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Unification of Four Fundamental Forces By Means of High Dimensional Spacetimes

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Abstract We propose unification of four fundamental forces in nature by using high-dimensional spacetimes. It is shown that step of passage from one dimension to other one is determined by the Planck length $L_{Pl} = \sqrt{G\hbar/c^3}$. It turns out that potentials and corresponding forces are conserved those forms and are given by unified way in increasing step by step of numbers of spacetime dimensions.

Keywords Newtonian, Coulomb, Yukawa weak potentials and forces. High-dimensional space, Planck length, coupling constants, Photon, Gravitation and scalar particle propagators

1. Introduction

In previous paper [1] we have shown that

1) the Newtonian and the Coulomb potentials

$$U_D^{N,C}(r) \simeq const\,\frac{1}{r^{D-3}}, \quad (1)$$

depending on spacetime dimensions satisfy the Laplacian equation

$$\Delta_D U_D^{U,C}(r_D) = 0 \quad (2)$$

where

$$\Delta_D = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \ldots + \frac{\partial^2}{\partial x_{D-1}^2}$$

and

$$r_D = \sqrt{x_1^2 + x_2^2 + \ldots + x_{D-1}^2} \quad .$$

2) These potentials are determined by Fourier transforms of the photon and graviton propagators

$$D^{\gamma,g}(p) = \frac{1}{\overleftarrow{p}^2}$$

in the static limit by the formulas

$$U_D^{N,C}(r) = const\int d^{D-1}p\,\frac{1}{\overleftarrow{p}^2}\,e^{ipr}, \quad (3)$$

Here

$$ipr = ip_1x_1 + ip_2x_2 + \ldots + ip_{D-1}x_{D-1} \quad .$$

The Yukawa potential is also given by the similar formula:

$$U_D^Y(r) = const\int d^{D-1}p\,\frac{1}{\overleftarrow{p}^2 + m^2}\,e^{ipr} \quad (4)$$

Purpose of this paper is to determine coupling constants of four fundamental interactions in unified way by means of the Planck length L_{pl}.

2. Modification of the Newtonnian Potentials and Corresponding Forces in High-Dimensions of Spacetime

2.1. Usual four-dimensional case

$D = 4$:

$$U_4^N(r) = \frac{G}{r}, \tag{5}$$

$$\overleftarrow{F}_4^N(r) = -\overleftarrow{\nabla}_4 U_4^N M_1 M_2 = \frac{G}{r^2} M_1 M_2 \overleftarrow{n}$$

for two bodies with masses M_1 and M_2, $\overleftarrow{n} = \overleftarrow{r}/r$ is the unit vector.

Here

$$G = 6.6743 \cdot 10^{-11} \frac{m^3}{kg \cdot sec^2} \tag{6}$$

2.2. Five-dimensional case

$$U_5^N(r_5) = \frac{1}{2} G \cdot L_{Pl} \frac{1}{r_5^2}, \tag{7}$$

$$\overleftarrow{F}_5^N(r_5) = -\nabla_5 U_5^N \cdot M_1 \cdot M_2 = G L_{Pl} \frac{M_1 \cdot M_2}{r_5^3} \overleftarrow{n} \tag{8}$$

where

$$r_5 = \sqrt{x_1^2 + x_2^2 + \ldots + x_4^2},$$

$$g_5^N = G L_{Pl} = 1.08 \times 10^{-45} \frac{m^4}{kg \cdot sec^2} \tag{9}$$

2.3 Six-dimensional case

$$U_6^N(r_6) = \frac{1}{3} G L_{Pl}^2 \cdot \frac{1}{r_6^3} \tag{10}$$

$$\overleftarrow{F}_6^N = -\nabla_6 U_6^N \cdot M_1 \cdot M_2 = G L_{Pl}^2 \frac{1}{r_6^4} \cdot M_1 \cdot M_2 \overleftarrow{n} \tag{11}$$

where

$$g_6^N = G L_{Pl}^2 = 1.74 \times 10^{-80} \frac{m^5}{kg \cdot sec^2} \quad . \tag{12}$$

2.4 Seven-dimensional case

$$U_7^N(r_7) = \frac{1}{4} G L_{Pl}^3 \frac{1}{r^4}, \tag{13}$$

$$\overleftarrow{F}_7^N = -\nabla_7 U_7^N \cdot M_1 \cdot M_2 = G L_{Pl}^3 \frac{1}{r_7^5} \cdot M_1 \cdot M_2 \overleftarrow{n} \tag{14}$$

where

$$g_7^N = G L_{Pl}^3 = 2.818 \times 10^{-115} \frac{m^6}{kg \cdot sec^2} \tag{15}$$

2.5. Eight-dimensional case

$$U_8^N(r_8) = \frac{1}{5} G L_{Pl}^4 \cdot \frac{1}{r_8^5} \tag{16}$$

$$\overleftarrow{F}_8^N = -\nabla_8 U_8^N \cdot M_1 \cdot M_2 = G L_{Pl}^4 \frac{1}{r_8^6} M_1 \cdot M_2 \cdot \overleftarrow{n} \tag{17}$$

here,

$$g_8^N = G L_{Pl}^4 = 4.554 \times 10^{-150} \frac{m^7}{kg \cdot sec^2} \tag{18}$$

2.6. Nine-dimensional case

$$U_9^N(r_9) = \frac{1}{6} G \cdot L_{Pl}^5 \cdot \frac{1}{r_9^6}, \tag{19}$$

$$\overleftarrow{F}_9^N = -\nabla_9 U_9^N \cdot M_1 \cdot M_2 = G L_{Pl}^5 \frac{1}{r_9^7} \cdot M_1 \cdot M_2 \overleftarrow{n} \tag{20}$$

where

$$g_9^N = G L_{Pl}^5 = 7.36 \times 10^{-185} \frac{m^8}{kg \cdot sec^2} \quad . \tag{21}$$

2.7. Ten-dimensional case (the string theory)

$$U_{10}^N(r_{10}) = \frac{G}{7} L_{Pl}^6 \cdot \frac{1}{r_{10}^7}, \tag{22}$$

$$\overleftarrow{F}_{10}^N = -\nabla_{10} U_{10}^N \cdot M_1 \cdot M_2 = G L_{Pl}^6 \frac{M_1 \cdot M_2}{r_{10}^8}, \overleftarrow{n} \tag{23}$$

where

$$g_{10}^N = G L_{Pl}^6 = 1.1895 \times 10^{-219} \frac{m^9}{kg \cdot sec^2} \quad . \tag{24}$$

2.8. Eleven-dimensional case (M-theory)

$$U_{11}^N(r_{11}) = \frac{1}{8} G \cdot L_{Pl}^7 \cdot \frac{1}{r_{11}^8}, \tag{25}$$

$$\overleftarrow{F}_{11}^N = -\nabla_{11} U_{11}^N \cdot M_1 \cdot M_2 = G L_{Pl}^7 \frac{M_1 \cdot M_2}{r_{11}^9}, \overleftarrow{n} \tag{26}$$

where

$$g_{11}^N = G L_{Pl}^7 = 1.923 \times 10^{-254} \frac{m^{10}}{kg \cdot sec^2} \tag{27}$$

3. Modification of the Coulomb Potentials and Corresponding Forces in High-Dimensions of Spacetime

3.1. Traditional four-dimensional case

$$U_4^C(r_4) = \frac{k_C}{r} \tag{28}$$

$$\overleftarrow{F}_4^C(r_4) = -q_1 \cdot q_2 \overleftarrow{\nabla}_4 U_4^C(r_4) = \frac{k_C}{r^2} q_1 \cdot q_2 \overleftarrow{n} \tag{29}$$

for two bodies with charges q_1 and q_2.

Here

$$k_C = \frac{1}{4} \cdot 10^7 \cdot c^2 \cdot \frac{m^3}{kg} = 2.2469 \times 10^{23} \frac{m^3}{kg} \quad (30)$$

in the SI-system.

3.2. Five-dimensional case

$$U_5^C(r_5) = \frac{1}{2} k_C \cdot L_{Pl} \frac{1}{r_5^2}, \quad (31)$$

$$\overleftarrow{F}_5^C(r_5) = -q_1 \cdot q_2 \overleftarrow{\nabla}_5 U_5^C(r_5) = k_C \cdot L_{Pl} \cdot \frac{q_1 \cdot q_2}{r_5^3} \overleftarrow{n}, \quad (32)$$

where

$$g_5^C = k_C L_{Pl} = 3.63 \times 10^{-12} \frac{m^4}{kg} \quad . \quad (33)$$

3.3. Six-dimensional case

$$U_6^C(r_6) = \frac{1}{3} k_C \cdot L_{Pl}^2 \frac{1}{r_6^3}, \quad (34)$$

$$\overleftarrow{F}_6^C(r_6) = -q_1 \cdot q_2 \overleftarrow{\nabla}_6 U_6^C(r_6) = k_C \cdot L_{Pl}^2 \cdot \frac{q_1 \cdot q_2}{r_6^4} \overleftarrow{n}, \quad (35)$$

where

$$g_6^C = k_C \cdot L_{Pl}^2 = 5.86 \times 10^{-47} \frac{m^5}{kg} \quad . \quad (36)$$

3.4. Seven-dimensional case

$$U_7^C(r_7) = \frac{1}{4} k_C \cdot L_{Pl}^3 \frac{1}{r_7^4}, \quad (37)$$

$$\overleftarrow{F}_7^C(r_7) = -q_1 \cdot q_2 \overleftarrow{\nabla}_7 U_7^C(r_7) = k_C \cdot L_{Pl}^3 \cdot \frac{q_1 \cdot q_2}{r_7^5} \overleftarrow{n}, \quad (38)$$

where

$$g_7^C = k_C \cdot L_{Pl}^3 = 9.48 \times 10^{-82} \frac{m^6}{kg}. \quad (39)$$

3.5. Eight-dimensional case

$$U_8^C(r_8) = \frac{1}{5} k_C^2 \cdot \frac{\hbar}{c} \frac{1}{r_8^5}, \quad (40)$$

$$\overleftarrow{F}_8^C(r_8) = -q_1 \cdot q_2 \overleftarrow{\nabla}_8 U_8^C(r_8) = k_C^2 \cdot \frac{\hbar}{c} \cdot \frac{q_1 \cdot q_2}{r_8^6} \overleftarrow{n}, \quad (41)$$

where

$$g_8^C = k_C^2 \cdot \frac{\hbar}{c} = 1.78 \times 10^4 \frac{m^7}{kg} \quad . \quad (42)$$

3.6. Nine-dimensional case

$$U_9^C(r_9) = \frac{1}{6} \frac{k_C^2}{c^3} \cdot \hbar \sqrt{G\hbar c} \frac{1}{r_9^6}, \quad (43)$$

$$\overleftarrow{F}_9^C(r_9) = -q_1 \cdot q_2 \overleftarrow{\nabla}_9 U_9^C(r_9) = \frac{k_C^2}{c^3} \hbar \cdot \sqrt{G\hbar c} \frac{q_1 \cdot q_2}{r_9^7} \overleftarrow{n}, \quad (44)$$

Here

$$g_9^C = \frac{k_C^2}{c^3}\hbar\sqrt{G\hbar c} = 2.88\times10^{-31}\frac{m^8}{kg} \quad . \tag{45}$$

3.7. Ten-dimensional case (the string theory)

$$U_{10}^C(r_{10}) = \frac{1}{7}k_C^2\frac{\hbar^2 G}{c^4}\frac{1}{r_{10}^7}, \tag{46}$$

$$\overleftarrow{F}_{10}^C(r_{10}) = -\overleftarrow{\nabla}_{10}U_{10}^C(r_{10}) = \frac{k_C^2}{c^4}\hbar^2 G\frac{q_1\cdot q_2}{r_{10}^8}\overleftarrow{n}, \tag{47}$$

Where

$$g_{10}^C = \frac{k_C^2}{c^4}\hbar^2 G = 4.6\times10^{-66}\frac{m^9}{kg} \quad . \tag{48}$$

3.8. Eleven-dimensional case (M-theory)

$$U_{11}^C(r_{11}) = \frac{1}{8}k_C^2\hbar\frac{1}{c^7}(G\hbar c)^{3/2}\frac{1}{r_{10}^8}, \tag{49}$$

$$\overleftarrow{F}_{11}^C(r_{11}) = -\overleftarrow{\nabla}_{11}U_{11}^C(r_{11}) = k_C^2\frac{\hbar}{c^7}(G\hbar c)^{3/2}\frac{q_1\cdot q_2}{r_{11}^9}\overleftarrow{n}, \tag{50}$$

where

$$g_{11}^C = k_C^2\frac{\hbar}{c^7}(G\hbar c)^{3/2} = 7.5\times10^{-101}\frac{m^{10}}{kg} \quad . \tag{51}$$

We notice that at the Plank scale

$$F_4^C = F_5^C = F_6^C = F_7^C and F_9^C = F_{10}^C = F_{11}^C, \tag{52}$$

while for the Newton force:

$$F_4^N = F_5^N = F_6^N = \ldots = F_9^N = F_{10}^N = F_{11}^N \tag{53}$$

4. Modification of the Weak Potentials and Corresponding Forces in High-Dimensions of Spacetime

Point of view of the language of high-dimensional spacetimes, Fermi-weak potential and its corresponding force at the beginning act on six-dimensional spacetime.

4.1. Six-dimensional case ($as \quad D = 4$)

Thus the Fermi potential and its force take the form

$$U_4^F(r) = \frac{1}{3}\frac{G_F}{r^3} \tag{54}$$

$$\overleftarrow{F_4^F} = -\overleftarrow{\nabla}_4 U_4^F = \frac{G_F}{r^4}\overleftarrow{n} \tag{55}$$

where the Fermi constant is

$$G_F = 1.436\times10^{-62}\frac{m^5\cdot kg}{sec^2}. \tag{56}$$

4.2. Seven-dimensional case ($as \quad D = 5$)

$$U_5^F(r) = \frac{1}{4}G_F\frac{\sqrt{G\hbar c}}{c^2}\frac{1}{r_5^4}, \tag{57}$$

$$\overleftarrow{F_5^F}(r) = -\overleftarrow{\nabla}_5 U_5^F = G_F \frac{\sqrt{G\hbar c}}{c^2}\frac{1}{r_5^5}\overleftarrow{n} = G_F \cdot L_{Pl}\frac{1}{r_5^5}\overleftarrow{n} \quad (58)$$

where

$$g_F^5 = G_F \frac{\sqrt{G\hbar c}}{c^2} = 2.32\times 10^{-97}\frac{m^6 \cdot kg}{sec^2} \quad . \quad (59)$$

4.3. Eight-dimensional case ($as \quad D = 6$ **)**

$$U_6^F(r) = \frac{1}{5}G_F \frac{G\hbar}{c^3}\frac{1}{r_6^5}, \quad (60)$$

$$\overleftarrow{F_6^F} = -\overleftarrow{\nabla}_6 U_6^F(r) = \frac{G_F}{c^3} G\cdot\hbar \frac{1}{r_6^6}\overleftarrow{n} \quad (61)$$

where

$$g_F^6 = \frac{G_F}{c^3}\cdot G\hbar = 3.75\times 10^{-132}\frac{m^7\cdot kg}{sec^2} \quad . \quad (62)$$

4.4 Nine-dimensional case ($as \quad D = 7$ **)**

$$U_7^F(r) = \frac{1}{6}G_F \frac{(G\hbar c)^{3/2}}{c^6}\frac{1}{r_7^6}, \quad (63)$$

$$\overleftarrow{F_7^F} = -\overleftarrow{\nabla}_7 U_7^F(r) = \frac{G_F}{c^6}(G\hbar c)^{3/2}\cdot\frac{1}{r_7^7}\overleftarrow{n}, \quad (64)$$

where

$$g_F^7 = \frac{G_F}{c^6}(G\hbar c)^{3/2} = 6.06\times 10^{-167}\frac{m^8\cdot kg}{sec^2} \quad . \quad (65)$$

4.5. Ten-dimensional case ($as \quad D = 8$ **)**

$$U_8^F(r) = \frac{1}{7}G_F \frac{(G\hbar c)^2}{c^8}\frac{1}{r_8^7}, \quad (66)$$

$$\overleftarrow{F_8^F} = -\overleftarrow{\nabla}_8 U_8^F(r) = \frac{G_F}{c^8}(G\hbar c)^2\cdot\frac{1}{r_8^8}\overleftarrow{n}. \quad (67)$$

Here

$$g_F^8 = \frac{G_F}{c^8}(G\hbar c)^2 = 9.8\times 10^{-202}\frac{m^9\cdot kg}{sec^2}. \quad (68)$$

4.6 Eleven-dimensional case ($as \quad D = 9$ **)**

$$U_9^F(r) = \frac{1}{8}\frac{(G_F)}{c^{10}}(G\hbar c)^{5/2}\cdot\frac{1}{r_9^8}, \quad (69)$$

$$\overleftarrow{F_9^F} = -\overleftarrow{\nabla}_9 U_9^F(r) = \frac{G_F}{c^{10}}(G\hbar c)^{5/2}\cdot\frac{1}{r_{10}^9}\overleftarrow{n} \quad (70)$$

Where

$$g_9^F = \frac{G_F}{c^{10}}(G\hbar c)^{5/2} = 1.58\times 10^{-236}\frac{m^{10}\cdot kg}{sec^2} \quad (71)$$

5. Modification of the Yukawa Potentials in High-Dimensions of Spacetime

5.1 Four-dimensional case

By definition the static limit

$$1)\quad U_4^Y(r) = g_4^Y \cdot c_4 \int \frac{d^3p}{m^2+p^2} e^{i\vec{p}\vec{r}} = 2\pi^2 \cdot c_4 \cdot g_4^Y \cdot \frac{1}{r} e^{-mr}, \tag{72}$$

where c_4 is a normalization constant, so that

$$U_4^Y = g_4^Y \cdot \frac{1}{r} e^{mr} = \frac{\hbar}{M_W c} \frac{1}{r} e^{-r/\lambda_w}, \tag{73}$$

Here

$$\lambda_w = \frac{\hbar}{M_w} \text{ and } M_w \text{ is the } W-\text{boson mass}$$

5.2. Let D=5, then we have analogously formula

$$2)\quad U_5^Y(r) = g_5^Y \cdot c_5 \int \frac{d^4p}{m^2+p^2} e^{i\vec{p}\vec{r}} = 4\pi g_5^Y \int_0^\infty dp \frac{p^3}{m^2+p^2} \times \int_{-1}^1 dx\sqrt{1-x^2} e^{iprx}, \tag{74}$$

Where

$$e_5 = \int_{-1}^1 dx\sqrt{1-x^2} e^{iprx} = \frac{2\pi}{2pr} J_1(pr), \tag{75}$$

and

$$\lambda_5 = \int_0^\infty \frac{dp \cdot p^2}{m^2+p^2} J_1(pr) = mK_1(mr) \tag{76}$$

Here $J_1(x)$ and $K_1(x)$ are well-known cylinder functions, and we have used the integral form [2,3]:

$$\int_0^\infty dx \frac{J_\nu(bx)x^{\nu+1}}{(a^2+x^2)^{1+\mu}} = \frac{a^{\nu-\mu}b^\mu}{2^\mu \Gamma(1+\mu)} = \begin{cases} K_{\nu-\mu}(ab), \\ -1 < Re\,\nu < Re(2\mu+\frac{3}{2}), \quad a,b>0. \end{cases} \tag{77}$$

Thus, after normalization procedure, one gets

$$U_5^Y(r_5) = g_5^Y \cdot \frac{m}{r_5} K_1(mr_5), \qquad r_5 = \sqrt{x_1^2+x_2^2+x_3^2+x_4^2}. \tag{78}$$

Similar calculations for $D = 6,7,8,9,10,11$ read

$$3)\quad U_6^Y(r_6) = g_6^Y \cdot (\frac{m}{r_6})^{3/2} K_{3/2}(mr_6), \tag{79}$$

$$4)\quad U_7^Y(r_7) = g_7^Y \cdot (\frac{m}{r_7})^2 K_2(mr_7), \tag{80}$$

$$5)\quad U_8^Y(r_8) = g_8^Y \cdot (\frac{m}{r_8})^{5/2} K_{5/2}(mr_8), \tag{81}$$

$$6)\quad U_9^Y(r_9) = g_9^Y \cdot (\frac{m}{r_9})^3 K_3(mr_9), \tag{82}$$

$$7)\quad U_{10}^Y(r_{10}) = g_{10}^Y \cdot (\frac{m}{r_{10}})^{7/2} K_{7/2}(mr_{10}), \tag{83}$$

and

$$8)\quad U^{Y}_{11}(r_{11}) = g^{Y}_{11}\cdot\left(\frac{m}{r_{11}}\right)^4 K_4(mr_{11}), \tag{84}$$

respectively. Here we have used the integral form

$$\int_0^1 dx\cos(ax)(1-x^2)^{\nu-\frac{1}{2}} = \frac{\sqrt{\pi}}{2}\left(\frac{2}{a}\right)^{\nu}\times$$

$$\Gamma\left(\nu+\frac{1}{2}\right)J_{\nu}(a) \quad .$$

In conclusion, we notice that Newtonian, Coulomb and Fermi weak forces are unified at the Planck scale and given by numerical values:

$$F^N = 2.56\times10^{59} M_1\cdot M_2\cdot\frac{m}{kg\cdot sec^2}, \tag{85}$$

$$F^C = 1\times10^{213} q_1\cdot q_2\cdot\frac{m}{kg}, \tag{86}$$

$$F^F = 2.1\times10^{77}\frac{mkg}{sec^2}, \tag{87}$$

6. Uniform of Writing down of Potentials and Those Corresponding Forces in Different Spacetime Dimensions

Let D, G, k_C, r_D and L_{Pl} are number of dimensions of spacetime, the Newtonian and Coulomb constants, $r_D = \sqrt{x_1^2 + x_2^2 + \ldots + x_{D-1}^2}$ and the Planck length then we have following unified formulas for potentials and those forces in high-dimensional spacetimes:

6.1. The Newtonian case

$$U_D^N(r_D) = \frac{1}{D-3} G\cdot L_{Pl}^{D-4}\frac{1}{r_D^{D-3}}, \tag{88}$$

$$\overleftarrow{F}_D^N(r_D) = -\overleftarrow{\nabla}_D U_D^N(r_D) M_1\cdot M_2 = G L_{Pl}^{D-4}\frac{M_1 M_2}{r_D^{D-2}}\overleftarrow{n} \tag{89}$$

At the Planck scale:

$$\overleftarrow{F_4}^N = \overleftarrow{F_5}^N = \ldots = \overleftarrow{F_{11}}^N(L_{Pl}) = G\cdot\frac{M_1 M_2}{L_{Pl}^2} = 2.56\times10^{59} M_1\cdot M_2\cdot\frac{m}{kg\cdot sec^2} \quad .$$

6.2. The Coulomb case

$$U_D^C(r_D) = \frac{1}{D-3} k_C L_{Pl}^{D-4}\frac{1}{r_D^{D-3}}, \tag{90}$$

$$\overleftarrow{F}_D^C(r_D) = -\overleftarrow{\nabla}_D U_D^c(r_D) q_1\cdot q_2 = k_C L_{Pl}^{D-4}\frac{q_1\cdot q_2}{r_D^{D-2}}\overleftarrow{n}, \tag{91}$$

where $D = 4,5,6,7.$

For $D = 8$, we have

$$U_8^C(r_8) = \frac{1}{5}k_C^2\frac{\hbar}{c}\cdot\frac{1}{r_8^5}, \tag{92}$$

$$\overleftarrow{F}_8^C(r_8) = -q_1\cdot q_2\overleftarrow{\nabla} U_8^C(r_8) = k_C^2\cdot\frac{\hbar}{c}\frac{q_1\cdot q_2}{r_8^6}\overleftarrow{n}, \tag{93}$$

In the case of $D = 9,10,11$ one gets

$$U_D^C(r_D) = \frac{1}{D-3}k_C^2\frac{\hbar}{c}L_{Pl}^{D-8}\cdot\frac{1}{r_D^{D-3}} \tag{94}$$

$$\overleftarrow{F}_D^C(r_D) = -q_1 \cdot q_2 \overleftarrow{\nabla}_D U_D^C(r_D) = k_C^2 \frac{\hbar}{c} \cdot L_{Pl}^{D-8} \cdot \frac{q_1 \cdot q_2}{r_D^{D-2}} \overleftarrow{n}, \tag{95}$$

At the Planck scale

$$\overleftarrow{F}_4^C = \overleftarrow{F}_5^C = \overleftarrow{F}_6^C = \overleftarrow{F}_7^C, \qquad \overleftarrow{F}_9^C = \overleftarrow{F}_{10}^C = \overleftarrow{F}_{11}^C,$$

$$\overleftarrow{F}_{11}^C(L_{Pl}) = 1 \times 10^{213} q_1 \cdot q_2 \cdot \frac{m}{kg} \overleftarrow{n}. \tag{96}$$

6.3. The Fermi case

$$U_D^F(r_D) = \frac{1}{D-3} G_F \cdot L_{Pl}^{D-6} \frac{1}{r_D^{D-3}}, \tag{97}$$

where $D = 6,7,...,11,$

$$\overleftarrow{F}_D^F(r_D) = -\overleftarrow{\nabla}_D U_D^F(r_D) = G_F L_{Pl}^{D-6} \cdot \frac{1}{r_D^{D-2}} \overleftarrow{n}, \tag{98}$$

At the Planck scale:

$$\overleftarrow{F}_6^F = \overleftarrow{F}_7^F = ... = \overleftarrow{F}_{11}^F, \quad \overleftarrow{F}_{11}^C(L_{Pl}) = 2.1 \times 10^{77} \frac{mkg}{sec^2} \tag{99}$$

6.4. The Yukawa case

$$U_D^Y(r_D) = g_D^Y \left(\frac{m}{r_D}\right)^{\left(\frac{D-3}{2}\right)} K_{\frac{D-3}{2}}(mr_D) \tag{100}$$

$$\overleftarrow{F_D}(r_D) = -\overleftarrow{\nabla_D} U_D^Y(r_D), \tag{101}$$

where

$$K_{\frac{D-3}{2}}(mr_D)$$

is the Mack'Donald function and g_D^Y is coupling constants for the Yukawa theory.

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