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# Irregular Total Labelling of Möbius Ladder $M_n$

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**Abstract**The total edge irregularity strength $tes(G)$ and total vertex irregularity strength $tvs(G)$ are invariants analogous to irregular strength $s(G)$ of a graph $G$ for total labellings. Concerning of these parameters, Bača et al [Discrete Mathematics, 307:1378–1388, (2007)] determined the bounds and precise values for some families of graphs. In this paper, we show the exact values of the total edge irregularity strength is $tes(M_n) = n + 1$ and total vertex irregularity strength is $tvs(M_n) = \lceil\frac{n}{2}\rceil + 1$ for the Möbius Ladder $(M_n)$.

**Keywords** Irregular total labelling; Möbius Ladder; Total labelling; Irregularity Strength
**Mathematics Subject Classification:** 05C50, 05C78

## 1.Introduction

We consider only finite undirected graphs without loops or multiple edges. Let $G = (V, E)$ be a graph with vertex set $V$ and edge set $E$.

An edge irregular total $k$-labelling of a graph G is a labelling of the vertices and edges with labels 1, ..., $k$ that for every two different edges their weights are distinct where the weight of an edge is the sum of its label and the labels of its two endvertices. A vertex irregular total $k$-labelling of a graph G is a labelling of the vertices and edges with labels 1, ..., $k$ that for every two different vertices their weights are distinct where the weight of a vertex is the sum of its label and the labels of its incident edges. The minimum $k$ for which the graph $G$ has an edge irregular total $k$-labelling is called the total edge irregularity strength of the graph $G$, $tes(G)$. Analogously, the minimum $k$ for which there exists a vertex irregular total $k$-labelling is called the total vertex irregularity strength of $G$, $tvs(G)$.

The notions of the total edge irregularity strength and total vertex irregularity strength were first introduced by Bača et al. [1]. They may be taken as an extension of the irregularity strength of a graph [3, 2, 5, 7, 8, 10, 11]. In [1], the authors put forward the lower bounds of $tes(G)$ and $tvs(G)$ in terms of the maximum degree $\Delta$, minimum degree $\delta$, $|E(G)|$ and $|V(G)|$, which may be stated as the Theorem 1.1 and 1.2:

**Theorem 1.1.**$tes(G) \geq max\{\lceil\frac{\Delta+1}{2}\rceil, \lceil\frac{|E(G)|+2}{3}\rceil\}$.

**Theorem 1.2.**$tvs(G) \geq \lceil\frac{|V(G)|+\delta}{\Delta+1}\rceil$.

Bača et al. [1] then determined the exact values of the total edge irregularity strength for Path $P_n$, Cycle $C_n$, Star $S_n$, Wheel $W_n$ and friendship graph $F_n$, and obtained the exact values of the total vertex irregularity strength for Tree $T$ with $n$ pendant vertices and no vertex of degree 2, Star $S_n$, complete graphs $K_n$, cycle $C_n$ and prism $D_n$. The author proved that irregular total labellings of Generalized Petersen graphs $P(n, k)$ in [9]. We refer the readers for some recent result [3, 5, 8].

In this paper, we deal with the Möbius Ladder $M_n$.

The Möbius ladder $M_n$ is defined to be a graph on $2n(n \geq 3)$ vertices with $V(M_n) = \{v_i, u_i: 0 \leq i \leq n-1\}$ and $E(M_n) = \{v_i u_i: 0 \leq i \leq n-1\} \cup \{v_i v_{i+1}, u_i u_{i+1}: 0 \leq i \leq n-2\} \cup \{u_{n-1}v_0, v_{n-1}u_0\}$.

In Figure 1.1, we show the Möbius Ladder $M_5$.

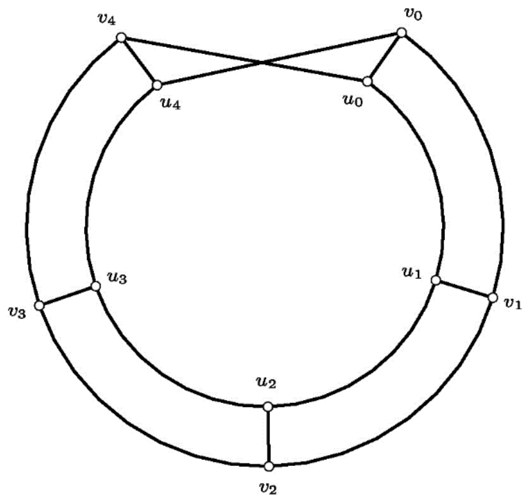


*Figure 1.1: Möbius Ladder $M_5$*

**2.Irregular total labelling of $M_n$**

**Theorem 2.1.** $tes(M_n) = n+1$.

*Proof.*

We construct the function $f$ as follows:

$$
\begin{aligned}
f(u_i) &= 1, && 0 \leq i \leq n-1,\\
f(v_i) &= \begin{cases} n, & i = 0,\\ n+1, & 1 \leq i \leq n-1,\end{cases}\\
f(u_i u_{i+1}) &= i+1, && 0 \leq i \leq n-2,\\
f(u_{n-1}v_0) &= 1,\\
f(v_{n-1}u_0) &= 1,\\
f(u_i v_i) &= \begin{cases} i+3, & i = 0,\\ i+2, & 1 \leq i \leq n-1,\end{cases}\\
f(v_i v_{i+1}) &= \begin{cases} i+3, & i = 0,\\ i+2, & 1 \leq i \leq n-2.\end{cases}
\end{aligned}
$$

Observe that

$$
\begin{aligned}
wt(u_i u_{i+1}) &= 1 + (i+1) + 1\\
&= \{3, 4, \ldots, n+1\}, && 0 \leq i \leq n-2,\\
wt(u_{n-1}v_0) &= 1 + 1 + n = n+2,\\
wt(v_{n-1}u_0) &= (n+1) + 1 + 1 = n+3,\\
wt(u_i v_i) &= \begin{cases} 1 + (i+3) + n = n+4, & i = 0,\\ 1 + (i+2) + (n+1) & \\ = \{n+5, n+6, \ldots, 2n+3\}, & 1 \leq i \leq n-1,\end{cases}\\
wt(v_i v_{i+1}) &= \begin{cases} n + (i+3) + (n+1) = 2n+4, & i = 0,\\ (n+1) + (i+2) + (n+1) & \\ = \{2n+5, 2n+6, \ldots, 3n+2\}, & 1 \leq i \leq n-2.\end{cases}
\end{aligned}
$$

So the weights of edges of $M_n$ under the labelling $f$ constitute the set {3, 4, ... , 3n+2} and the function $f$ is a map from $M_n \cup E(M_n)$ into $\{1,2,\dots,n+1\}$.

It is clearly that the total labellings $f$ have the required properties of an edge irregular total labelling. Hence $tes(M_n) \le n+1$. By Theorem 1.1, $tes(M_n) \ge \lceil\frac{|E(G)|+2}{3}\rceil = \lceil\frac{3n+2}{3}\rceil = n+1$. This concludes the proof.

In Figure 2.1, we show the edge irregular total labellings for $M_{10}$ .

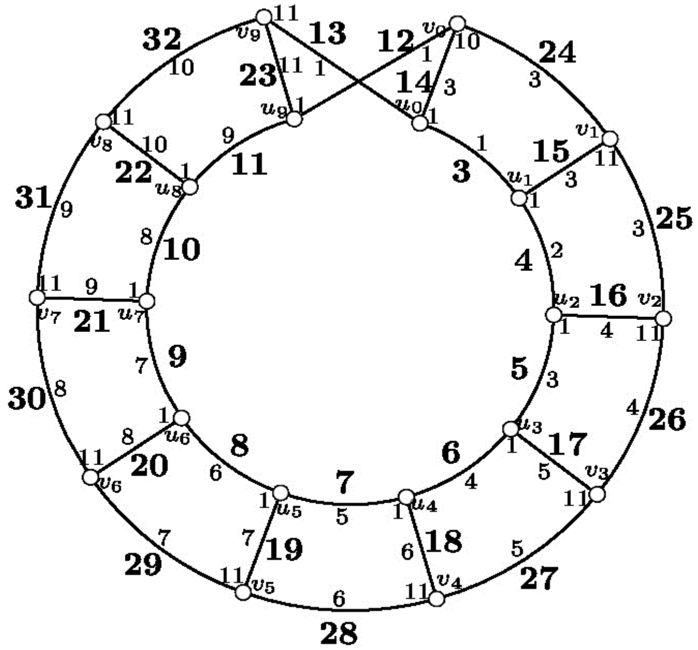


*Figure 2.1: Edge irregular total labellings for $M_{10}$*

**Theorem 2.2.** $tvs(M_n) = \lceil\frac{n}{2}\rceil + 1$.

*Proof.*

We construct the function $f$ as follows:

**Case** 1. $n mod 2 = 0$.

$$
\begin{aligned}
f(u_i u_{i+1}) &= \begin{cases} \frac{i}{2}+1, & 0 \le i \le n-2 and i mod 2 = 0, \\ 1, & 1 \le i \le n-3 and i mod 2 = 1, \end{cases} \\
f(u_{n-1} v_0) &= 1, \\
f(v_i v_{i+1}) &= \begin{cases} \frac{i}{2}+2, & 0 \le i \le n-2 and i mod 2 = 0, \\ \frac{n}{2}+1, & 1 \le i \le n-3 and i mod 2 = 1, \end{cases} \\
f(v_{n-1} u_0) &= \frac{n}{2}+1, \\
f(u_i v_i) &= \begin{cases} \frac{n}{2}, & i = 0 \\ \frac{i}{2}, & 2 \le i \le n-2 and i mod 2 = 0, \\ \frac{i+1}{2}, & 1 \le i \le n-1 and i mod 2 = 1, \end{cases} \\
f(u_i) &= 1, \qquad 0 \le i \le n-1, \\
f(v_i) &= \frac{n}{2}+1, \qquad 0 \le i \le n-1.
\end{aligned}
$$

Observe that

$$wt(u_i)=\begin{cases}(\frac{n}{2}+1)+\frac{n}{2}+(\frac{i}{2}+1)+1=n+3, & i=0,\\ 1+\frac{i}{2}+(\frac{i}{2}+1)+1 & \\ =\{5,7,\dots,n+1\}, & 2\le i\le n-2 andimod2=0,\\ (\frac{i-1}{2}+1)+\frac{i+1}{2}+1+1 & \\ =\{4,6,\dots,n+2\}, & 1\le i\le n-1 andimod2=1,\end{cases}$$

$$wt(v_i)=\begin{cases}1+\frac{n}{2}+(\frac{i}{2}+2)+(\frac{n}{2}+1) & \\ =n+4, & i=0,\\ (\frac{n}{2}+1)+\frac{i}{2}+(\frac{i}{2}+2)+(\frac{n}{2}+1) & \\ =\{n+6,n+8,\dots,2n+2\}, & 2\le i\le n-2 andimod2=0,\\ (\frac{i-1}{2}+2)+\frac{i+1}{2}+(\frac{n}{2}+1)+(\frac{n}{2}+1) & \\ =\{n+5,n+6,\dots,2n+3\}, & 1\le i\le n-1 andimod2=1.\end{cases}$$

So the weights of vertices of $M_n$ under the labeling $f$ constitute the set {4, 5, ... , 2n+3} and the function $f$ is a map from $V(M_n)\cup E(M_n)$ into $\{1,2,\dots,\frac{n}{2}+1\}$ for even $n$.

**Case** 2. $nmod2=1$.

$$f(u_iu_{i+1}) = \begin{cases}\frac{i}{2}+1, & 0\le i\le n-3 andimod2=0,\\ 1, & 1\le i\le n-2 andimod2=1,\end{cases}$$

$$f(u_{n-1}v_0) = \frac{n-1}{2}+1=\lceil\frac{n}{2}\rceil,$$

$$f(v_iv_{i+1}) = \begin{cases}\frac{i}{2}+1, & 0\le i\le n-3 andimod2=0,\\ \lceil\frac{n}{2}\rceil+1, & 1\le i\le n-2 andimod2=1,\end{cases}$$

$$f(v_{n-1}u_0) = \frac{n-1}{2}+1=\lceil\frac{n}{2}\rceil,$$

$$f(u_iv_i) = \begin{cases}1, & i=0\\ \frac{i}{2}, & 2\le i\le n-1 andimod2=0,\\ \frac{i+1}{2}, & 1\le i\le n-1 andimod2=1,\end{cases}$$

$$f(u_i) = \begin{cases}\lceil\frac{n}{2}\rceil, & i=0,\\ 1, & 1\le i\le n-1,\end{cases}$$

$$f(v_i) = \lceil\frac{n}{2}\rceil+1, \qquad 0\le i\le n-1.$$

Observe that

$$wt(u_i)=\begin{cases} \lceil\frac{n}{2}\rceil+1+(\frac{i}{2}+1)+\lceil\frac{n}{2}\rceil=n+3, & i=0,\\ 1+\frac{i}{2}+(\frac{i}{2}+1)+1 & \\ =\{5,7,\ldots,n+2\}, & 2\le i\le n-1 and imod2=0,\\ (\frac{i-1}{2}+1)+\frac{i+1}{2}+1+1 & \\ =\{4,6,\ldots,n+1\}, & 1\le i\le n-2 and imod2=1,\end{cases}$$

$$wt(v_i)=\begin{cases} \lceil\frac{n}{2}\rceil+1+(\frac{i}{2}+1)+(\lceil\frac{n}{2}\rceil+1) & \\ =n+4, & i=0,\\ (\frac{n}{2}+1)+\frac{i}{2}+(\frac{i}{2}+2)+(\lceil\frac{n}{2}\rceil+1) & \\ =\{n+6,n+8,\ldots,2n+3\}, & 2\le i\le n-1 and imod2=0,\\ (\frac{i-1}{2}+1)+\frac{i+1}{2}+(\lceil\frac{n}{2}\rceil+1)+(\lceil\frac{n}{2}\rceil+1) & \\ =\{n+5,n+7,\ldots,2n+2\}, & 1\le i\le n-2 and imod2=1.\end{cases}$$

So the weights of vertices of $M_n$ under the labelling $f$ constitute the set {4, 5, ... , 2n+3} and the function $f$ is a map from $V(M_n)\cup E(M_n)$ into $\{1,2,\ldots,\lceil\frac{n}{2}\rceil+1\}$ for odd $n$.

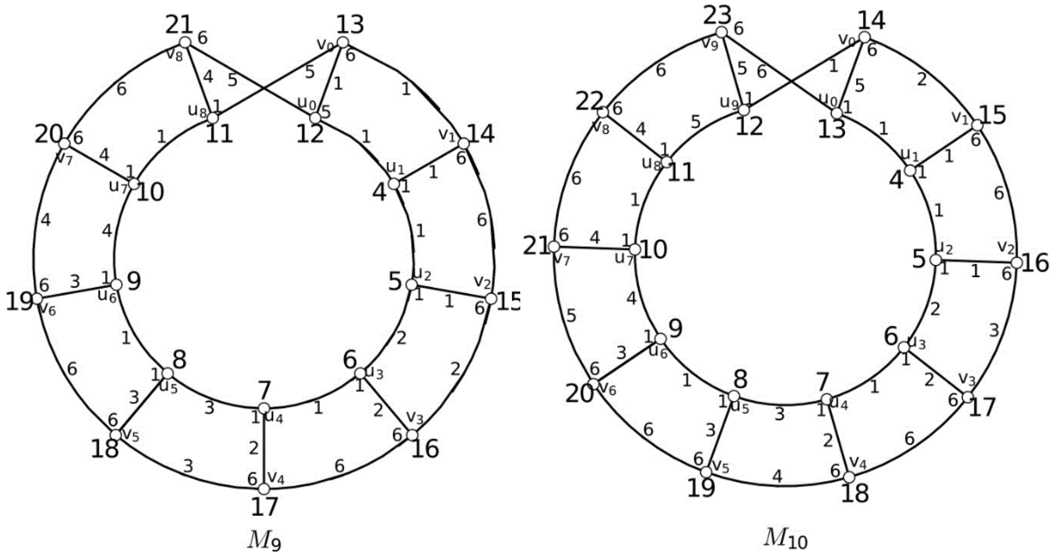


*Figure 2.2: Vertex irregular total labellings for $M_9$ and $M_{10}$*

In Figure 2.2. , we show the vertex irregular total labeling for $M_9$ and$M_{10}$.

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