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Extended Charge and Indication of a Scale of the Supersymmetry

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Abstract We propose simple model of a rigid string charge and its electromagnetic interaction.

Keywords Rigid string charge, nonlocal quantum electrodynamics, propagator for a rigid string

1. Introduction

The study of nonlocal (or extended) and supersymmetric theories of quantized fields aimed to construct unified theory of elementary particles interactions including gravitation plays a vital role in the contemporary physics. Among them the string theory is physicist's dream to construct the unified theory of every thing (see for example, Green, Schwarz and Witten, 1987; Polchinski, 1998; Weinberg, 2000; Efimov, 1977 and Namsrai, 1986 and references therein).

However, in spite of enormous successes and developments of these theories until now there absent clear working models allowing us to predict a range of energetic interval or scale at which experimental prediction and observation of physical processes and consequences arisen from these theories will be expected.

In this paper, we would like to consider a simple model of a rigid string-stick charge configuration. As shown in Figure 1 three charges q_i with different signs are located on it. Here sum of these charges gives desired charge value of a particle under consideration.

$$\sum_i q_i = e \tag{1}$$

Notice that it is quite possible to consider fractional charge configuration as composed particle consisting of three fractional charges like quarks. However, for now it is not important for our calculation purpose.

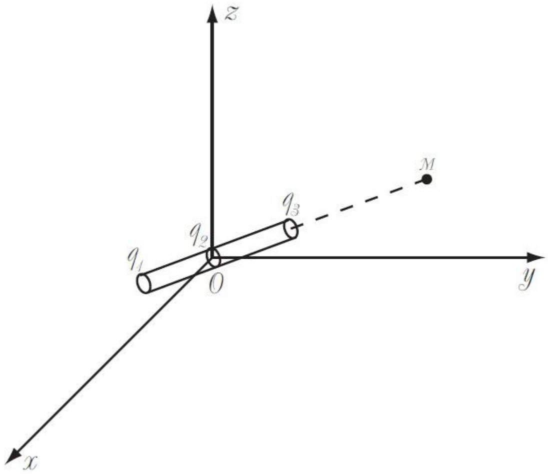

Figure 1: One dimensional extended charge configuration consisting of three charges with different signs (two plus charges and one minus charge and vice versa)

Let $q_3 = q_2 = -e$ and $q_1 = e$ and l is a half size of the stick. Then a potential of this configuration takes the form

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{e}{r}\left(1+\frac{2l}{r}\right), \quad \varepsilon_0 = 1 \tag{2}$$

Here we have assumed that $r = OM$ is much larger than l ($r>>l$) and that an orientation of the stick in space is not important and therefore one can always choose its position along the radius vector of observation. We see that the text book formula (2) is the sum of potentials of a point-like charge and a dipole, as it should be.

Our main goal is to construct electromagnetic interaction of this charge configuration and to obtain restriction on a size l of the stick.

2. The corresponding principle in the static limit

It is well known that in the static limit there exists relationship between the potentials of charges and the form of the propagators of force carrying particles or messengers. For example, for the Coulomb and Yukawa potential cases it takes the form in the static limit;

$$\frac{1}{p^2} = \frac{1}{e}\int d^3 r e^{ipr}\left(\frac{e}{4\pi}\frac{1}{r}\right) \tag{3}$$

$$\frac{1}{m^2+p^2} = \frac{1}{g}\int d^3 r e^{ipr}\left(\frac{g}{4\pi}\frac{e^{-mr}}{r}\right) \tag{4}$$

Here, the inverse Fourier transforms are also valid. Thus, according to the rule (3) from the potential (2) one gets

$$D(p) = \frac{1}{e}\int d^3 r e^{ipr} V(r) = \frac{1}{p^2} + \pi l \frac{1}{\sqrt{p^2}} \tag{5}$$

Relativistic extension of this formula in p-space acquires the form

$$D(p^2) = \frac{1}{p^2 - i\varepsilon} + \pi l \frac{\sqrt{p^2 - i\varepsilon}}{p^2 - i\varepsilon} \tag{6}$$

In this article, we have used Weinberg's text book metric form $p^2 = g^{\mu\nu} p^\mu p^\nu = -p_0^2 + p^2$. Thus, the propagator of the force carrying particle, corresponding to the potential of the rigid string in x-space is

$$D_{\mu\nu}(x) = \frac{-i}{(2\pi)^4} g_{\mu\nu}\int d^4 p e^{ipx}\left[\frac{1}{p^2 - i\varepsilon} + \pi l \frac{\sqrt{p^2 - i\varepsilon}}{p^2 - i\varepsilon}\right] \tag{7}$$

Mathematical nature of the last term in (6) and (7) is very interesting if we write it in the Dirac form

$$\pi l \frac{\sqrt{p^2 - i\varepsilon}}{p^2 - i\varepsilon} = \pi l \frac{-i\hat{p}}{p^2 - i\varepsilon} \tag{8}$$

where we have used the well-known relation $p^2 = (-i\hat{p})^2$, $\hat{p} = g_{\mu\nu}\gamma^\mu \rho^\nu = -\gamma^0 p^0 + \vec{\gamma} p$, γ^μ is the Dirac γ-matrices.

It is obviously that expression (8) is exactly equal to the propagator of a massless spinor, which we call supersymmetric partner of photon or **photino**. In the language of the radiation theory i.e., in terms of the quantum field theory it means that the rigid string charge configuration is radiated or absorbed photons and photinos simultanuously but a portion of photinos in this mixed or supersymmetric states (fields) is determined by a size of the extended charge i.e. the parameter l.

3. Possible form of a superfield corresponding to the rigid string charge and its radiation mixture
3.1. Vector like superfield
Owing to the sum of propagators (6)–(8) one can propose that extended composed charges are radiated or absorbed vector like superfield $\mathsf{A}_\mu(x)$ consisting of photons and photinos. There are many equivalent forms of represention of a superfield. For example, in the language of the supersymmetry, photon and photino fields can be formally present in the superfield form

$$\phi = \begin{pmatrix} \bar{\theta}\bar{\sigma}^\mu\theta A_\mu \\ \sqrt{\pi l}\,\bar{\theta}\psi_{\tilde{\gamma}} \\ \end{pmatrix}, \qquad \phi^* = (\theta\sigma^\mu\bar{\theta}A_\mu, \sqrt{\pi l}\,\bar{\psi}_{\tilde{\gamma}}\bar{\theta}\theta\theta) \tag{9}$$

where A_μ and $\psi_{\tilde{\gamma}}$ are the photon and the massless spinor fields, θ and $\bar{\theta}$ are Grassman variables. By means of these fields Green function (6) with using (8) in x-space can be written in the form

$$\begin{aligned} &D_\phi(x-y) = \int d^2\theta \int d^2\bar{\theta} \langle 0 | T\{\phi^*(x)\phi(y)\} | 0\rangle \\ &= g^{\mu\nu} g_{\mu\nu} D_{ph}(x-y) + \pi l D^{\tilde{f}}(x-y) \end{aligned} \tag{10}$$

Symbols $\tilde{\gamma}$ and $\tilde{f}$ in expressions (9) and (10) are applied to quantities belonging to supersymmetric partners of photons. Here we have used the definitions

$$\begin{aligned} &\sigma^\mu\bar{\sigma}^\nu = g^{\mu\nu} + 2\sigma^{\mu\nu} \\ &\theta\sigma^{\mu\nu}\theta = 0 \\ &\int d^2\theta\theta^2 = \int d^2\bar{\theta}\bar{\theta}^2 = 1 \end{aligned} \tag{11}$$

It is natural that the Fourier components of $D_{ph}(x)$ and $\pi l D^{\tilde{f}}(x)$ are given by the first and second terms in (6), respectively.
An another form of representation of vector like superfield $\mathsf{A}_\mu(x)$ is also valid:

$$\begin{aligned} &\mathsf{A}^\mu(x) = A^\mu(x) + \sqrt{\pi l}\,(\theta\sigma^\mu\bar{\theta})\psi_{\tilde{\gamma}}(x) \\ &\mathsf{A}^{*\nu} = A^\nu(x) + \sqrt{\pi l}\,\bar{\psi}_{\tilde{\gamma}}(x)(\bar{\theta}\bar{\sigma}^\nu\theta) \end{aligned}$$

or in a short notation:

$$\begin{aligned} &\mathsf{A}^\mu(x) = A^\mu(x) + \tilde{A}^\mu(x) \\ &\mathsf{A}^\nu(x) = A^\nu(x) + \tilde{A}^\nu(x) \\ &\tilde{A}^\mu(x) = \sqrt{\pi l}\,(\theta\sigma^\mu\bar{\theta})\psi_{\tilde{\gamma}}(x) \\ &\tilde{A}^\nu(x) = \sqrt{\pi l}\,\bar{\psi}_{\tilde{\gamma}}(x)(\bar{\theta}\bar{\sigma}^\nu\theta) \end{aligned} \tag{12}$$

By definition

$$\begin{aligned} &\mathsf{D}_{\mu\nu}(x-y) = \overbrace{\mathsf{A}^\mu(x)\mathsf{A}^{*\nu}(y)} \\ &= \int d^2\theta \int d^2\bar{\theta} \langle 0 | \{T[A^\mu(x) + \sqrt{\pi l}\,(\theta\sigma^\mu\bar{\theta})\psi_{\tilde{\gamma}}(x)] \\ &\times [A^\nu(y) + \sqrt{\pi l}\,\bar{\psi}_{\tilde{\gamma}}(y)(\bar{\theta}\bar{\sigma}^\nu\theta)]\} | 0\rangle \end{aligned}$$

Here it is assumed that pairing between fields $A^\mu(x)$ and $\psi_{\tilde{\gamma}}(x)$ is identically zero and as a result we obtain the propagator of the composed or nonlocal photon

$$\mathrm{D}_{\mu\nu}(x-y)=\frac{-i}{(2\pi)^4}g^{\mu\nu}\int d^4pe^{ipx}\frac{1}{p^2-i\varepsilon}\left(1+\pi l\sqrt{p^2-i\varepsilon}\right) \tag{13}$$

due to the equalities (11). Notice that pairing between local fields $A^{\mu}(x)$ and $A^{\nu}(y)$ gives the local propagator of photons, and pairing between spinor fields $\psi_{\tilde{\gamma}}(x)$ and $\overline{\psi}_{\tilde{\gamma}}(x)$ yields the propagator of the massless spinor field.

3.2. Spinor like superfield

We assume that an extended rigid string-stick is also itself behaviour as spinor like superfield consisting of spinor and its supersymmetric partner-slepton (selectron and smuon etc.). Notice that the supersymmetric partners of the leptons are spin-zero bosons: charged sleptons. In general speaking for a given fermion f, there are two supersymmetric partners, $\tilde{f}_L$ and $\tilde{f}_R$, which are scalar partners of the corresponding left- and right-handed fermion. Let us consider simplest spinor superfield consisting of spinor and its supersymmetric partner-boson with mass m:

$$\begin{aligned}\Psi(x)&=\psi(x)+m\sqrt{l}\theta\varphi_s(x)\\ \overline{\Psi}(y)&=\overline{\psi}(y)+m\sqrt{l}\theta\overline{\theta}^2\varphi_s(x)\end{aligned} \tag{14}$$

Here, the lower case s belongs to the supersymmetric partners of the leptons, i.e. to the spin-zero bosons. Thus, the propagator of this superfield is

$$\begin{aligned}S(x-y)&=\overbrace{\Psi(x)\overline{\Psi}(y)}\\ &=\int d^2\theta\int d^2\overline{\theta}\langle 0|T\{\Psi(x)\overline{\Psi}(y)\}|0\rangle\\ &=\frac{-i}{(2\pi)^4}\int d^4pe^{ipx}\left[\frac{m-i\hat{p}}{m^2+p^2-i\varepsilon}+(ml)\frac{m}{m^2+p^2-i\varepsilon}\right]\end{aligned} \tag{15}$$

Parameter m^2l in the second term in (15) is arisen from the dimensional argument and the assumption that in the limit $l\to 0$ contribution due to supersymmetric scalar part of the theory goes to zero, i.e. it corresponds to the almost local theory of QED, in the approximation $O(m^2l^2)$.

Thus, electromagnetic interaction between two superfield (12) and (14) leads to:

1. The usual local QED of leptons;
2. Interaction between leptons and photinos at the order of $O(\sqrt{ml})$;
3. Interaction between supersymmetric partners of leptons, i.e. charged spin-zero bosons and photons at the order of $O(\sqrt{ml})$;
4. Pure supersymmetric interaction between photinos and charged spin-zero bosons at the order of $O(ml)$.

From the point of view of direct experimental predicting study of the supersymmetric theory it is natural to consider first two cases together. Now we turn to this problem.

4. Nonlocal QED

4.1. The interaction Lagrangian

Let us consider interaction between leptons and nonlocal photon like superfield $\mathrm{A}_{\mu}(x)$ (12) the propagator of which is given by the formula (7) or (13). Such model we call the nonlocal QED (NQED). Since, here a spinor field of leptons does not charge and only mixed photon like fields $\mathrm{A}_{\mu}(x)$ (12) instead of the local photon field $A_{\mu}(x)$ are entered into the scheme of the construction for the interacting theory. This scheme allows us to construct nonlocal interaction of these fields by using usual procedure for studying the local QED. Thus, the Lagrangian density of these interacting fields $\psi(x)$ and $A_\mu(x)$ is taken the form;

$$L_I(x) = -\frac{1}{4}F_B^{*\mu\nu}F_{B\mu\nu} - \overline{\psi}_B(x)[\gamma_\mu(\partial^\mu + ie_B A_B^\mu(x) + m_B)]\psi_B(x) \tag{16}$$

where

$$F_B^{\mu\nu} = \partial^\mu A_B^\nu - \partial^\nu A_B^\mu - ie_B(A^\mu A^\nu - A^\nu A^\mu)$$

and A_B^μ and ψ_B are the bare (unrenormalized) field of the nonlocal photon and electron (or muon), and $-e_B$ and m_B are the bare charge and mass of the electron (or muon). As in the local field theory, we introduce renormalized field, charge and mass

$$\begin{aligned} \psi &\equiv Z_2^{-1/2}\psi_B \\ A^\mu &\equiv Z_3^{-1/2}A_B^\mu \end{aligned} \tag{17}$$

$$\begin{aligned} e &\equiv Z_2^{1/2}e_B \\ m &= m_B + \delta m \end{aligned} \tag{18}$$

with the constants Z_2, Z_3 and δm.

As usually, the Lagrangian may then be written in terms of remormalized quantities

$$L = L_0 + L_1 + L_2 \tag{19}$$

where

$$L_0 = -\frac{1}{4}F^{*\mu\nu}F_{\mu\nu} - \overline{\psi}[\gamma_\mu\partial^\mu + m]\psi \tag{20}$$

$$L_1 = -ieA_\mu(x)\overline{\psi}(x)\gamma^\mu\psi(x) \tag{21}$$

and L_2 is determined by as a sum of "counterterms"

$$\begin{aligned} L_2 &= -\frac{1}{2}(Z_3-1)F^{*\mu\nu}F_{\mu\nu} - (Z_2-1)\overline{\psi}[\gamma_\mu\partial^\mu + m]\psi \\ &+ Z_2\delta m\overline{\psi}\psi - ie(Z_2-1)A_\mu(x)\overline{\psi}(x)\gamma^\mu\psi(x) \end{aligned} \tag{22}$$

Notice that all of the terms in L_2 are of second order and higher order in e, and that these terms ensure to cancel the ultraviolet divergences that arise from loop graphs in the nonlocal QED.

It is well known that the Lagrangian density (16) is invariant under the gauge transformations:

$$\begin{aligned} \psi_B(x) &\Rightarrow e^{i\lambda(x)}\psi'_B(x) \\ \overline{\psi}_B(x) &\Rightarrow \overline{\psi}'_B(x)e^{-i\lambda(x)} \end{aligned} \tag{23}$$

and

$$A_B^\mu(x) \Rightarrow \left[A'^\mu_B(x) - \frac{1}{e_B}\frac{\partial\lambda(x)}{\partial x^\mu}\right] \tag{24}$$

We now would like to calculate Feynman diagrams in the nonlocal electrodynamics defined by the Lagrangians (20), (21) and (22).

4.2. Vacuum polarization

Since, in this concrete scheme the propagator $S(x-y)$ of the charged lepton spinor does not changed and therefore the diagrams of the vacuum polarization i.e. closed spinor propagators of leptons in the nonlocal QED are investigated by the same way as in the local theory. For completeness we calculate it in e^2-order in detail.

In the coordinate space, the matrix element of the S-matrix, corresponding to the diagram in Figure 2 has the form

$$-i:\mathrm{A}_\mu(x)\{-ie^2\gamma^\mu S(x-y)\gamma^\nu S(y-x)\}\mathrm{A}_\mu(y):=-i:\mathrm{A}_\mu(x)\Pi^{\mu\nu}(x-y)\mathrm{A}_\mu(y):$$

where

$$\Pi^{\mu\nu}(x-y)=-ie^2Tr\{\gamma^\mu S(x-y)\gamma^\nu S(y-x)\} \tag{25}$$

In the momentum p-space the vacuum polarization (25) takes the form

$$\Pi^{\rho\sigma}(q)=\frac{-ie^2}{(2\pi)^4}\int d^4p\frac{Tr(-i\hat{p}+m)\gamma^\rho(-i(\hat{p}-\hat{q})+m)\gamma^\delta]}{(p^2+m^2-i\varepsilon)((p-q)^2+m^2-i\varepsilon)} \tag{26}$$

Next we would like to act as follows from the standard local theory.

1. Take the Feynman parameterization

$$\frac{1}{(p^2+m^2-i\varepsilon)((p-q)^2+m^2-i\varepsilon)}=$$
$$\int_0^1 dx\frac{1}{[(p-qx)^2+m^2+q^2x(1-x)-i\varepsilon]^2}$$

2. Carry out shift of the variable of integration in momentum space $p\to p+qx$.
3. Calculate the trace as

$$\Lambda^{\rho\sigma}(p,q)=Tr\{[-i(\hat{p}+\hat{q}x)+m]\gamma^\rho[-i(\hat{p}-\hat{q}(1-x))+m]\gamma^\sigma\}$$
$$=4[-(p+qx)^\rho(p-q(1-x))^\sigma+(p+qx)(p-q(1-x))g^{\rho\sigma}$$
$$-(p+qx)^\sigma(p-q(1-x))^\rho+m^2g^{\rho\sigma}]$$

Our next step is called a Wick rotation $p^0\to+ip^4$, $d^4p\to(d^4p)_E=dp^1dp^2dp^3dp^4$ and all scalar products are evaluated using the Euclidean norm $a\cdot b=a^1b^1+a^2b^2+a^3b^3+a^4b^4$ with $q^4=-iq^0$.

It turns out that the integral of the type

$$\Pi^{\rho\sigma}(q)=\frac{e^2}{(2\pi)^4}\int_0^1 dx\int(d^4p)_E\frac{1}{[p^2+m^2+q^2x(1-x)]^2}\Lambda^{\rho\sigma}(p,q) \tag{27}$$

is badly divergent, which is calculated by using the dimensional regularization technique introduced in ’t Hooft and Veltman, 1972, based a continuation from four ($d=4$) to an arbitrary number d of spacetime dimensions.

For calculation purpose, we take account following formulas in d-spacetime:

1. All variables and trace are taken in d-dimensional spacetime with

$$\{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu}$$
$$g_{\mu\mu}=d$$
$$Tr(I)=N(d)$$

$$Tr\gamma_\mu \gamma_\nu = N(d) g_{\mu\nu} \tag{28}$$

where N is a regular function of d only and $N(4) = 4$. We have also

$$\gamma^\mu \gamma_\mu = d, \qquad \gamma_\mu \gamma_\nu \gamma^\mu = (2-d)\gamma_\nu \tag{29}$$

2.

$$Tr(\gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta) = N(d)[g_{\mu\nu} g_{\alpha\beta} + g_{\nu\alpha} g_{\mu\beta} - g_{\mu\alpha} g_{\nu\beta}] \tag{30}$$

3.

$$\gamma_\nu \gamma_\rho \gamma_\mu \gamma_\sigma \gamma^\nu = (2-d)\gamma_\rho \gamma_\mu \gamma_\sigma + 2(\gamma_\mu \gamma_\sigma \gamma_\rho - \gamma_\rho \gamma_\sigma \gamma_\mu) \tag{31}$$

4.

$$\gamma_\nu \gamma_\rho \gamma_\mu \gamma^\rho \gamma^\nu = (2-d)^2 \gamma_\mu \tag{32}$$

5.

$$p^\mu p^\nu \Rightarrow p^2 \frac{g^{\mu\nu}}{d} \tag{33}$$

6.

$$p^\mu p^\nu p^\rho p^\sigma \Rightarrow (p^2)^2 [g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}] \frac{1}{d(d+2)} \tag{34}$$

We use also the well-known formulas;

7. $d^d p_E \Rightarrow \Omega_d k^{d-1} dk$, where $k \equiv \sqrt{p^2}$ and Ω_d is the area of a unit sphere in d-dimensions

$$\Omega_d = 2\pi^{d/2}\Gamma(d/2) \tag{35}$$

8. There is an infinity in the one-loop contribution to the the vacuum polarization in the nonlocal QED, arising from the limiting behavior of the Gamma functions

$$\Gamma\left(2 - \frac{d}{2}\right) \Rightarrow \frac{1}{2 - d/2} - \gamma \tag{36}$$

where γ is the Euler number (constant), $\gamma = 0.5772157$.

9. Make use of the limiting behavior:

$$\lim_{\varepsilon \to 0} a^\varepsilon = \lim_{\varepsilon \to 0} e^{\varepsilon \ln a} = 1 + \varepsilon \ln a \tag{37}$$

where as usually we choose $\varepsilon = 2 - d/2$, for $d \to 4$.

10. To evaluate the resulting integral like

$$\int d^4 k \frac{(k^2)^n}{[k^2 + \nu^2]^m}$$

with $(k^2 + \nu^2)^m$ coming from the combined propagator denominators in Feynman diagrams, and $(k^2)^n$ coming from the propagator numerators and vertex momentum factors, we use the standard formula

$$\int_0^\infty dk \frac{k^{l-1}}{[k^2+\nu^2]^m} = \nu^{l-2m} \frac{\Gamma(l/2)\Gamma(m-l/2)}{2\Gamma(m)} \tag{38}$$

where $l = d + 2m$. In this work, we used this formula in special cases $n = 0$, $n = 2$, $n = 4$ and $m = 2$, $m = 3/2$, $m = 3$, $m = 5/2$.

11. Finally, we are needed in some properties of the Gamma functions

$$x\Gamma(x) = \Gamma(1+x)$$

$$\psi_1(x) = \frac{d\ln\Gamma(x)}{dx} = \frac{\Gamma'(x)}{\Gamma(x)}$$

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon}\Gamma(1+\varepsilon) = \frac{1}{\varepsilon} - \gamma + O(\varepsilon)$$

$$\Gamma(-1+\varepsilon) = -\left[\frac{1}{\varepsilon} + 1 - \gamma + O(\varepsilon)\right]$$

or in the general case

$$\Gamma(-n+\varepsilon) = \frac{(-1)^n}{n!}\left[\frac{1}{\varepsilon} + \left(1 + \frac{1}{2} + \cdots + \frac{1}{n} - \gamma\right) + O(\varepsilon)\right] \tag{39}$$

Above listed formulas are very useful to construct gauge invariant and finite nonlocal QED in any order in l, where l is the half size of the rigid string - stick. Here we are restricted in its first order in l. After such general mathematical preparation, we go to study expression (27) for the vacuum polarization diagram

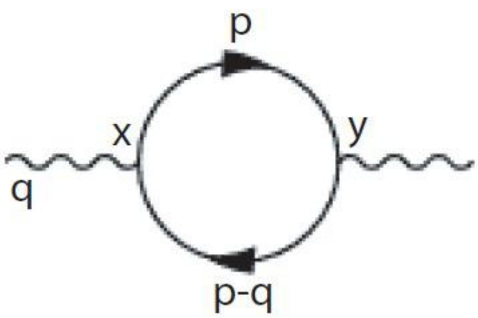

Figure 2: The one-loop diagram for the vacuum polarization in the nonlocal quantum electrodynamics arisen from the superfield concept

To carry out angular averages in (27) we drop all terms that are **odd** in p, and replace the terms that have **even numbers** of p-factors with using (33) and (34). Also, after writing the integrand in this way as a function only of p^2, the volume element $d^4 p_E$ is to be replaced in accordance with (35). Thus, expression $\Lambda^{\rho\sigma}(p,q)/4$ acquires the form

$$\Lambda^{\rho\sigma}(p,q)/4 = [-\frac{2k^2}{d} g^{\rho\sigma} + 2q^\rho q^\sigma x(1-x)$$
$$+ (k^2 - q^2 x(1-x))g^{\rho\sigma} + m^2 g^{\rho\sigma}] \tag{40}$$

where $k \equiv p$.

We now use integrals of the type of (38):

1.

$$\int_0^\infty dk k^{d-1}[k^2+\nu^2]^{-2} = \frac{1}{2}(\nu^2)^{d/2-2}\Gamma\left(\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)$$

2.

$$\int_0^{\infty} dk k^{(d+2)-1}[k^2+\nu^2]^{-2} = \frac{1}{2}(\nu^2)^{d/2-1}\Gamma\left(1+\frac{d}{2}\right)\Gamma\left(1-\frac{d}{2}\right)$$

Then, expression (27) takes the standard-gauge invariant form in d -dimensions:

$$\Pi^{\rho\sigma}(q) = \frac{4e^2\Omega_d}{(2\pi)^4}\Gamma\left(\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)(q^\rho q^\sigma - q^2 g^{\rho\sigma})$$
$$\times\int_0^1 dx x(1-x)[m^2+q^2x(1-x)]^{d/2-2} \tag{41}$$

We note the very remarkable result that this contribution satisfies relation

$$q_\rho \Pi^{\rho\sigma}(q) = 0 \tag{42}$$

that is the basis of the conservation and neutrality of the electric current in NQED in which dimensional regularization gives also this result of the conservation of current that does not depend on the dimensionality of spacetime.

Owing to (36) the Gamma function $\Gamma(2-d/2)$ in (41) has singularity at the limit $d \to 4$. Moreover, as shown in Subsection 4.1., there is another term that must be added to $\Pi^{\rho\sigma}(q)$, arising from the term $-\frac{1}{4}(Z_3-1)F^{*\mu\nu}F_{\mu\nu}$ in the interaction Lagrangian. This term has a structure like (41)

$$\Pi^{\rho\sigma}_{L_2}(q) = -(Z_3-1)(q^2 g^{\rho\sigma} - q^\rho q^\sigma) \tag{43}$$

so to order e^2, the full $\Pi^{\rho\sigma}_f$ has the form

$$\Pi^{\rho\sigma}_f = (q^2 g^{\rho\sigma} - q^\rho q^\sigma)\Pi^f_l(q^2) \tag{44}$$

with

$$\Pi^f_l(q^2) = -\frac{4e^2\Omega_d}{(2\pi)^4}\Gamma(\frac{d}{2})\Gamma(2-\frac{d}{2})\int_0^1 dx x(1-x)$$
$$\times[m^2+q^2x(1-x)]^{d/2-2} - (Z_3-1) \tag{45}$$

As in the local QED, the definition of the renormalized electromagnetic superfield requires that $\Pi_l(0)=0$. Therefore, to order e^2,

$$Z_3 = 1 - \frac{4e^2\Omega_d}{(2\pi)^4}\Gamma\left(\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)(m^2)^{d/2-2}$$
$$\times\int_0^1 dx x(1-x)\left[(m^2+q^2x(1-x))^{d/2-2} - (m^2)^{d/2-2}\right] \tag{46}$$

Now we can remove the regularization allowing d to approach its physical value $d=4$. There is an infinity in the one-loop contribution, arising from the limiting behavior of the Gamma function (36). According to the local QED a finite part of $\Pi_l(q^2)$ is extracted from the mathematical prescription

$$\Pi^f_l(q^2) = \Pi_l(q^2) - \Pi_l(0) - \left.\frac{\partial\Pi_l(q^2)}{\partial q^2}\right|_{q^2=0}\cdot q^2 \tag{47}$$

A direct calculation gives

1.

$$\Pi_l(0) = I\cdot(m^2)^{d/2-2}x(1-x)$$

2.

$$\left.\frac{\partial \Pi_l(q^2)}{\partial q^2}\right|_{q^2=0} q^2 = I\left[\left(\frac{d}{2}-2\right)(m^2)^{d/2-3}x^2(1-x)^2 q^2\right]$$

where

$$I = -\frac{4e^2\Omega_d}{(2\pi)^4}\Gamma\left(\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)\int_0^1 dx$$

The poles at $d=4$ obviously cancel in $\Pi_l(q^2)$ because for $d=4$ both $(m^2+q^2x(1-x))^{d/2-2}$ and $(m^2)^{d/2-2}$ have the same limit, unity (see formula (37)). For the same reason, the term $-\gamma$ in $\Gamma(2-d/2)$ cancels in the total $\Pi_l^f(q^2)$, though γ it does make a finite contribution to Z_3-1. There are other finite contribution to Z_3-1, that arise from the product of the pole in $\Gamma(2-d/2)$ with the linear terms in the expression of $\Omega_d\Gamma(d/2)$ around $d=4$, but there also cancel in the total $\Pi_l^f(q^2)$.

The only terms that do contribute to $\Pi_l(q^2)$ in the limit $d\to 4$ are those arising from the product of the pole in $\Gamma(2-d/2)$ with the linear terms in the expression of $(m^2+q^2x(1-x))^{d/2-2}$ and $(m^2)^{d/2-2}$ in powers of $d-4$:

$$(m^2+q^2x(1-x))^{d/2-2}-(m^2)^{d/2-2}\to\left(\frac{d}{2}-2\right)\ln\left(1+\frac{q^2x(1-x)}{m^2}\right)$$

due to the formula (37) and are also those arising from the product $\Gamma(2-d/2)(d/2-2)$ in (47).

Finally, all these simpler calculations read

$$\Pi_l(q^2)=\frac{e^2}{2\pi^2}\int_0^1 dx x(1-x)\ln\left(1+\frac{q^2x(1-x)}{m^2}\right) \tag{48}$$

The physical importance of the vacuum polarization in NQED can be explored by considering its effect on the scattering of two charged particles of spin $1/2$ (see, Weinberg,1995, for detail).

4.3. Electron self-energy in the nonlocal quantum electrodynamics

The complete electron propagator in NQED is given by the sum

$$[-i(2\pi)^{-4}S_l'(p)]=[-i(2\pi)^{-4}S(p)]$$
$$+[-i(2\pi)^{-4}S(p)][i(2\pi)^4\Sigma_l(p)][-i(2\pi)^{-4}S(p)]+\cdots$$

where

$$S(p)=\frac{-i\hat{p}+m_e}{p^2+m_e^2-i\varepsilon}$$

The sum is trivial, and gives

$$S_l'(p)=[i\hat{p}+m_e-\Sigma_l-i\varepsilon]^{-1} \tag{49}$$

In lowest order there is a one-loop contribution to Σ_l given in Figure 3:

$$:\overline{\psi}(x)\Sigma_l(x-y)\psi(y):$$

where

$$\Sigma_l(x-y)=-ie^2\mathrm{D}_l(x-y)\gamma^\mu S(x-y)\gamma_\mu$$

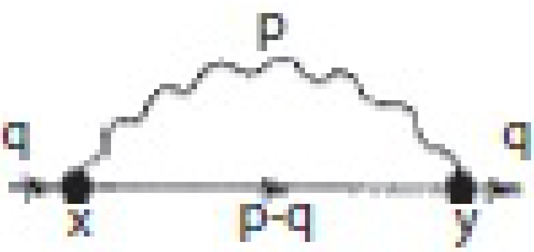

Figure 3: The one loop diagram for the electron self-energy function in NQED

Similar to the vacuum polarization, the electron self-energy function has the from

$$\Sigma_l(q) = \frac{ie^2}{(2\pi)^4}\int d^4p\left[\frac{1}{p^2-i\varepsilon}+\frac{\pi l}{\sqrt{p^2-i\varepsilon}}\right]$$

$$\times\left[\frac{\gamma^\rho(-i\hat{q}+i\hat{p}+m)\gamma_\rho}{(q-p)^2+m^2-i\varepsilon}\right] \tag{50}$$

here and below we have used notation $m = m_e$.

Making use of the general Feynman parametric formula

$$a^{-n_1}b^{-n_2} = \frac{\Gamma(n_1+n_2)}{\Gamma(n_1)\Gamma(n_2)}\int_0^1 dx_1\int_0^1 dx_2\delta(1-x_1-x_2)$$

$$\times x_1^{n_1-1}x_2^{n_2-1}(ax_1+bx_2)^{-n_1-n_2} \tag{51}$$

and the shift $p \to p+qx$, and the formula (29), one gets

$$\Sigma_l(q) = \frac{ie^2}{(2\pi)^4}\int_0^1 dx\int d^4p[-i(\hat{q}-\hat{p})(2-d)+md]$$

$$\times\{\frac{1}{[(p-qx)^2+q^2x(1-x)+m^2x-i\varepsilon]^2}+\frac{\pi l}{2}\frac{1}{\sqrt{1-x}}$$

$$\times\frac{1}{[(p-qx)^2+q^2x(1-x)+m^2x-i\varepsilon]^{3/2}}\} \tag{52}$$

Going to the Wick rotation and using the d-dimensional procedure as before, we obtain

$$\Sigma_l(q) = -\frac{e^2\pi^{d/2}\Gamma^2(d/2)}{(2\pi)^4}\int_0^1 dx[-i(2-d)(1-x)\hat{q}+md]$$

$$\times\{\Gamma(2-\frac{d}{2})[q^2x(1-x)+m^2x]^{d/2-2}$$

$$+\frac{\pi l}{2}\frac{\Gamma\left(\frac{3}{2}-\frac{d}{2}\right)}{\Gamma\left(\frac{3}{2}\right)\sqrt{1-x}}[q^2x(1-x)+m^2x]^{d/2-3/2}\}$$

$$= \Sigma_{local}(q)+\Sigma_l'(q) \tag{53}$$

The interaction (22) also contributes a renormalization counter term $-(Z_2-1)(i\hat{q}+m)+Z_2\delta m$ in $\Sigma_l(q)$, with Z_2 and δm determined by the condition that the complete propagator $S_l'(p)$ regarded as a function of $i\hat{q}$ should have a pole at $i\hat{q} = -m$ with residue unity.

In order to remove the regularization, allowing d to go to its limit $d \to 4$, we act as follows. We calculate the quantity $\Sigma_l(q)$ and its derivatives $\partial\Sigma_{local}/\partial i\hat{q}$ at the point $i\hat{q} = -m$, and use the Taylor series:

$$\Sigma_l(q) - \Sigma_{local}(q)\big|_{i\hat{q}=-m} + i\frac{\partial\Sigma_{local}}{\partial\hat{q}}\bigg|_{i\hat{q}=-m}(i\hat{q}+m) \quad (54)$$

We notice that at the limit $d \to 4$ the second part $\Sigma_l'(q)$ in (53) arisen from the supersymmetric contribution to order l is finite.

We know that a renormalization counterterm in $\Sigma_l(q)$ has the general form

$$-(Z_2^l - 1)(i\hat{q}+m) + Z_2^l m$$

as should expected. Expressions in the definition of (54) have the limits for $d \to 4$:

$$\Sigma_{local}(q) = -\Lambda\int_0^1 dx[2i\hat{q}(1-x)+4m]\Gamma\left(2-\frac{d}{2}\right)$$
$$\times\left[1+\left(\frac{d}{2}-2\right)\ln A\right] \quad (55)$$

$$\Sigma_{local}(q)\big|_{i\hat{q}=-m} = -\Lambda\int_0^1 dx(-2m(1-x)+4m)\Gamma\left(2-\frac{d}{2}\right)$$
$$\times\left[1+\left(\frac{d}{2}-2\right)\ln m^2x^2\right] \quad (56)$$

and

$$i\frac{\partial\Sigma_{local}(q)}{\partial\hat{q}}\bigg|_{i\hat{q}=-m}(i\hat{q}+m) = -\Lambda i(i\hat{q}+m)\int_0^1 dx$$
$$\times\{2i(1-x)\Gamma\left(2-\frac{d}{2}\right)\left[1+\left(\frac{d}{2}-2\right)\ln m^2x^2\right]$$
$$+(-2m(1-x)+4m)\Gamma\left(2-\frac{d}{2}\right)\left(\frac{d}{2}-2\right)$$
$$\times(m^2x^2)^{d/2-3}2imx(1-x)\} \quad (57)$$

where we have used the short notation

$$\Lambda = \frac{e^2\pi^{d/2}\Gamma^2\left(\frac{d}{2}\right)}{(2\pi)^4}$$
$$A = q^2x(1-x)+m^2x$$

Making use of equalities (55)–(57) it is easy to show that the poles at $d = 4$ cancel in expression (54) and therefore the local part of (53) has the standard form

$$\Sigma_{local}(q) = -\frac{2e^2\pi^2}{(2\pi)^4}\int_0^1 dx\{2\frac{1-x^2}{x}(i\hat{q}+m)$$
$$+(i\hat{q}(1-x)+2m)\ln\frac{m^2x^2}{q^2x(1-x)+m^2x}\} \quad (58)$$

While, as mension above the second part $\Sigma_l'(q)$ in (53) due to supersymmetric extension of the theory is finite at the limit $d=4$. That is

$$\Sigma_l'(q) = -\frac{2e^2\pi^2}{(2\pi)^4}\int_0^1 dx(i\hat{q}(1-x)+2m)(-2\pi l)$$
$$\times\frac{1}{\sqrt{x(1-x)}}\frac{1}{\sqrt{q^2(1-x)+m^2}} \quad (59)$$

where we have used the equality $\Gamma(-1/2) = -2\sqrt{\pi}$. By using the formula

$$\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}(1-\beta x)^{-\nu}dx = B(\lambda,\mu)_2F_1(\nu,\lambda;\lambda+\mu;\beta)$$

$$[Re\lambda>0, Re\mu>0, (\beta)<1]$$

one can express last expression (59) by means of the hypergeometric function

$$\Sigma_l'(q) = \frac{e^2 l}{2}\frac{1}{\sqrt{m^2+q^2}}\left[\frac{i}{4}\hat{q}_2F_1\left(\frac{1}{2},\frac{1}{2};2;\beta\right)+m_2F_1\left(\frac{1}{2},\frac{1}{2};1;\beta\right)\right] \quad (60)$$

where

$$\beta = \frac{q^2}{m^2+q^2}, \qquad q^2\neq -m^2$$

Finally, notice that the self-energy of the lepton in the NQED

$$\Sigma_l(q) = \Sigma_{local}(q)+\Sigma_l'(q)$$

is still a divergence from the behavior of the first term in (58) as $x\to 0$, which can be traced to the singular behavior of the integral over the photon momentum p in (50) at $p^2=0$, when we take q^2 at the point $q^2=-m^2$, where we evaluated Z_2-1. Such infrared divergences have common root as in the local QED.

4.4. Anomalous magnetic moments of the leptons

Let us consider contributions due to a size of the extended charges or supersymmetric extension of the theory to the magnetic moment of the leptons.

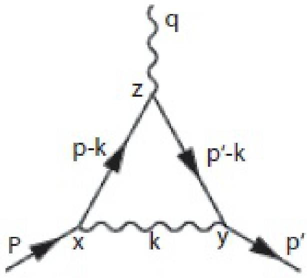

Figure 4: One-loop diagram for the (nonlocal) photon-lepton vertex function Γ^μ in NQED.

Here we need to calculate the matrix element corresponding to the one-loop graph in Figure 4. In NQED, one-loop graph (Figure 4) gives the matrix element

$$\Gamma_l^\mu(p',p) = \int d^4k[e\gamma^\rho(2\pi)^4]\left[\frac{-i}{(2\pi)^4}\frac{-i(\hat{p}'-\hat{k})+m}{(p'-k)^2+m^2-i\varepsilon}\right]$$

$$\times[\gamma^{\mu}]\left[\frac{-i}{(2\pi)^4}\frac{-i(\hat{p}-\hat{k})+m}{(p-k)^2+m^2-i\varepsilon}\right][e\gamma_{\rho}(2\pi)^4]$$
$$\times\left[\frac{-i}{(2\pi)^4}\left(\frac{1}{k^2-i\varepsilon}+\pi l\frac{1}{\sqrt{k^2-i\varepsilon}}\right)\right] \quad (61)$$

where p' and p are the final and initial lepton four-momenta, respectively. This integral has ultraviolet divergence, here we do use the dimensional regularization procedure. To combine denominators, we use the Feynman parameterization prescriptions:

$$\frac{1}{(p'-k)^2+m^2-i\varepsilon}\cdot\frac{1}{(p-k)^2+m^2-i\varepsilon}\frac{1}{k^2-i\varepsilon}$$

$$=2\int_0^1 dx\int_0^x dy[((p'-k)^2+m^2-i\varepsilon)y+((p-k)^2+m^2-i\varepsilon)(x-y)$$

$$+(k^2-i\varepsilon)(1-x)]^{-3}=2\int_0^1 dx\int_0^x dy[(k-p'y-p(x-y))^2$$

$$+m^2x^2+q^2y(x-y)-i\varepsilon]^{-3}$$

$$\frac{1}{(p'-k)^2+m^2-i\varepsilon}\cdot\frac{1}{(p-k)^2+m^2-i\varepsilon}\frac{1}{\sqrt{k^2-i\varepsilon}}=\frac{\Gamma(2+\frac{1}{2})}{\Gamma(\frac{1}{2})}$$

$$\times\int_0^1 dx\int_0^x dy\frac{1}{\sqrt{1-x}}[((p'-k)^2+m^2-i\varepsilon)y+((p-k)^2+m^2-i\varepsilon)(x-y)$$

$$+(k^2-i\varepsilon)(1-x)]^{-5/2}=\frac{3}{4}\int_0^1 dx\frac{1}{\sqrt{1-x}}\times\int_0^x dy[(k-p'y-p(x-y))^2$$

$$+m^2x^2+q^2y(x-y)-i\varepsilon]^{-5/2}$$

Here $q=p-p'$ is the momentum transferred to the (nonlocal) photon, which is mixed the usual photon and its supersymmetric partner-photino field with a portion of $\sqrt{\pi l}$.

Shifting the variable of the integration

$$k\to k+p'y+p(x-y) \quad (62)$$

the integral (61) becomes

$$\Gamma_l^{\mu}(p',p)=\frac{2ie^2}{(2\pi)^4}\int_0^1 dx\int_0^x dy\int d^4k\{\frac{1}{[k^2+m^2x^2+q^2y(x-y)-i\varepsilon]^3}$$
$$+\frac{3}{8}\pi l\frac{1}{\sqrt{1-x}}\frac{1}{[k^2+m^2x^2+q^2y(x-y)-i\varepsilon]^{5/2}}\}$$
$$\times\gamma^{\rho}[-i(\hat{p}'(1-y)-\hat{k}-\hat{p}(x-y))+m]\gamma^{\mu}$$

$$\times[-i(\hat{p}'(1-x+y)-\hat{k}-\hat{p}y)+m]\gamma_\rho \quad (63)$$

Next step is a Wick rotation, replace the volume element $d^4k_E=\Omega_d k^{d-1}dk$ and use the formulas (28)–(34) and (35). Putting this all together, Eq.(63) now reads

$$\Gamma_l^\mu(p',p)=\frac{-2e^2\Omega_d}{(2\pi)^4}\int_0^1 dx\int_0^x dy\int_0^\infty k^{d-1}dx$$
$$\times\left[\frac{1}{(k^2+L)^3}+\frac{3\pi l}{8\sqrt{1-x}}\frac{1}{(k^2+L)^{5/2}}\right]$$
$$\times\left\{[-k^2\gamma^\rho\gamma^\sigma\gamma^\mu\gamma_\sigma\gamma_\rho/d]+A+B+C+D\right\} \quad (64)$$

where we have used short notation:

$$L=m^2x^2+q^2y(x-y)$$
$$A=\gamma^\rho[-i(\hat{p}'(1-y)-\hat{p}(x-y))+m]\gamma^\mu[-i(\hat{p}'(1-x+y)-\hat{p}y)+m]\gamma_\rho$$
$$B=-\gamma^\rho\hat{k}\gamma^\mu\hat{k}\gamma_\rho$$
$$C=\gamma^\rho[-i(\hat{p}'(1-y)-\hat{p}(x-y))+m]\gamma^\mu(i\hat{k})\gamma_\rho$$
$$D=\gamma^\rho(i\hat{k})\gamma^\mu[-i(\hat{p}'(1-x+y)-\hat{p}y)+m]\gamma_\rho \quad (65)$$

As in the case of the local theory, we are interested here only in the matrix element $\bar{u}(p')\Gamma_l^\mu(p',p)u(p)$ of the vertex function between Dirac spinors that satisfy the relations

$$\bar{u}(p')(i\hat{p}'+m)=0 \qquad (i\hat{p}+m)u(p)=0$$

We able therefore to simplify this expression by using the anticommutation relations of the Dirac matrices to move all factors $\hat{p}'$ to the left and all factors $\hat{p}$ to the right, replacing them when they arrive on the left or right with im. We take into account the following standard relations between two Dirac spinors $\bar{u}(p')$ and $u(p)$:

$$a_1=-(1-y)(1-x+y)\gamma^\rho\hat{p}'\gamma^\mu\hat{p}\gamma_\rho$$
$$=-2(1-y)(1-x+y)[-3m^2\gamma^\mu-q^2\gamma^\mu-2im(p'^\mu+p^\mu)]$$
$$a_2=(x-y)(1-x+y)\gamma^\rho\hat{p}\gamma^\mu\hat{p}\gamma_\rho$$
$$=(x-y)(1-x+y)[-4imp^\mu-2m^2\gamma^\mu]$$
$$a_3=-im(1-x+y)\gamma^\rho\gamma^\mu\hat{p}\gamma_\rho=-4im(1-x+y)p^\mu$$
$$b_1=y(1-y)\gamma^\rho\hat{p}'\gamma^\mu\hat{p}'\gamma_\rho=y(1-y)[-4imp'^\mu-2m^2\gamma^\mu]$$
$$b_2=-y(x-y)\gamma^\rho\hat{p}\gamma^\mu\hat{p}'\gamma_\rho=-2m^2y(x-y)\gamma^\mu$$
$$b_3=im\gamma^\rho\gamma^\mu\hat{p}\gamma_\rho y=4imyp^\mu$$
$$c_1=-im(1-y)\gamma^\rho\hat{p}'\gamma^\mu\gamma_\rho=-4im(1-y)p'^\mu$$
$$c_2=im(x-y)\gamma^\rho\hat{p}\gamma^\mu\gamma_\rho=4im(x-y)p^\mu$$
$$c_3=m^2\gamma^\rho\gamma^\mu\gamma_\rho=-2m^2\gamma^\mu$$

We would like to sum up these expressions and obtain

$$A=2m^2\gamma^\mu(x^2-4x+2)+2(1-y)(1-x+y)q^2\gamma^\mu$$
$$+4im(y-x+xy)p'^\mu+4im(x^2-xy-y)p^\mu \quad (66)$$

This is the result of the local theory.

Thus, in the nonlocal QED, the vertex function corresponding to the diagram shown in Figure 4 takes the form by means of short notation:

$$\Gamma_l^{\mu}(p',p)=\frac{-2e^2\Omega_d}{(2\pi)^4}\int_0^1 dx\int_0^x dy\int_0^{\infty} dk k^{d-1}[A+B+C+D]$$
$$\times\left[\frac{1}{(k^2+L)^3}+\frac{3\pi l}{8}\frac{1}{\sqrt{1-x}}\frac{1}{(k^2+L)^{5/2}}\right] \quad (67)$$

Here, L, A, B, C and D are given by expressions (65). According to the above calculations:

$$\Lambda=\bar{u}(p')Au(p)=2m^2(x^2-4x+2)\gamma^{\mu}$$
$$+2(1-y)(1-x+y)q^2\gamma^{\mu}-2imx(1-x)[p^{\mu}+p'^{\mu}] \quad (68)$$

$$\bar{u}(p')Bu(p)=-\frac{(2-d)^2}{d}k^2\gamma^{\mu} \quad (69)$$

We know that integration of terms C and D with odd k-variables goes to zero. Notice that in Eqs.(68) and (69) we have exploited the symmetry of the vertex function (or the diagram) under the reflection $p\to p'$ (or $y\to x-y$), that gives the factor $p'^{\mu}+p^{\mu}$ exactly.

We next use the integral formula (38), the Gamma-matrix function algebra (28)–(34) and the limiting procedure like (36) and (35) for removal of the d-dimensional regularization as before. According to the local theory there are other diagrams that need to be taken into account. There is the zeroth-order term γ^{μ} in Γ_l^{μ}. The term proportional to Z_2-1 in the counterterms (22) gives contribution in Γ_l^{μ}:

$$\Gamma_{lL_2}^{\mu}=(Z_3-1)\gamma^{\mu} \quad (70)$$

Also, the effect of insertions of corrections to the external (mixed) photon propagator is a term:

$$\Gamma_{l,vac.pol}^{\mu}(p',p)=\left[\frac{1}{(p'-p)^2-i\varepsilon}+\pi l\frac{1}{\sqrt{(p'-p)^2-i\varepsilon}}\right]$$
$$\times\Pi_l^{\mu\nu}(p'-p)\gamma_{\nu} \quad (71)$$

The form of each of these terms (67), (70) and (71) is in agreement with the general rule:

$$\bar{u}(p')\Gamma_l^{\mu}(p',p)u(p)=\bar{u}(p')\left[\gamma^{\mu}F_l(q^2)-\frac{i}{2m}(p+p')^{\mu}G_l(q^2)\right]u(p) \quad (72)$$

To order e^2, the form factors are

$$F_l(q^2)=Z_2+\Pi_l(q^2)+F_{local}(q^2)+F_{1l}(q^2) \quad (73)$$

and

$$G_l(q^2)=G_{local}(q^2)+G_{1l}(q^2) \quad (74)$$

where $\Pi_l(q^2)$ is the vacuum polarization function (48),

$$F_{local}(q^2)=\frac{-2e^2\pi^2}{(2\pi)^4}\int_0^1 dx\int_0^x dy[\ln\frac{m^2x^2+q^2y(x-y)}{m^2x^2}$$
$$+\frac{m^2(x^2-4x+2)+q^2(1-y)(1-x+y)}{m^2x^2+q^2y(x-y)}] \quad (75)$$

$$F_{1l}(q^2) = -\frac{2e^2\pi^3 l}{(2\pi)^4}\int_0^1 dx \frac{1}{\sqrt{1-x}}\int_0^x dy[2\sqrt{m^2x^2+q^2y(x-y)} + \frac{m^2(x^2-4x+2)+q^2(1-y)(1-x+y)}{\sqrt{m^2x^2+q^2y(x-y)}}] \quad (76)$$

$$G_{local}(q^2) = -\frac{e^2m^2}{4\pi^2}\int_0^1 dx\int_0^x dy \frac{x(1-x)}{m^2x^2+q^2y(x-y)} \quad (77)$$

and

$$G_{1l}(q^2) = -\frac{e^2m^2}{4\pi} l\int_0^1 dx \frac{1}{\sqrt{1-x}}\int_0^x dy \frac{x(1-x)}{\sqrt{m^2x^2+q^2y(x-y)}} \quad (78)$$

We see that Eqs.(77) and (78) are finite. It makes to calculate the anomalous magnetic moment of the leptons, which is expressed as

$$\mu_l = \frac{e}{2m}[1-G_{local}(0)-G_{1l}(0)] \quad (79)$$

where

$$-G_{local}(0) = \frac{\alpha}{2\pi} \quad (80)$$

$$-G_{1l}(0) = \frac{4}{15}\alpha l m \quad (81)$$

The contribution (80) is the famous $\alpha/2\pi$ correction first obtained by Schwinger (1948). While expression (81) is arisen from the supersymmetric extention of the theory within the extended charge configuration. It seems that the present experimental values of the anomalous magnetic moment of the electron and muon (Carey et.al., 1999, Particle Data Group,2002; Barger et.al.,2005; Yao, 2006):

$$\Delta\mu_{exp}^{(e)} = \frac{\mu_e}{\mu_B} - 1 = (g-2)/2 =$$

$$= (1159.6521810 \pm 0.0000007)\times 10^{-6}$$

$$\Delta\mu_{exp}^{(\mu)} = \frac{\mu_\mu}{(e\hbar/2m_\mu)} - 1 = (g_\mu - 2)/2 =$$

$$= (11659208.0 \pm 5.4 \pm 3.3)\times 10^{-10}$$

are reliably confirmed by local quantum electrodynamics (see, for example, Heine-meyer et.al.,2004). It is natural to suppose that the absolute value of the contributions calculated here should be of an order or not greater than the experimental errors. This makes it possible to establish the following restriction on a scale of the supersymmetry or size of the rigid string stick:

$$l \lesssim \begin{cases} 1.4\times 10^{-20} cm & for \quad e \\ 4.3\times 10^{-20} cm & for \quad \mu \end{cases} \quad (82)$$

Finally, notice that formulas (7), (8) and (13) allow us to suppose that the rigid string-stick charges are radiated or absorbed complicated (super-) fields consisting of photons and its supersymmetric partners-photinos-massless spinor fields. We are called these fields super fields. However, as seen above in this mixed (or super) fields, radio $\gamma = n_\gamma / n_{\tilde{\gamma}} = n_\gamma / n_{photino}$ of numbers of photons and massless spinors generated by the charge of the stick simultaneously is of order $:1/\sqrt{\pi l}\,:10^9$. It means that among billion photons a few photinos may be appeared if the supersymmetry exists in nature.

4.5. The gauge invariance of the theory and Ward-Takahashi identity

Electromagnetic interaction of charged leptons with the nonlocal photons, propagator of which is determined by the formula (7) is gauge invariant. In the language of the perturbation theory that is exposed in previous sections, the **gauge invariance** of NQED means that **every matrix elements of the** S **-matrix** defining the concrete electromagnetic processes have a definite structure and algebraical relations exist between them. For example, in the momentum representation for the so-called vacuum polarization diagram in the second order of the perturbation theory the following form (see Section 4.2) exists:

$$\Pi_l^{\mu\nu}(q^2) = (q^2 g^{\mu\nu} - q^\nu q^\mu)\Pi_l(q^2) \tag{83}$$

Such form of the vacuum polarization satisfies the relation (42) that is the basis of the gauge invariance of the theory. Moreover, the following relation between the vertex function $\tilde{\Gamma}_\mu(p,q)$ and the self-energy of the electron $\tilde{\Sigma}(p)$ is also valid:

$$\frac{\partial\tilde{\Sigma}(p)}{\partial p_\mu} = -\tilde{\Gamma}_\mu(p,q)\Big|_{q=0}, \quad (q = p' - p) \tag{84}$$

This relation is called the **Ward-Takahashi** identity. The explicit forms of these functions are (see, Sections 4.3 and 4.4 and also Bogolubov and Shirkov, 1980):

$$\tilde{\Sigma}(p) = \frac{-ie^2}{(2\pi)^4}\int d^4k \mathrm{D}(k^2)\gamma_\mu S(\hat{p}-\hat{k})\gamma_\mu \tag{85}$$

and

$$\tilde{\Gamma}_\mu(p,q) = \frac{ie^2}{(2\pi)^4}\int d^4k \mathrm{D}((p-k)^2)\gamma_\nu S(\hat{q}+\hat{k})\gamma_\mu S(\hat{k})\gamma_\nu \tag{86}$$

and the identity (84) proved where in the momentum space $S(\hat{p}) = (m-\hat{p})^{-1}$, and

$$\mathrm{D}(k^2) = \frac{1}{-k^2 - i\varepsilon} + \pi l\frac{1}{\sqrt{-k^2 - i\varepsilon}}$$

are local spinor and nonlocal photon propagators written in Bogolubov and Shirkov's text book form. For the proof of relation (84) consider the identity

$$\frac{\partial S(\hat{p})}{\partial p_\mu} = S(\hat{p})\gamma_\mu S(\hat{p}) \tag{87}$$

where the vertex γ_μ is given by

$$\gamma_\mu = -\frac{\partial}{\partial p_\mu}S^{-1}(\hat{p}) \tag{88}$$

Further, differentiating $\tilde{\Sigma}(p)$ over p_μ and making use of the identity (87), we have

$$\frac{\partial\tilde{\Sigma}(p)}{\partial p_\mu} = -\frac{ie^2}{(2\pi)^4}\int d^4k \mathrm{D}(k^2)\gamma_\nu S(\hat{p}-\hat{k})\gamma_\mu S(\hat{p}-\hat{k})\gamma_\nu \tag{89}$$

Choosing other momentum variables in (86) and assuming $q=0$, $p' = p+q = p$, we get

$$\tilde{\Gamma}_\mu(p,0) = \frac{ie^2}{(2\pi)^4}\int d^4k \mathrm{D}(k^2)\gamma_\nu S(\hat{p}-\hat{k})\gamma_\mu S(\hat{p}-\hat{k})\gamma_\nu$$

Comparing this expression with (89) we obtain the *Ward-Takahashi identity* (84). The relation of the type (see, Section 4.4):

$$q_\mu \Gamma^\mu(p,q)\Big|_{p'^2=p^2=-m^2} = 0$$

follows from the identity

$$q_\mu \left\{ S(\hat{p}_1)\gamma^\mu S(\hat{p}_2) \right\} = S(\hat{p}_1) - S(\hat{p}_2)$$

if $q = p_1 - p_2$.

It is important to notice that in the nonlocal QED the vertex γ^μ in any Feynman diagrams is connected with the propagator $S(\hat{p})$ of the charged particle by the relation (88).

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