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Gravitational Effects Generated by the Curvature of Space on the Earth's Surface

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Abstract Although the spatial curvature at the surface of the Earth is very small value, i.e., $1.71\times10^{-23}(1/m^2)$, it is enough value to produce 1G (9.8 m/s^2) acceleration. This paper is an attempt to explain the cause of gravitation by applying the mechanical structure of space to General Relativity. The concept of mechanical structure of space was primarily intended for application to space propulsion physics. Author has proposed new space propulsion concept at international conferences and peer-reviewed journals since 1988 [1]. However, this paper focuses on the cause of gravitational effects. The mass on the Earth will not be pulled by the Earth and fall, but will be pushed and fall in the direction of the Earth due to the pressure of the field in the curved space area around the Earth.

Since this paper mainly describes the concept of the principle, most of the mathematical expressions are reduced. See the references for theoretical formulas [11, 12].

Keywords Gravitation; curvature; General Relativity; space-time; continuum mechanics; acceleration field; curved space; flat space

1. Introduction

Given a priori assumption that space as a vacuum has a physical fine structure like continuum, it enables us to apply a continuum mechanics to the so-called "vacuum" of space. Minami proposed a hypothesis for mechanical property of space-time in 1988 [1]. A primary motive was to research in the realm of space propulsion theory. His propulsion principle using the substantial physical structure of space-time is based on this hypothesis [1-7].

In this paper, a fundamental concept of space-time that focuses on theoretically innate properties of space including strain and curvature is described. Assuming that space as vacuum is an infinite continuum, space can be considered as a kind of transparent elastic field. That is, space as a vacuum performs the motions of deformation such as expansion, contraction, elongation, torsion and bending. The latest expanding universe theories (Friedmann, de Sitter, inflationary cosmological model) support this assumption. Space can be regarded as an elastic body like rubber. This conveniently coincides with the precondition of a mechanical structure of space.

General Relativity implies that space is curved by the existence of energy (mass energy or electromagnetic energy and etc.). General relativity is based on Riemannian geometry. If we admit this space curvature, space is assumed as an elastic body. According to continuum mechanics, the elastic body has the property of the motion of deformation such as expansion, contraction, elongation, torsion and bending.

When we make a comparison between the space on the Earth and outer space, although there seems to be no difference, obviously a different phenomenon occurs. Simply put, an object moves radially inward, that is, drops straight down on the Earth, but in the outer space, the object floats and does not move.

The difference between the two phenomena can be explained by whether space is curved or not, that is, whether 20 independent components of a Riemann curvature tensor is zero or not. In essence, the existence of spatial curvature and curved extent region determine whether the object drops straight down or not. Although the spatial curvature at the surface of the Earth is very small value, i.e., $1.71\times10^{-23}(1/m^2)$, it is enough value to produce 1G (9.8 m/s^2) acceleration. Conversely, the spatial curvature in the universe is zero, therefore any acceleration is not produced. Accordingly, if the spatial curvature of a localized area containing object is controlled to curvature of $1.71\times10^{-23}(1/m^2)$ with a sufficiently large curved space area, the object moves and receives 1G acceleration in the universe. Of course, we are required to control both the magnitude of the curvature and the size of the curved space area.

The following Chapter 2 explains the space curvature and induced acceleration, and Chapter 3 briefly introduces the concept of gravitational mechanism [8-12].

2. Spatial Curvature and Derived Gravitational Acceleration

2.1 Overview of the Linear Approximation of Weak Static Gravitational Fields

The acceleration α and major curvature R^{00} are given by

$$R^{00} = \frac{1}{2} g^{ij} h_{00,ij} \, , \alpha = c^2 \Gamma^i{}_{00} = \frac{1}{2} c^2 h_{00,i} \quad . \tag{2.1}$$

respectively from the weak field approximation of the gravitational field equation.

Here, h_{00} is deviation between metric tensor g_{00} of curved space and Minkowski metric tensor η_{00} of flat space, that is,

$$g_{00} = \eta_{00} + h_{00} = -1 + h_{00} \quad . \tag{2.2}$$

The notation of the symbol is as follows:

$$h_{00,ij} = \partial_i \partial_j h_{00} = \frac{\partial h_{00}}{\partial x^i \partial x^j} \quad . \tag{2.3}$$

As is well known, the partial derivative $u_{i,j} = \partial_j u_i = \frac{\partial u_i}{\partial x^j}$ is not tensor equation. The covariant derivative $u_{i:j} = u_{i,j} - u_k \Gamma^k_{ij}$ is tensor equation and can be carried over into all coordinate systems.

If the gravitational field is time-invariant, or static, and the gravitational field is not very strong, Ricci tensor $R_{\mu\nu}$ is given by:

$$R_{\mu\nu} = \frac{1}{2}\left(\Box h_{\mu\nu} + h_{,\mu\nu} - h_{\mu\rho,\rho\nu} - h^{\rho}_{\nu,\mu\rho}\right) = \frac{1}{2}\left(\Box h_{\mu\nu} + \partial_\mu \partial_\nu h - \partial_\rho \partial_\nu h_{\mu\rho} - \partial_\mu \partial_\rho h^{\rho}_{\nu}\right) \; ,$$

where $\Box = \eta^{\mu\nu}\partial_\mu \partial_\nu = \nabla^2 - (\partial_0)^2$. (2.4)

Since all are static $h_{\mu\nu,0} = 0$ ($\partial_0 h_{\mu\nu} = 0$) and now setting μ = ν = 0, this component R_{00} is obtained

$$R_{00} = \frac{1}{2}\left(\nabla^2 h_{00} - (h_{00,0})^2 + h_{,00} - h_{0\rho,\rho 0} - h^{\rho}_{0,0\rho}\right) = \frac{1}{2}\nabla^2 h_{00} \quad . \tag{2.5}$$

On the other hand,

$$g_{00} = \eta_{00} + h_{00} = -1 + h_{00} = -1 - \frac{2}{c^2}\phi \quad . \tag{2.6}$$

As is well known, potential ϕ is

$$\phi = -\frac{GM}{R} \quad , \tag{2.7}$$

where M is the Earth mass, R is the Earth radius, G is the Gravity constant.

We get, $h_{00} = -\frac{2}{c^2}\phi = -\frac{2}{c^2} \times -\frac{GM}{R} = \frac{2GM}{c^2 R}$. (2.8)

Then,

$$R_{00} = \frac{1}{2}\nabla^2 h_{00} = \frac{1}{2}\nabla^2\left(-\frac{2\phi}{c^2}\right) = -\frac{1}{c^2}\nabla^2\phi \quad . \tag{2.9}$$

Curvature R_{00} can be described by the following approximation:

$$R_{00} = \frac{1}{2}\nabla^2 h_{00} = \frac{1}{2}\left(\frac{\partial^2}{\partial x^2}h_{00} + \frac{\partial^2}{\partial y^2}h_{00} + \frac{\partial^2}{\partial z^2}h_{00}\right) \approx \frac{1}{2}\frac{d^2 h_{00}}{dx^2} \approx \frac{1}{2}h_{00} \Big/ R^2 , \tag{2.10}$$

where x is toward the Earth center.

A similar result is obtained from Eq.(2.1) as:

$$R^{00} = 1/2 \cdot g^{ij} h_{00},_{ij} = 1/2 \cdot g^{33}\partial^2 h_{00} / \partial x^3 \partial x^3 = 1/2 \cdot g^{33}\partial^2 h_{00} / \partial x \partial x = 1/2 \cdot \partial^2 h_{00} / \partial r^2 \approx 1/2 \cdot h_{00} / R^2 \tag{2.11}$$

The approximate expression for gravitational acceleration is:

$$\alpha = \frac{1}{2}c^2 h_{00,i} = \frac{1}{2}c^2 h_{00,x} = \frac{1}{2}c^2\frac{dh_{00}}{dx} \approx \frac{1}{2}c^2 h_{00} \Big/ R \quad . \tag{2.12}$$

On the other hand, as will be described in detail later, the gravitational acceleration is also given by the following equation:

$$\alpha = \sqrt{-g_{00}}\, c^2 \int_a^b R^{00}(r)dr \quad . \tag{2.13}$$

Considering $g_{00} = -1$, substituting Eq.(2.10) or Eq.(2.11) into Eq.(2.13), we get

$$\alpha = c^2 \int_R^\infty \frac{1}{2}\frac{h_{00}}{r^2}dr = \frac{c^2}{2}\frac{h_{00}}{R} \quad . \tag{2.14}$$

Eq.(2.14) matches Eq.(2.12), and the equation of gravitational acceleration expressed by Eq.(2.13) gives the mechanism of gravitation. This physical concept becomes clear in the next section.

Further, major curvature of Ricci tensor ($\mu = \nu = 0$) is calculated as follows:

$$R^{00} = g^{00}g^{00}R_{00} = -1 \times -1 \times R_{00} = R_{00} \quad . \tag{2.15}$$

Here for convenience, raise the index and use it in the notation of R^{00} instead of R_{00}.

$$R^{00} = \frac{1}{2}g^{ij}h_{00,ij} = \frac{1}{c^2}g^{ij}\left(\frac{1}{2}c^2 h_{00,i}\right)_{,j} = \frac{1}{c^2}g^{ij}\alpha_{i,j} = \frac{1}{c^2}\alpha^j{}_{,j} \quad , \tag{2.16}$$

where $h_{00,ij} = \dfrac{\partial h_{00}}{\partial x^i \partial x^j}$.

2.2. Gravitational Acceleration on the Earth's Surface and Its Generation Mechanism

The accumulation of surface forces in a curved area of space from the Earth's surface R to the point at infinity (∞) gives gravitational acceleration on the Earth's surface.

$$\alpha = c^2\int_R^\infty R^{00}(r)dr = c^2\int_R^\infty \frac{1}{2}h_{00}\frac{1}{r^2}dr = -\frac{1}{2}c^2 h_{00}\left[\frac{1}{r}\right]_R^\infty = -\frac{1}{2}c^2 h_{00}(0-\frac{1}{R}) = \frac{1}{2}\frac{c^2 h_{00}}{R} \quad . \tag{2.17}$$

From Eq. (2.8), the deviation h_{00} of the metric tensor g_{00} from the flat space (η_{00} = -1) on the Earth's surface becomes:

$$h_{00} = \frac{2GM}{c^2 R} \quad . \tag{2.18}$$

The curvature of the space is (see Eq.2.10):

$$R_{00} = \left(\frac{1}{2}h_{00}\right) / R^2 \quad . \tag{2.19}$$

The gravitational acceleration is (see Eq.2.12):

$$\alpha = (\frac{1}{2}c^2 h_{00}) / R \quad . \tag{2.20}$$

Substitute the values of the Earth radius R=6.378×10^{3}km, GM=3.986×10^{5}km^3/s^2, c=3×10^{5}km, we get the following values respectively:

$$h_{00} = \frac{2GM}{c^2 R} = \frac{2\times 3.986\times 10^5}{(3\times 10^5)^2 \times 6.378\times 10^3} = \frac{2\times 3.986\times 10^5}{9\times 6.378\times 10^{13}} = 1.389\times 10^{-9}$$

$$R_{00} = \left(\frac{1}{2}h_{00}\right)\times\frac{1}{R^2} = \frac{1}{2}\times 1.389\times 10^{-9}\times\frac{1}{(6.378\times 10^3)^2} = 1.71\times 10^{-2}\times 10^{-9}\times 10^{-6}$$

$$= 1.71\times 10^{-17}(1/km)^2 = 1.71\times 10^{-23}(1/m^2)$$

$$\alpha = \left(\frac{1}{2}c^2 h_{00}\right)\times\frac{1}{R} = \frac{1}{2}\times(3\times 10^5)^2\times 1.389\times 10^{-9}\times\frac{1}{6.378\times 10^3}$$

$$= \frac{1}{2}\times 9\times 1.389\times\frac{1}{6.378}\times 10^{10}\times 10^{-9}\times 10^{-3} = 0.98\times 10^{-2}\,km/s^2 = 9.8m/s^2$$

In this way, the following approximate values can be obtained:

The amount of displacement of the space on the Earth's surface: $h_{00} = 1.389\times 10^{-9}$.

Curvature of space on the Earth's surface: $R_{00} = 1.71\times 10^{-23} / m^2$.

Gravitational acceleration on the Earth's surface: $\alpha = 9.8m/s^2$.

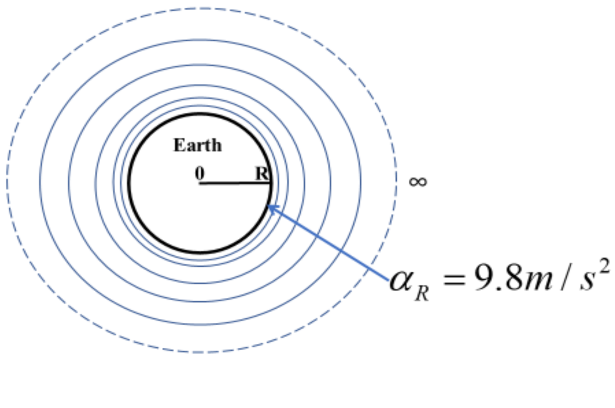
Next, a description will be given using the drawings.

Figure 2.1 shows a concentric curved space area around the Earth. Distance of radius R from the center of the Earth is the surface of the Earth.

At a distance from the Earth to an infinite point, the space becomes flat space without being affected by the gravitation of the Earth. The point at infinity is indicated by a symbol ∞ and a dotted line.

The accumulation of surface forces in a curved area of space from the Earth's surface R to the point at infinity (∞) gives gravitational acceleration on the Earth's surface, i.e., $\alpha_R = 9.8m/s^2$.

$$\alpha = c^2\int_R^\infty R^{00}(r)dr = c^2\int_R^\infty \frac{1}{2}h_{00}\frac{1}{r^2}dr = -\frac{1}{2}c^2 h_{00}\left[\frac{1}{r}\right]_R^\infty = -\frac{1}{2}c^2 h_{00}(0-\frac{1}{R}) = \frac{1}{2}\frac{c^2 h_{00}}{R} \tag{2.21}$$



$$\alpha = c^2\int_R^\infty R^{00}(r)dr = c^2\int_R^\infty \frac{1}{2}h_{00}\frac{1}{r^2}dr = -\frac{1}{2}c^2 h_{00}\left[\frac{1}{r}\right]_R^\infty = -\frac{1}{2}c^2 h_{00}(0-\frac{1}{R}) = \frac{1}{2}\frac{c^2 h_{00}}{R}$$

Figure 2.1: Mechanism of gravitational acceleration generation on the Earth's surface

As well, Figure 2.2 shows a concentric curved space area around the Earth. Distance of radius R from the center of the Earth is the surface of the Earth. Here consider the gravitational acceleration at a height **h** away from the Earth's surface.

At a distance from the Earth to an infinite point, the space becomes flat space without being affected by the gravitation of the Earth. The point at infinity is indicated by a symbol ∞ and a dotted line.

The accumulation of surface forces in a curved area of space from the Earth's surface R+**h** to the point at infinity (∞) gives gravitational acceleration at the Earth's height **h**, i.e., $\alpha_{R+h} < \alpha_R = 9.8m/s^2$.

$$\alpha = c^2 \int_{R+h}^{\infty} R^{00}(r)dr = c^2 \int_{R+h}^{\infty} \frac{1}{2} h_{00} \frac{1}{r^2} dr = -\frac{1}{2} c^2 h_{00} \left[\frac{1}{r}\right]_{R+h}^{\infty} = -\frac{1}{2} c^2 h_{00} (0 - \frac{1}{R+h}) = \frac{1}{2} \frac{c^2 h_{00}}{(R+h)}$$

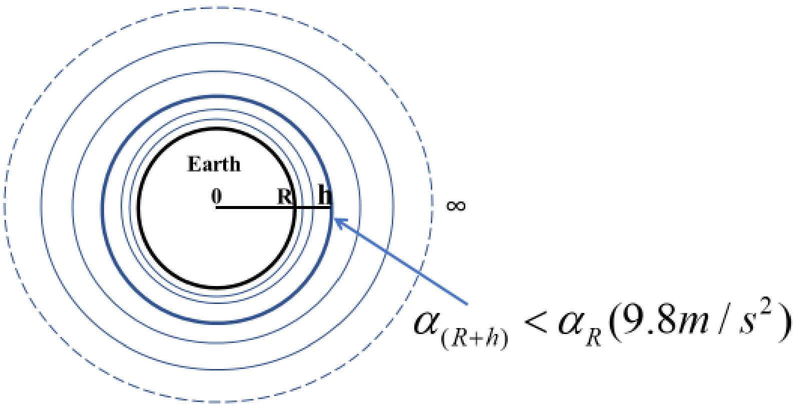


Figure 2.2: Mechanism of gravitational acceleration generation on the Earth's surface height h

Next, consider the universal gravitational force of a well-known apple falling to the Earth.

Although the attraction between the Earth and the apple by universal gravitation can be explained by a mathematical formula, $F = G\frac{Mm}{r^2}$, there is no explanation of the mechanism of the attraction, that is, the principle of operation.

The mechanism can be understood by interpreting that the Earth and the apple are pushed toward each other from behind the curved space area around the Earth and the curved space area around the apple.

A phenomenon is that an apple is not pulled and falls by the Earth, but the apple is pushed toward the Earth under the pressure of the vast curved space area of the Earth.

Figure 2.3 shows the mechanism.

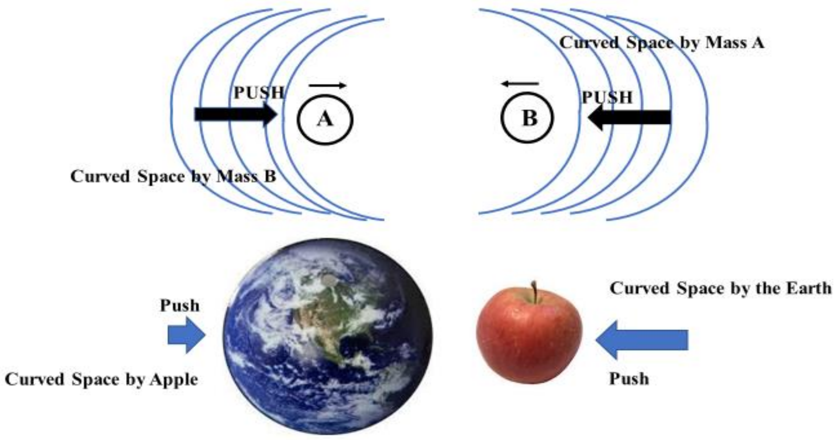
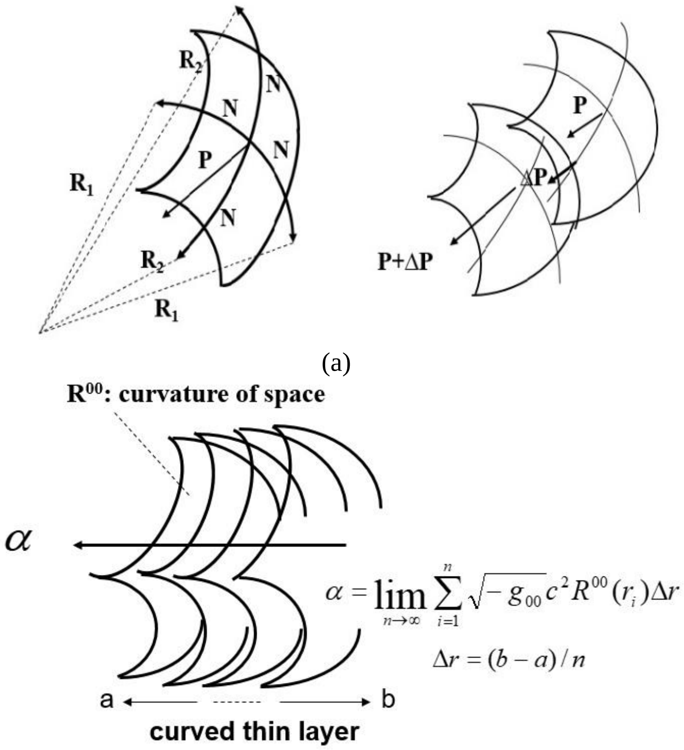


Figure 2.3: Apple and the Earth are pushed out of a curved space and collide

Apple is pushed from the vast curved space area of the Earth and go straight to the Earth. On the other hand, the Earth is also pushed from the narrow curved space area of the apple and go straight to the apple. Since the mass

of an apple is smaller than that of the Earth, the range of the curved space is small and the acceleration with respect to the Earth is almost zero.
In effect, it looks like an apple is pulled by the Earth and falls.
Concerning the detail theory, refer to the appendix A and B for an explanation of why curved space and its curved space regions generate acceleration fields.
APPENDIX A: Generation of Surface Force Induced by Spatial Curvature, APPENDIX B: Acceleration induced by Spatial Curvature.

2.3. Brief Concept of Pressure Field for Gravitational Acceleration
However, here is a brief explanation of the concept using APPENDIX A.
Figure 2.4 shows the fundamental principle of curved space.
On the supposition that space is an infinite continuum, continuum mechanics can be applied to the so-called "vacuum" of space. This means that space can be considered as a kind of transparent field with elastic properties. If space curves, then an inward normal stress "$-P$" is generated. This normal stress, i.e. surface force serves as a sort of pressure field.



(a)

(b)

Figure 2.4: Curvature of Space: (a)curvature of space plays a significant role. If space curves, then inward stress (surface force) "P" is generated $\Longrightarrow$ A sort of pressure field; (b) a large number of curved thin layers form the unidirectional surface force, i.e. acceleration field α .

$$-P = N \cdot (2R^{00})^{1/2} = N \cdot (1/R_1 + 1/R_2), \tag{2.22}$$

where N is the line stress, R_1, R_2 are the radius of principal curvature of curved surface, and R^{00} is the major component of spatial curvature.
A large number of curved thin layers form the unidirectional surface force, i.e. acceleration field. Accordingly, the spatial curvature R^{00} produces the acceleration field α .

The fundamental three-dimensional space structure is determined by quadratic surface structure. Therefore, a Gaussian curvature K in two-dimensional Riemann space is significant. The relationship between K and the major component of spatial curvature R^{00} is given by:

$$K = \frac{R_{1212}}{(g_{11}g_{22} - g_{12}{}^{2})} = \frac{1}{2} \cdot R^{00}, \tag{2.23}$$

where R_{1212} is non-zero component of Riemann curvature tensor.

It is now understood that the membrane force on the curved surface and each principal curvature generates the normal stress"–*P*" with its direction normal to the curved surface as a surface force. The normal stress "–*P*"acts towards the inside of the surface as shown in Figure 2.4 (a).

A thin-layer of curved surface will take into consideration within a spherical space having a radius of R and the principal radii of curvature that are equal to the radius ($R_1=R_2=R$).Since the membrane force N (serving as the line stress) can be assumed to have a constant value, Eq.(2.22) indicates that the curvature R^{00} generates the inward normal stress P of the curved surface. The inwardly directed normal stress serves as a pressure field.

When the curved surfaces are included in a great number, some type of unidirectional pressure field is formed. A region of curved space is made of a large number of curved surfaces and they form the field as a unidirectional surface force (i.e. normal stress). Since the field of the surface force is the field of a kind of force, the force accelerates matter in the field, i.e. we can regard the field of the surface force as the acceleration field. A large number of curved thin layers form the unidirectional acceleration field (Figure 2.4 (b)). Accordingly, the spatial curvature R^{00} produces the acceleration field α .Therefore, the curvature of space plays a significant role to generate pressure field.

For example, consider a soap bubble.

The pressure P due to the membrane force on the surface of a soap bubble of radius R is directed inward. The membrane force on the surface of the soap bubble corresponds to N in the Figure 2.4 (a).

$$-P = N \cdot (1/R_1 + 1/R_2) = N \cdot (1/R + 1/R) = 2N/R \quad . \tag{2.24}$$

This pressure "P" keeps the soap bubbles from breaking due to the expansion force of the internal air.

Refer to the **APPENDIX A: Generation of Surface Force Induced by Spatial Curvature** in detail.

3. Consideration of Gravitational Mechanism

Let us consider about gravitation. Why does apple fall in the Earth? A well-known answer is that there exists gravitation between Earth and apple. Apple is because it's pulled by a law of universal gravitation $F = G\dfrac{Mm}{r^2}$ to the Earth. Here, M is the mass of the Earth, m is the mass of apple, G is the gravitational constant, r is the distance between the Earth and apple, F is the gravitational force. From a phenomenological standpoint, it is a sufficient explanation.

However, what is the mechanism? According to General Relativity, it is said that apple moves geodesic line formed by curved space near the Earth. This is seen as lacking in sufficient explanation. The following explanation may allow someone to understand the mechanism of gravitation.

If we were to visualize the curvature of space around the Earth (M), we would describe it as having an aggregation of curved surface. A great number of thin curved surfaces are arranged in a spherical concentric pattern. This curvature would gradually become smaller as we moved away from the Earth in what we could imagine as layers of an onion. The surrounding space becomes a flat space of curvature of 0 at an imagined immense distance from the Earth (Figure 3.1).

In the following thought experiment, an apple of mass m positioned at a distance r apart from the Earth would receive a pressure of the field formed by an accumulation of the normal stress (Figure3.1). As was described earlier, with reference to Figure 2.4, the membrane force on the curved surface and each principal curvature generates the normal stress"–*P*" with its direction normal to the curved surface as a surface force. The normal stress "–*P*" acts towards the inside of the surface as shown in Figure 2.4 (a).

A thin-layer of curved surface will take into consideration within a spherical space having a radius of R and the principal radii of curvature that are equal to the radius (R_1=R_2=R).Since the membrane force N (serving as the line stress) can be assumed to have a constant value, the inwardly directed normal stress serves as a pressure field. When the curved surfaces are included in a great number, some type of unidirectional pressure field is formed. That is, a sort of graduated pressure field is generated by the curved range from an imaginary point "a" in curved space to a point "b" (the point at which space is absent of curvature, i.e., flat space of curvature 0) (Figure 3.1). Then apple moves directly towards the center of the Earth, that is, the apple falls. Falling acceleration of apple in curved space is proportional to both the value of spatial curvature and the size of curved space.

If the Earth (M) were to disappear instantly, the curved surface of space close to the Earth would return to the flat surface. Because an external action causing curvature (i.e., mass energy) disappears. The change from a curved surface to a flat surface would advance the position r of the apple at the speed of light (i.e., the strain rate of space-time). The propagation velocity of the change from flat space to curved space and the propagation velocity of change from curved space to flat space are both the same, i.e. the velocity of light.

However, in our thought experiment, the apple would still receive pressure from the surrounding field by the accumulation of the normal stress.

Because, since there still exists the curved region behind the apple from a to b (the remote flat space), the apple continues falling. The pressure continues to push the apple to the center of the Earth (Figure 3.2).

However, as soon as the change from a curved surface to a flat surface passes through the point of the apple (i.e., "a" point), the pressure at point "a" disappears and the apple would only float without falling (or keep falling due to inertia)(Figure 3.3).

The above discussion provides a basis to consider the following thought experiment. Even if the Sun instantly disappeared, the Earth would still continue to revolve around the Sun until 8 minutes 32 seconds, or the time at which it takes light to advance between the Sun and Earth. However, as soon as the change from curved surface to flat surface passes through the point of the Earth, or at 8 minutes 32 seconds after the event, the pressure pushing the Earth would disappear, and the Earth would fly away in a direction tangential to the revolution orbit.

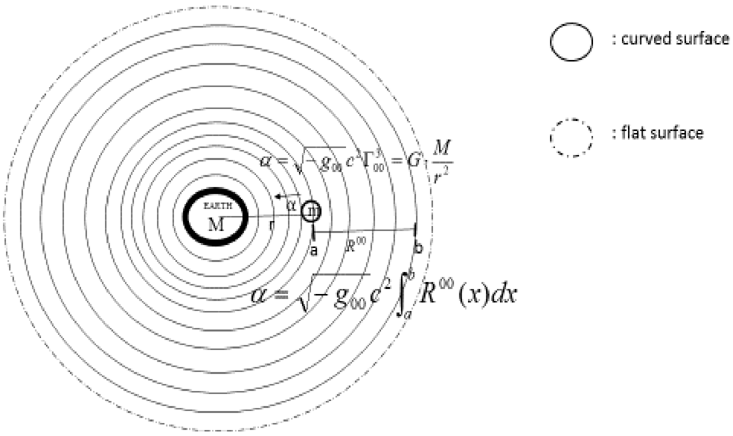


Figure 3.1: Apple falls receiving a pressure of the field

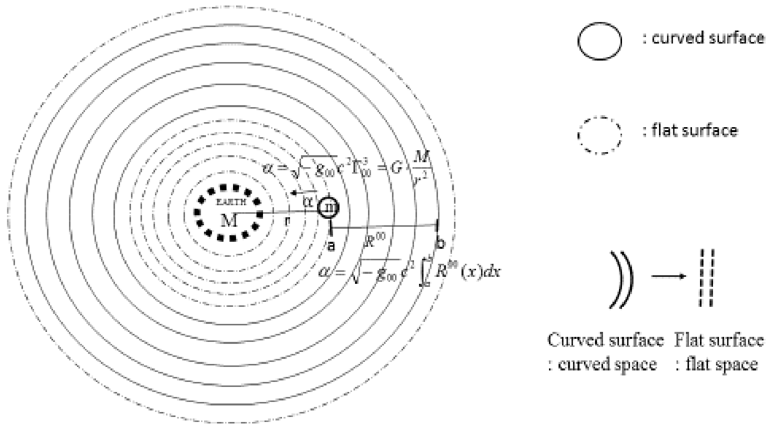


Figure 3.2: Apple still continues falling receiving a pressure of the field

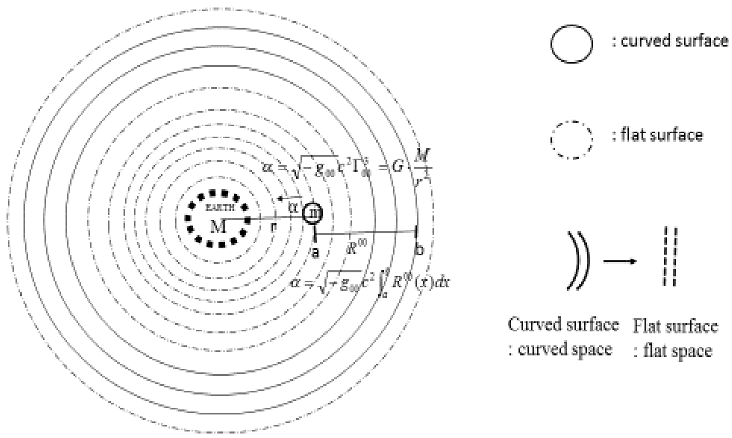


Figure 3.3: Apple only floats without falling due to the lack of pressure of the field

In view of this, gravitation may be considered as a pressure generated in a region of curved space.

4. Conclusion

Assuming that space is an infinite continuum, a mechanical concept of space became identified. Space can be considered as a kind of transparent elastic field. The pressure field derived from the geometrical structure of space is newly obtained by applying both continuum mechanics and General Relativity to space. As a result, a fundamental concept of space-time is described that focuses on theoretically innate properties of space including strain and curvature.

The mass on the Earth will not be pulled by the Earth and fall, but will be pushed and fall in the direction of the Earth due to the pressure of the field in the curved space area around the Earth. Although the spatial curvature at the surface of the Earth is very small value, i.e., $1.71 \times 10^{-23} (1/m^2)$, it is enough value to produce 1G (9.8 m/s^2) acceleration.

Gravitation can be explained as a pressure field induced by the curvature of space.

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Appendix A:

Generation of Surface Force Induced by Spatial Curvature

On the supposition that space is an infinite continuum, continuum mechanics can be applied to the so-called "vacuum" of space. This means that space can be considered as a kind of transparent field with elastic properties.

If space curves, then an inward normal stress "$-P$" is generated. This normal stress, i.e. surface force serves as a sort of pressure field.

$$-P = N \cdot (2R^{00})^{1/2} = N \cdot (1/R_1 + 1/R_2), \tag{A1}$$

where N is the line stress, R_1, R_2 are the radius of principal curvature of curved surface, and R^{00} is the major component of spatial curvature.

A large number of curved thin layers form the unidirectional surface force, i.e. acceleration field. Accordingly, the spatial curvature R^{00} produces the acceleration field α.

The fundamental three-dimensional space structure is determined by quadratic surface structure. Therefore, a Gaussian curvature K in two-dimensional Riemann space is significant. The relationship between K and the major component of spatial curvature R^{00} is given by:

$$K = \frac{R_{1212}}{(g_{11}g_{22} - g_{12}{}^2)} = \frac{1}{2} \cdot R^{00}, \tag{A2}$$

where R_{1212} is non-zero component of Riemann curvature tensor.

<Reference matters for Eq.(A2)>

For a two-dimensional surface, from the Bianchi identity, the Riemann curvature tensor is given by

$R_{\nu\mu\lambda\kappa} = K(g_{\nu\lambda}g_{\mu\kappa} - g_{\nu\kappa}g_{\mu\lambda})$, that is, $R_{1212} = K(g_{11}g_{22} - g_{12}{}^2)$.

And, for a spherical surface of radius r, its Gaussian curvature K is 1 / r^2.

The scalar curvature R and the Gaussian curvature K on the quadratic surface are as follows:

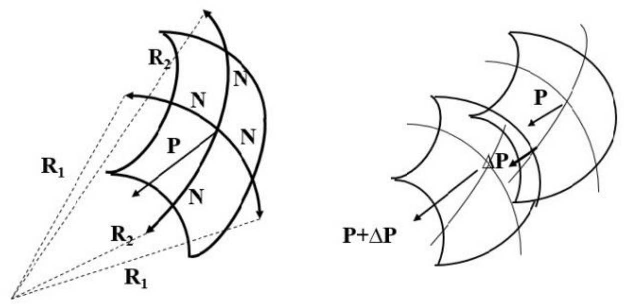
$$R = R_i{}^i = g^{ij}R_{ij} = g^{11}R_{11} + g^{22}R_{22} = \frac{1}{r^2}(-1) + \frac{1}{r^2\sin^2\theta}(-\sin^2\theta) = -\frac{2}{r^2} = -2K$$

The scalar curvature R (1/m^2) on a four-dimensional surface is given by

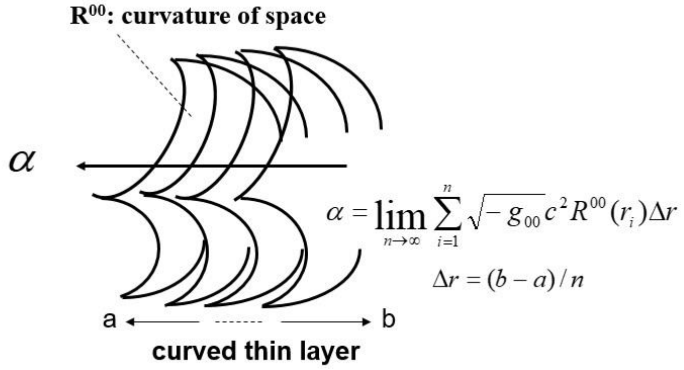
$$R = R_i^{\ i} = g_{ij}R^{ij} = g_{00}R^{00} + g_{11}R^{11} + g_{22}R^{22} + g_{33}R^{33} \approx g_{00}R^{00} = -R^{00}\ (g_{00} \approx -1:\ weak\ field)$$

Thus $K = \frac{1}{2} \cdot R^{00}$ is obtained.

It is now understood that the membrane force on the curved surface and each principal curvature generates the normal stress"–*P*" with its direction normal to the curved surface as a surface force. The normal stress "–*P*" acts towards the inside of the surface as shown in Figure A1 (a).
A thin-layer of curved surface will take into consideration within a spherical space having a radius of *R* and the principal radii of curvature that are equal to the radius (R_1=R_2=R).Since the membrane force *N* (serving as the line stress) can be assumed to have a constant value, Eq. (A1) indicates that the curvature R^{00} generates the inward normal stress *P* of the curved surface. The inwardly directed normal stress serves as a pressure field.
When the curved surfaces are included in a great number, some type of unidirectional pressure field is formed. A region of curved space is made of a large number of curved surfaces and they form the field as a unidirectional surface force (i.e. normal stress). Since the field of the surface force is the field of a kind of force, the force accelerates matter in the field, i.e. we can regard the field of the surface force as the acceleration field. A large number of curved thin layers form the unidirectional acceleration field (Figure A1 (b)). Accordingly, the spatial curvature R^{00} produces the acceleration field α .Therefore, the curvature of space plays a significant role to generate pressure field.



(a)



(b)

Figure A1: Curvature of Space: (a) curvature of space plays a significant role. If space curves, then inward stress (surface force)"P" is generated$\Longrightarrow$ A sort of pressure field;(b) a large number of curved thin layers form the unidirectional surface force, i.e. acceleration field α .

Applying membrane theory, the following equilibrium conditions are obtained in quadratic surface, given by:

$$N^{\alpha\beta} b_{\alpha\beta} + P = 0, \tag{A3}$$

where $N^{\alpha\beta}$ is a membrane force, i.e. line stress of curved space, $b_{\alpha\beta}$ is second fundamental metric of curved surface, and P is the normal stress on curved surface [8].
The second fundamental metric of curved space $b_{\alpha\beta}$ and principal curvature $K_{(i)}$ has the following relationship using the metric tensor $g_{\alpha\beta}$,

$$b_{\alpha\beta} = K_{(i)} g_{\alpha\beta} . \tag{A4}$$

Therefore we get:

$$N^{\alpha\beta} b_{\alpha\beta} = N^{\alpha\beta} K_{(i)} g_{\alpha\beta} = g_{\alpha\beta} N^{\alpha\beta} K_{(i)} = N_{\alpha}{}^{\alpha} K_{(i)} = N \cdot K_{(i)} . \tag{A5}$$

From Eq. (A3) and Eq. (A5), we get:

$$N_{\alpha}{}^{\alpha} K_{(i)} = -P . \tag{A6}$$

As for the quadratic surface, the indices α and i take two different values, i.e. 1 and 2, therefore Eq. (A6) becomes:

$$N_1{}^1 K_{(1)} + N_2{}^2 K_{(2)} = -P , \tag{A7}$$

where $K_{(1)}$ and $K_{(2)}$ are principal curvature of curved surface and are inverse number of radius of principal curvature (i.e. $1/R_1$ and $1/R_2$).
The Gaussian curvature K is represented as:

$$K = K_{(1)} \cdot K_{(2)} = (1/R_1) \cdot (1/R_2) . \tag{A8}$$

Accordingly, suppose $N_1{}^1 = N_2{}^2 = N$, we get:

$$N \cdot (1/R_1 + 1/R_2) = -P . \tag{A9}$$

It is now understood that the membrane force on the curved surface and each principal curvature generate the normal stress"–*P*" with its direction normal to the curved surface as a surface force. The normal stress "–*P*" is towards the inside of surface as showing in Figure A1 (a).
A thin-layer of curved surface will be taken into consideration within a spherical space having a radius of R and the principal radii of curvature which are equal to the radius ($R_1=R_2=R$). From Eqs. (A2) and (A8), we then get:

$$K = \frac{1}{R_1} \cdot \frac{1}{R_2} = \frac{1}{R^2} = \frac{R^{00}}{2} . \tag{A10}$$

Considering $N \cdot (2/R) = -P$ of Eq.(A9), and substituting Eq.(A10) into Eq.(A9), the following equation is obtained:

$$-P = N \cdot \sqrt{2R^{00}} . \tag{A11}$$

Since the membrane force *N* (serving as the line stress) can be assumed to have a constant value, Eq. (A11) indicates that the curvature R^{00} generates the inward normal stress *P* of the curved surface. The inwardly directed normal stress serves as a kind of pressure field. Accordingly, the cumulated curved region of curvature R^{00} produces the acceleration field α.

Here, we give an account of curvature R^{00} in advance. The solution of metric tensor $g^{\mu\nu}$ is found by gravitational field equation as the following:

$$R^{\mu\nu} - \frac{1}{2} \cdot g^{\mu\nu} R = -\frac{8\pi G}{c^4} \cdot T^{\mu\nu} , \tag{A12}$$

where $R^{\mu\nu}$ is the Ricci tensor, R is the scalar curvature, *G* is the gravitational constant, *c* is the velocity of light, $T^{\mu\nu}$ is the energy momentum tensor.
Furthermore, we have the following relation for scalar curvature R :

$$R = R^{\alpha}{}_{\alpha} = g^{\alpha\beta} R_{\alpha\beta}, \; R^{\mu\nu} = g^{\mu\alpha} g^{\nu\beta} R_{\alpha\beta}, \; R_{\alpha\beta} = R^{j}{}_{\alpha j\beta} = g^{ij} R_{i\alpha j\beta} . \tag{A13}$$

Ricci tensor $R_{\mu\nu}$ is also represented by:

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\alpha,\nu} - \Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta} + \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} \quad (= R_{\nu\mu}) , \tag{A14}$$

where $\Gamma^{i}{}_{jk}$ is Riemannian connection coefficient.

If the curvature of space is very small, the term of higher order than the second can be neglected, and Ricci tensor becomes:

$$R_{\mu\nu} = \Gamma^{\alpha}_{\mu\alpha,\nu} - \Gamma^{\alpha}_{\mu\nu,\alpha} . \tag{A15}$$

The major curvature of Ricci tensor ($\mu = \nu = 0$) is calculated as follows:

$$R^{00} = g^{00} g^{00} R_{00} = -1 \times -1 \times R_{00} = R_{00} . \tag{A16}$$

As previously mentioned, Riemannian geometry is a geometry that deals with a curved Riemann space, therefore a Riemann curvature tensor is the principal quantity. All components of Riemann curvature tensor are zero for flat space and non-zero for curved space. If an only non-zero component of Riemann curvature tensor exists, the space is not flat space but curved space.Therefore, the curvature of space plays a significant role.

APPENDIX B:

Acceleration Induced by Spatial Curvature

A massive body causes the curvature of space-time around it, and a free particle responds by moving along a geodesic line in that space-time. The path of free particle is a geodesic line in space-time and is given by the following geodesic equation;

$$\frac{d^2 x^i}{d\tau^2} + \Gamma^{i}_{jk} \cdot \frac{dx^j}{d\tau} \cdot \frac{dx^k}{d\tau} = 0 , \tag{B1}$$

where $\Gamma^{i}{}_{jk}$ is Riemannian connection coefficient, τ is proper time, x^i is four-dimensional Riemann space, that is, three dimensional space ($x=x^1$, $y=x^2$, $z=x^3$) and one dimensional time ($w=ct=x^0$), where c is the velocity of light. These four coordinate axes are denoted as x^i (i=0, 1, 2, 3).

Proper time is the time to be measured in a clock resting for a coordinate system. We have the following relation derived from an invariant line element ds^2 between Special Relativity (flat space) and General Relativity (curved space):

$$d\tau = \sqrt{-g_{00}}\, dx^0 = \sqrt{-g_{00}}\, cdt . \tag{B2}$$

From Eq.(B1), the acceleration of free particle is obtained by

$$\alpha^i = \frac{d^2 x^i}{d\tau^2} = -\Gamma^{i}_{jk} \cdot \frac{dx^j}{d\tau} \cdot \frac{dx^k}{d\tau} . \tag{B3}$$

As is well known in General Relativity, in the curved space region, the massive body "*m* (kg)" existing in the acceleration field is subjected to the following force F^i(N) :

$$F^i = m\Gamma^{i}_{jk} \cdot \frac{dx^j}{d\tau} \cdot \frac{dx^k}{d\tau} = m\sqrt{-g_{00}}\, c^2 \Gamma^{i}_{jk} u^j u^k = m\alpha^i , \tag{B4}$$

where u^j,u^k are the four velocity, $\Gamma^{i}{}_{jk}$ is the Riemannian connection coefficient, and τ is the proper time.

From Eqs.(B3),(B4), we obtain:

$$\alpha^i = \frac{d^2 x^i}{d\tau^2} = -\Gamma^{i}_{jk} \cdot \frac{dx^j}{d\tau} \cdot \frac{dx^k}{d\tau} = -\sqrt{-g_{00}}\, c^2 \Gamma^{i}{}_{jk} u^j u^k . \tag{B5}$$

Eq.(B5) yields a more simple equation from the condition of linear approximation, that is, weak-field, quasi-static, and slow motion (speed v << speed of light *c*: $u^0 \approx 1$):

$$\alpha^i = -\sqrt{-g_{00}} \cdot c^2 \Gamma^{i}_{00} . \tag{B6}$$

On the other hand, the major component of spatial curvature R^{00} in the weak field is given by

$$R^{00} \approx R_{00} = R^{\mu}_{0\mu 0} = \partial_0 \Gamma^{\mu}_{0\mu} - \partial_{\mu} \Gamma^{\mu}_{00} + \Gamma^{\nu}_{0\mu} \Gamma^{\mu}_{\nu 0} - \Gamma^{\nu}_{00} \Gamma^{\mu}_{\nu\mu} \quad . \tag{B7}$$

In the nearly Cartesian coordinate system, the value of $\Gamma^{\mu}_{\nu\rho}$ are small, so we can neglect the last two terms in Eq.(B7), and using the quasi-static condition we get

$$R^{00} = -\partial_{\mu} \Gamma^{\mu}_{00} = -\partial_i \Gamma^{i}_{00} \, . \tag{B8}$$

From Eq.(B8), we get formally

$$\Gamma^{i}_{00} = -\int R^{00}(x^i) dx^i \ . \tag{B9}$$

Substituting Eq.(B9) into Eq.(B6), we obtain

$$\alpha^i = \sqrt{-g_{00}}\, c^2 \int R^{00}(x^i) dx^i \ . \tag{B10}$$

Accordingly, from the following linear approximation scheme for the gravitational field equation:(1) weak gravitational field, i.e. small curvature limit, (2) quasi-static, (3) slow-motion approximation (i.e., $v/c << 1$), and considering range of curved region, we get the following relation between acceleration of curved space and curvature of space:

$$\alpha^i = \sqrt{-g_{00}}\, c^2 \int_a^b R^{00}(x^i) dx^i \ , \tag{B11}$$

where α^i : acceleration (m/s^2), g_{00}: time component of metric tensor, a-b: range of curved space (m), x^i: components of coordinate (i=0,1,2,3), c: velocity of light, R^{00}: major component of spatial curvature(1/m^2).

Eq.(B11) indicates that the acceleration field α^i is produced in curved space. The intensity of acceleration produced in curved space is proportional to the product of spatial curvature R^{00} and the length of curved region.

Eq.(B4) yields more simple equation from above-stated linear approximation ($u^0 \approx 1$),

$$F^i = m\sqrt{-g_{00}}\, c^2 \Gamma^i_{00} u^0 u^0 = m\sqrt{-g_{00}}\, c^2 \Gamma^i_{00} = m\alpha^i = m\sqrt{-g_{00}}\, c^2 \int_a^b R^{00}(x^i) dx^i \ . \tag{B12}$$

Setting i=3(i.e., direction of radius of curvature: r), we get Newton's second law:

$$F^3 = F = m\alpha = m\sqrt{-g_{00}}\, c^2 \int_a^b R^{00}(r) dr = m\sqrt{-g_{00}}\, c^2 \Gamma^3_{00} \ . \tag{B13}$$

The acceleration (α) of curved space and its Riemannian connection coefficient (Γ^3_{00}) are given by:

$$\alpha = \sqrt{-g_{00}}\, c^2 \Gamma^3_{00} \ , \qquad \Gamma^3_{00} = \frac{-g_{00,3}}{2g_{33}} , \tag{B14}$$

where c: velocity of light, g_{00} and g_{33}: component of metric tensor, $g_{00,3}$: $\partial g_{00}/\partial x^3 = \partial g_{00}/\partial r$. We choose the spherical coordinates "$ct=x^0$, $r=x^3$, $\theta=x^1$, $\varphi=x^2$" in space-time. The acceleration α is represented by the equation both in the differential form and in the integral form. Practically, since the metric is usually given by the solution of gravitational field equation, the differential form has been found to be advantageous.

Concerning Field Propulsion, see the references for theoretical formulas [1-7].

Now in general, the line element is described in:

$$\begin{aligned} ds^2 &= g_{ij} dx^i dx^j = g_{00}(dx^0)^2 + g_{33}(dx^3)^2 + g_{11}(dx^1)^2 + g_{22}(dx^2)^2 \\ &= g_{00}(cdt)^2 + g_{33}(dr)^2 + g_{11} r^2 (d\theta)^2 + g_{22} r^2 \sin^2\theta (d\varphi)^2 \end{aligned} \ . \tag{B15}$$

We choose the spherical coordinates "ct=x^0, r=x^3,θ=x^1, φ=x^2" in space-time.

Next, let us consider External Schwarzschild Solution.

The line element is obtained as follows:

$$ds^2 = -(1-\frac{r_g}{r})c^2dt^2 + \frac{1}{1-\frac{r_g}{r}}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\ . \tag{B16}$$

The metrics are given by:

$$\begin{aligned} &g_{00} = -(1-r_g/r), g_{11} = g_{22} = 1, g_{33} = 1/(1-r_g/r), \\ &and\ other\, g_{ij} = 0\ . \end{aligned} \tag{B17}$$

where r_g is the gravitational radius (i.e. $r_g = 2GM/c^2$).

Combining Eq.(B17) with Eq.(B14) yields:

$$\alpha = G\cdot\frac{M}{r^2}, (r_g \langle\ r) \quad , \tag{B18}$$

where G is a gravitational constant and M is a total mass.

Eq.(B18) indicates the gravitational acceleration of the well-known the Earth mass M.