{\Delta_{\mu\nu,\rho\delta}}{(2\pi)^D i} \int d^D x e^{ipx} \frac{V(-p_D^2 L_{Pl}^2)}{-p_D^2 - i\varepsilon},$$

for photon and graviton fields,

$$D^m(x_D) = \frac{1}{(2\pi)^D i} \int d^D x e^{ipx} \frac{V_m(-p_D^2 L_{Pl}^2)}{m^2 - p_D^2 - i\varepsilon}$$

for scalar particles. Here

$$V(-p_D^2 L_{Pl}^2) = \frac{J_\lambda(P_D L_{Pl})}{(p_D L_{Pl})^\lambda} \cdot 2^\lambda = \frac{1}{2i} \int_{-\beta+i\infty}^{-\beta-\infty} d\xi \frac{\left(\frac{-p_D^2 L_{Pl}^2}{4}\right)^\xi}{\sin\pi\xi \cdot \Gamma(1+\xi)\Gamma(\lambda+\xi+1)}$$

$$V_m(-p_D^2 L_{Pl}^2) = \frac{1}{2i} \int_{-\beta+i\infty}^{-\beta-\infty} d\xi \frac{\left[\frac{(m^2 - p_D^2) L_{Pl}^2}{4}\right]^\xi}{\sin\pi\xi \cdot \Gamma(1+\xi)\Gamma(\lambda+\xi+1)}$$

where

$$p_D^2 = p_0^2 - p_2^2 - \ldots - p_{D-1}^2,$$

$$\lambda = \frac{D-3}{2}, \quad D = 4,5,6\ldots,10,11,\ldots.$$

This regularised quantum field theory is finite and free from ultraviolet divergences [4], except vacuum polarization diagrams which are reqularized by using D-dimensional procedure due to t'Hooft and Veltman [5].

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