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Effect of Hall Currents on the Flow and Heat Transfer of a Nanofluid over a Moving Surface with Suction/Injection

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Abstract The problem of a steady two-dimensional boundary layer flow over a moving surface in a nanofluid with suction/injection. The effect of magnetic field, Hall current on heat transfer and different types of nanoparticles Cu, Ag and Al_2O_3 particles will be studied. The governing partial differential equations are transformed into ordinary differential equations using a similarity transformation, before being solved numerically by a Rung-Kutta-Fehlberg method with shooting technique. The numerical result the velocity and temperature effect profiles are discussed graphically for different values of the parameters of physical the influence of magnetic field (M) , and nanoparticles volume fraction (φ) , and the effect of Hall current (m) , and slip parameter (δ).

Keywords nanofluid , Hall current, suction/injection , magnetic field

List of symbols		**Greek Letters**	
u, v, w	Velocity components	σ	Electrical conductivity
J	Current density	μ_{nf}	Nanofluid dynamic viscosity
B_o	Magnetic field	ρ_{nf}	Density of the nanofluid
m	Hall parameter	α_{nf}	Thermal diffusion of the nanofluid
E	Electric field	φ	Nano particle volume fraction
L	Slip length	ρ_f	Density of base fluid
p_r	Prandtl number	μ_f	Dynamic viscosity of water
U_w	Surface velocity	α_f	Thermal diffusion of water
V_w	Velocity of mass transfer	ρ_s	Solid fraction
T_w	Surface temperature	ν_f	Kinematic viscosity
Re	Reynolds number	c_p	Specific heat of fluid
M	Magnetic parameter	υ_{nf}	Kinematic viscosity of nanofluid
k_f	Thermal conductivity of the base fluid	δ	Slip parameter
T_∞	The ambient temperature	η	Similarity variable
k_{nf}	Thermal conductivity of nanofluid	ψ	Stream function
x, y, z	Dimensionless coordinates		

subscripts	
nf	Nanofluid
f	fluid
s	solid

1. Introduction

Thermal properties of liquid play a decisive role in heating as well as cooling application in industrial processes. Researcher in nanofluids have been trying to exploit the unique properties of nanoparticles to develop stable as well as highly conducting heat transfer fluids. The thermal conductivity of nanofluids depends on many factors such as particle volume fraction, particle material, particle size, particle shape, base fluid properties and temperature. It's important to know that choi [1] was the first introduce what's the meaning of "nanofluids" Enhancing thermal conductivity of fluids with nanoparticles, which "nanofluids" is a fluid containing nanometer-size particles called "nanoparticles" which improve the heat transfer of fluids. Lee et al, [2] and choi have shown that it is possible to break down the limits of conventional solid particle suspensions by conceiving the concept of nanoparticle-fluid suspensions. Nanoparticles of materials such as $(Al_2O_3\ , Cuo)$ have been used for the preparation of nanofluids. These suspensions can change the thermal properties of the base fluid. Das S.K., Choi S.U.S, Yu W., et al [3] and many references on nanofluids can found in review [4-10]. Kuznetsav and Nield [11] conducted a study to evaluate the effect of nanoparticles on natural convection boundary layer flow past a vertical plate. Aminreza N, Rashid P, Mohammed G [12] examine the effect of slip boundary condition in the presence of nanoparticles on the heat transfer characteristics of stretching sheet. Banks [13] the flow field of a stretching wall with a power low velocity variation and Elbashbeshy [14] extended the work of Banks [13] for porous stretching surface with different values of the injection parameter. Crane [15] studied steady boundary layer flow by a stretching plate, he examined the steady incompressible boundary layer flow of a Newtonian fluid. There are many reviews on nanofluid thermal conductivity research all studies reveal the fact that microstructural characteristics of nanofluids have a significant role in deciding the effective thermal conductivity of nanofluid. Hall effect is the production of a voltage difference across an electrical conductor, transverse to an electric current in the conductor and to an applied magnetic field perpendicular to the current, it was discovered by Edwin Hall in 1879. K. Sumathi, T. Arunachalam and R. Kavitha [16] they studied the effect of Hall current on mass transfer in a steady free-convective flow in the presence of a uniform transverse magnetic field in porous medium. Aziz et al [17] studied the effect of thermal radiation on steady MHD mixed convection flow past an exponentially stretching sheet with Hall currents assigning wall temperature and stretching velocity to vary according to specific exponential form. Ahmad et al [18] have studied the Hall effects on the free convection flow of a non-Newtonian power law fluid at a stretching surface. Vinod Kumar, Rajesh Johri, and Rajeev Jha [19] they studied the effects of Hall current effect on free convection on boundary layer flow of an electrically conducting visco-elastic incompressible fluid through porous medium over a continuously moving flat surface with heat source in the presence of a uniform magnetic field. The present of this paper is to extend the work by E.M.A. Elbashbeshy et al [20] to the case where the magnetic field in the cooling process has a negative effect on the mechanical properties of the surface. We will study the effect of magnetic field and Hall currents on a flow and heat transfer characteristics of a nanofluid over a steady moving surface in the presence of suction/injection.

Table 1: Thermophysical properties of fluid and nanoparticles

Properties	Fluid (water)	cu	Ag	Al_2O_3
$c_p(J/kg\ K)$	4,179	385	235	765
$\rho(kg/m^3)$	997.1	8,933	10,500	3,970
$K(W/m\ K)$	0.613	400	429	40
$\alpha \times 10^7\ (m^2/s)$	1.47	1,163.1	1,738.6	131.7

2. Mathematical Formulation

Consider a steady dimensional boundary layer flow over a continuous moving surface, the nanofluid is assumed incompressible and the flow is laminar. It is assumed that the base fluid is a water containing a different types of nanoparticles: Ag (silver), Cu (copper) and Al_2O_3 (aluminum oxide) and the nanoparticles are assumed to have a uniform shape and size. It is assumed that both the fluid phase and nanoparticles are in thermal equilibrium state and no slip occurs between them. The thermo physical properties of the fluid with nanoparticles are given in table 1. We assumed that the velocity of the surface is (U_w) and the velocity of mass transfer is (V_w) we consider a constant magnetic field of strength (B_o) which is perpendicular of the surface. It is further assumed that the induced magnetic field is negligible in comparison to the applied magnetic field this assumption because the magnetic Reynolds number is generally very small for partially ionized gases. We consider a Cartesian coordinate system (x,y,z), where x is measured along the stretching sheet in the direction of motion and y is measured normal to the sheet in the outward direction toward the fluid. The leading edge of the stretching sheet is taken as coincident with z-axis. The effect of Hall current gives rise to Lorentz force in the z- direction resulting in a cross flow in this direction. So the flow becomes three- dimensional in Fig 1. We assume that there is no variation of flow quantities in z- direction.

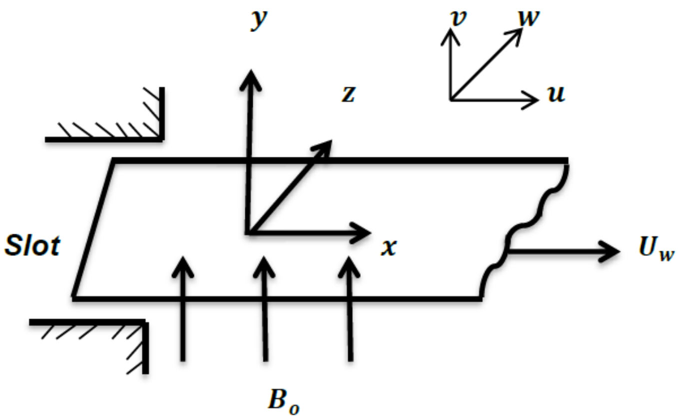

Figure 1: Schematic representation of the physical model and coordinates system

The generalized Ohm's law can be written as:

$$J + \frac{\omega_e \tau_e}{B_0}(J \times B) = \sigma_{nf}(E + V \times B) \quad (1)$$

Where $J = (J_x, J_y, J_z)$ is the current density vector, V is the velocity vector, E is the electric field vector, $B = (0, B_0, 0)$ is the magnetic induction vector, τ_e is the electron collision time, ω_e is the cyclotron frequency of electron σ_{nf} is the electrical conductivity of the nanofluid, we consider the case of short circuit problem in which the applied electric field $E = 0$. Assuming the plate to be electrically nonconducting, the generalized Ohm's law under the above condition gives $J_y = 0$ everywhere in the flow. Hence Eq. (1) reduces to:

$$J_x = \frac{\sigma_{nf} B}{(1+m^2)}(mu - w), \quad (2)$$

$$J_z = \frac{\sigma_{nf} B}{(1+m^2)}(u + mw) \quad (3)$$

The governing boundary layer equation for the steady two-dimensional laminar nanofluid flows over a moving surface can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = o \quad (4)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\frac{\mu_{nf}}{\rho_{nf}}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{nf} B_o^2}{\rho_{nf}(1+m^2)}(u + mw) - \left(\frac{\sigma_f B_o^2}{\rho_{nf}}\right)u \quad (5)$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \left(\frac{\mu_{nf}}{\rho_{nf}}\right)\frac{\partial^2 w}{\partial y^2} + \frac{\sigma_{nf} B_o^2}{\rho_{nf}(1+m^2)}(mu - w) \quad (6)$$

$$u\frac{\partial T}{\partial x} + V\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} \quad (7)$$

With boundary conditions

$$u = U_w, \quad v = V_w, \quad w = L\frac{\partial w}{\partial y}, \quad T = T_w \qquad at\ y = 0 \quad (8)$$

$$u = 0, \quad v = 0, \quad T = T_\infty, \quad w = 0 \qquad at\ y \to \infty$$

Where u, v and w are velocity components in the x, y and z directions, respectively, μ_{nf} is the nanofluid dynamic viscosity, ρ_{nf} is the density of the nanofluid, σ is the electrical conductivity, α_{nf} is the thermal diffusion of the nanofluid, m is the hall parameter, L is the slip length, T the temperature of the nanofluid, T_w is the surface temperature, and T_∞ is the ambient temperature.

We assumed that

$$U_w(x) = ax, \quad T_w(x) = T_\infty + bx, \quad (9)$$

where a and b are constants. The properties of nanofluid are given by Oztop and Abu-Nada [21]:

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}},$$

$$\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s ,$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}},$$

$$(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s ,$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s+2k_f) - 2\varphi(k_f-k_s)}{(k_s+2k_f)+\varphi(k_f-k_s)}$$

Where φ is nanoparticle volume fraction, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, k_{nf} is the thermal conductivity of the nanofluid, k_f and k_s are the thermal conductivities of the fluid and the solid fractions, respectively, and ρ_f and ρ_s are the densities of the fluid and solid fraction. We look for similarity solution of Eqs. (4)-(7) subjected to the boundary condition (8) of the following form

$$\eta = \sqrt{\frac{a}{\nu_f}}\, y, \qquad \psi = \sqrt{a\nu_f}\, x\, f(\eta), \qquad \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty}, \qquad w = ax\, h(\eta) \tag{10}$$

Where η is the similarity variable and θ is the dimensionless temperature, and ν_f is the kinematic viscosity of the base, $\psi(x,y)$ is the physical stream function and is defined in the usually way as $u=\frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, which identically satisfies Eq. (4) substituting Eq. (10) into Eqs. (5), (6) and (7). We obtain the following ordinary differential equations

$$f''' + (1-\varphi)^{2.5}B[ff'' - f'^2] - \frac{M\left(\frac{\sigma_{nf}}{\sigma_f}\right)(1-\varphi)^{2.5}}{(1+m^2)}[f' + mh] - M(1-\varphi)^{2.5}f' = 0 \tag{11}$$

$$h'' + (1-\varphi)^{2.5}\, B[fh' - f'h] - \frac{M\left(\frac{\sigma_{nf}}{\sigma_f}\right)}{(1+m^2)}(h - mf') = 0 \tag{12}$$

$$\theta'' + \frac{P_r}{L}(f\theta' - f'\theta) = 0 \tag{13}$$

The transformed boundary conditions (8) are:

$$f(0) = f_w, \quad f'(0) = 1 \quad \theta(0) = 1 \quad and\ f'(\infty) = 0, \quad \theta(\infty) = 0, \quad h(0) = \delta h'(0),$$
$$h(\infty) = 0 \tag{14}$$

where $M = \frac{\sigma\beta_0{}^2}{a\rho_f}$ is the magnetic parameter, $fw = -V_w\sqrt{\frac{1}{a\nu_f}}$ is the suction/injection, $\delta = L\sqrt{\frac{a}{\nu_f}}$ is the slip parameter ,

$B = \left(1-\varphi+\varphi\frac{\rho_s}{\rho_f}\right)$ parameter, $L = \frac{\left(\frac{k_{nf}}{k_f}\right)}{\left(1-\varphi+\varphi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)}$ and $p_r = \left(\frac{\nu\rho C_p}{k}\right)_f$ is the Prandtl number.

3. Results and Discussion

The nonlinear ordinary differential equation (4) and (7) subject to the boundary condition (8) were solved by using similarity transformation method, before solved numerically by a Rung-Kutta method with shooting technique. We consider Cu-water is the base fluid. The prandtl number of the base fluid (water) is kept constant at 6.2, the influence of magnetic field (M), and nanoparticles volume fraction (φ), and the effect of Hall current (m), and slip parameter (δ) on the velocity $f'(\eta)$ and transverse velocity $h(\eta)$ and temperature $\theta(\eta)$.

Figures (2-4) show the variation of $f'(\eta)$, $h(\eta)$ and $\theta(\eta)$ with different value of M which M=(0, 0.5 , 1 , 1.5 , 2) with $pr = 6.2$, $m = 1.5$, $\varphi = 0.1$ and $\delta = 0.2$.The effect of magnetic parameter (M) on the primary velocity $f'(\eta)$ and secondary velocity $h(\eta)$ and temperature $\theta(\eta)$. It is observed that marked reduction in velocity $f'(\eta)$ with increase of magnetic field but transverse velocity and temperature fluid increasing.

The effect of Hall parameter (m) on the profile of velocity $f'(\eta)$ and $h(\eta)$ and $\theta(\eta)$ of water-base Cu nano fluid in the boundary layer slip flow is presented in figs (5)-(7). When $m = 0,1,2,3,4,5$ and $pr = 6.2$, $\delta = 0.2$, $\varphi = 0.1$ and $M = 3$. It is observed that increase the Hall current decrease the velocity $f'(\eta)$ but the transverse velocity and temperature increasing.

In figures (8-10), the influence of slip parameter (δ) when $\delta = 0, 0.1, 0.2, 0.3, 0.4$ on the primary velocity $f'(\eta)$, secondary velocity $h(\eta)$ and temperature $\theta(\eta)$ with $pr = 6.2, M = 3, m = 1.5$ and $\varphi = 0.1$. The slip parameter (δ) increase in magnitude decrease the velocity $f'(\eta)$, transverse velocity $h(\eta)$ but increasing the temperature fluid.

In figures (11-13) for some value of the cu-nanoparticle volume fraction parameter φ, when $\varphi = 0.05, 0.1, 0.15, 0.2, 0.3$ and $pr = 6.2, M = 3, m = 1.5, \delta = 0.1$. The effect of nanoparticles volume fraction (φ) on the primary velocity $f'(\eta)$ and secondary velocity $h(\eta)$ and temperature $\theta(\eta)$ are shown in figs (11)-(13). It is observed that increase of nanoparticle volume fraction increasing the velocity $f'(\eta)$ and transverse velocity $h(\eta)$ but decrease the temperature fluid $\theta(\eta)$.

In figures (14-16) The Prandtl number (Pr) has some value 2, 3, 5, 6.2, 7.2 when $M = 3, \delta = 0.2, \varphi = 0.1$ and $m = 1.5$. The effect of pr on temperature $\theta(\eta)$ are shown in fig (16) , the increase of Prandtl number the temperature is decrease but there is no effect for velocity $f'(\eta)$ and transverse velocity $h(\eta)$.

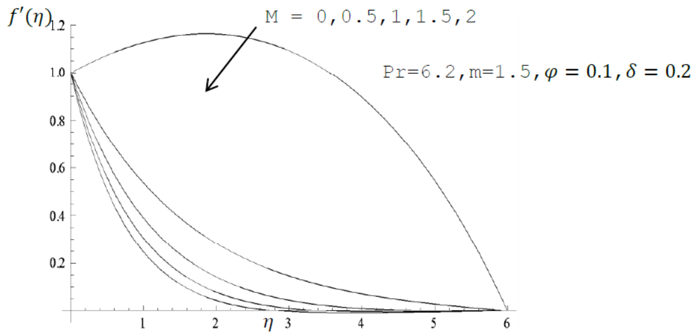

Figure 2: The velocity profiles with increasing of magnetic parameter

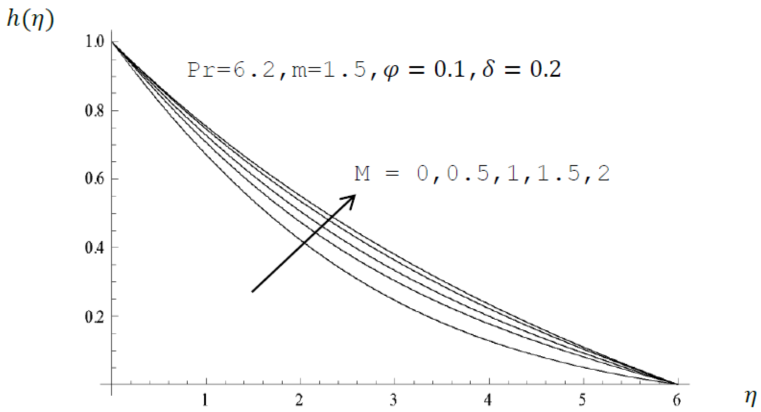

Figure 3: The transverse velocity profile with increasing of magnetic parameter

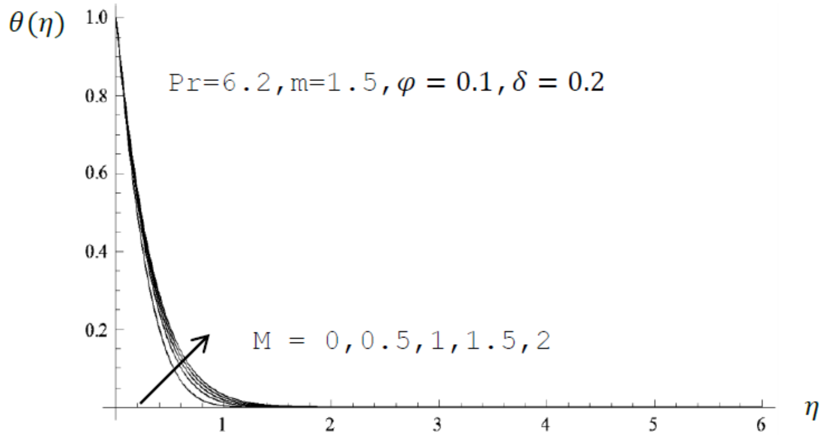

Figure 4: The temperature profile with increasing of magnetic parameter

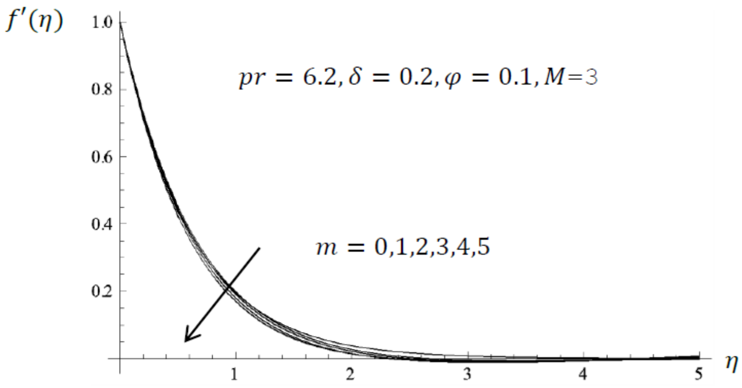

Figure 5: The velocity profiles with increasing of Hall parameter

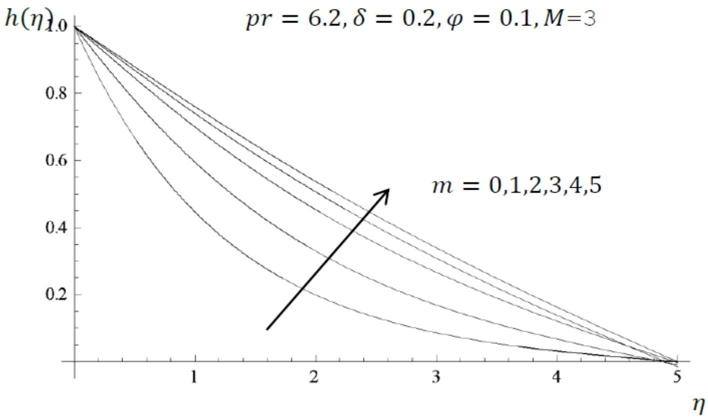

Figure 6: The transverse velocity profile with increasing of Hall parameter

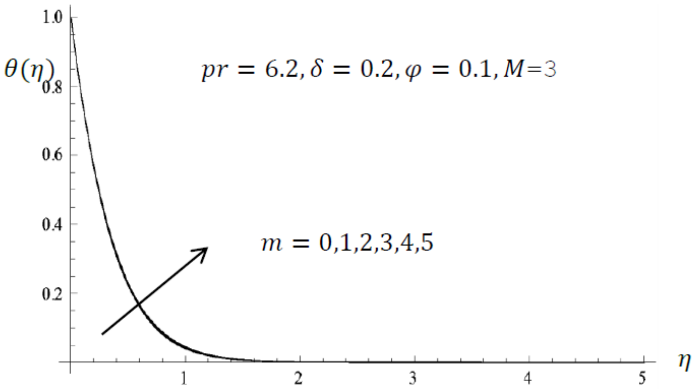

Figure 7: The temperature profiles with increasing of Hall parameter

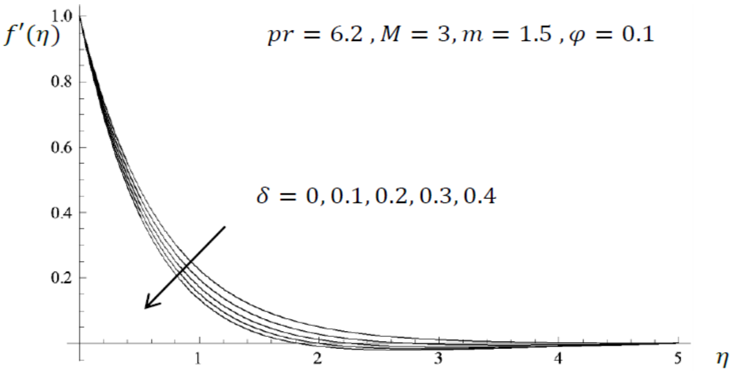

Figure 8: The velocity profiles with increasing of slip parameter

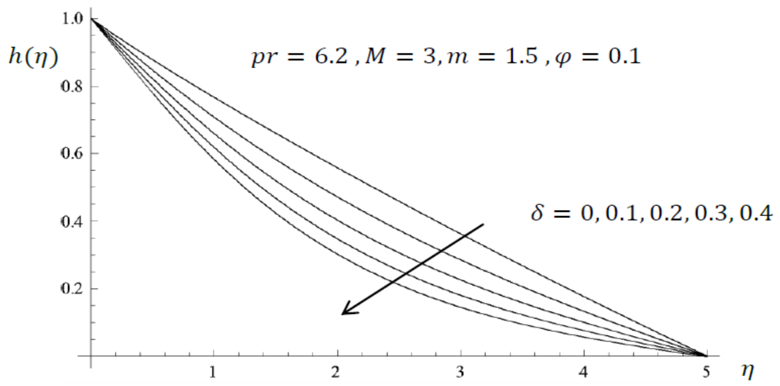

Figure 9: the transverse velocity profiles with increasing of slip parameter

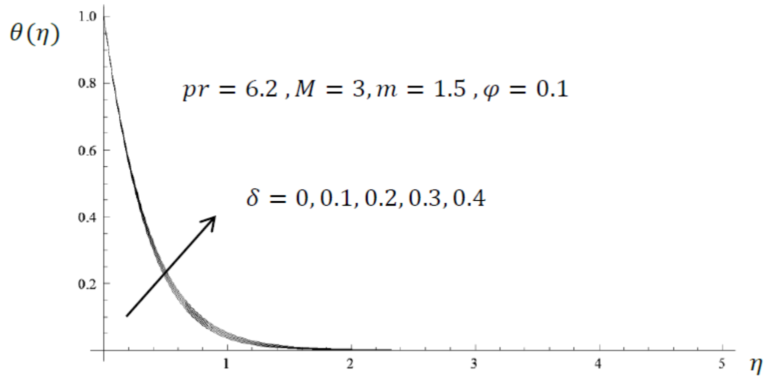

Figure 10: The temperature profiles with increasing of slip parameter

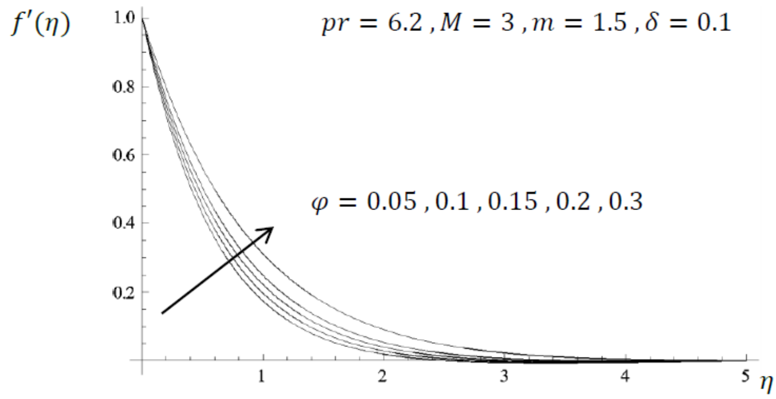

Figure 11: The velocity profiles with increasing of nanoparticles volume fraction

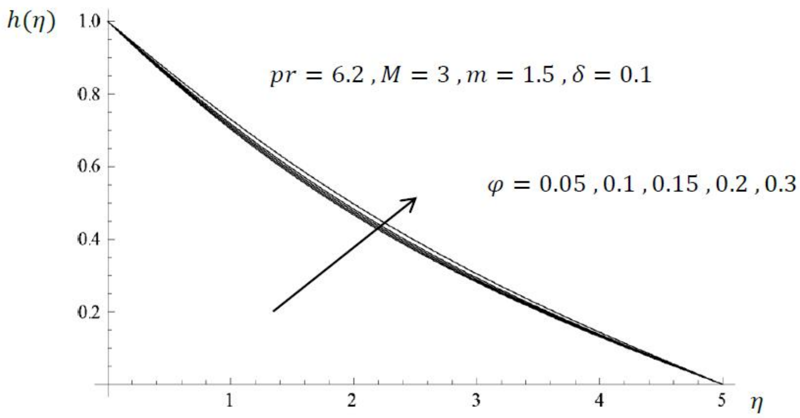

Figure 12: The transverse velocity profiles with increasing of nanoparticles volume fraction

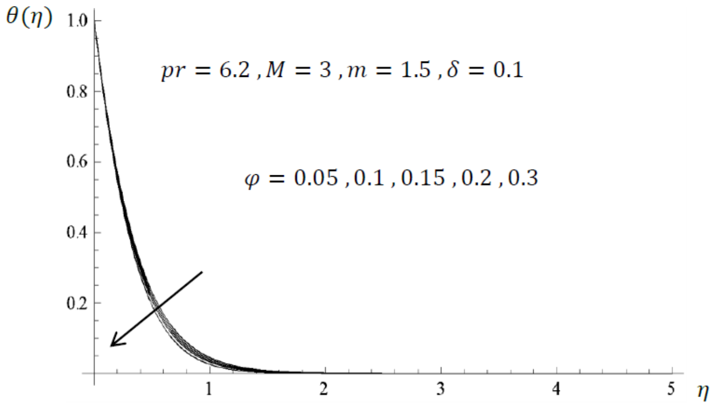

Figure 13: The temperature profiles with increasing of nanopartciles volume fraction

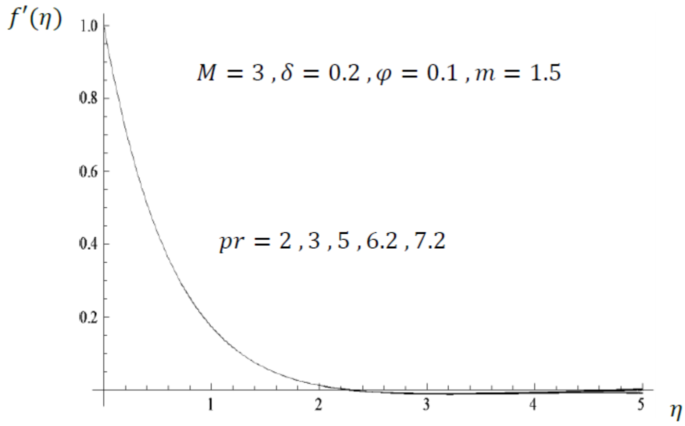

Figure 14: The velocity profiles with increasing of prandtl number

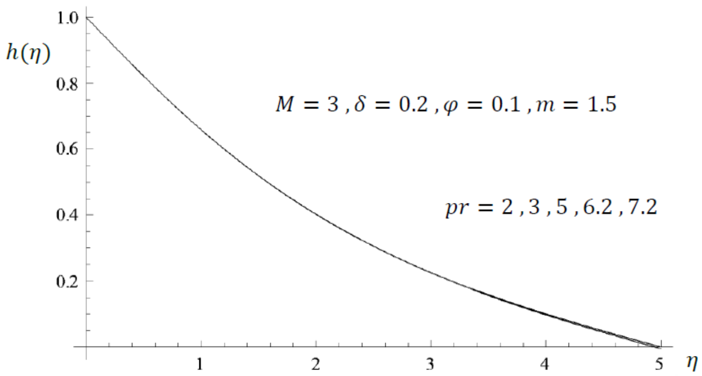

Figure 15: The transverse velocity profiles with increasing of prandtl number

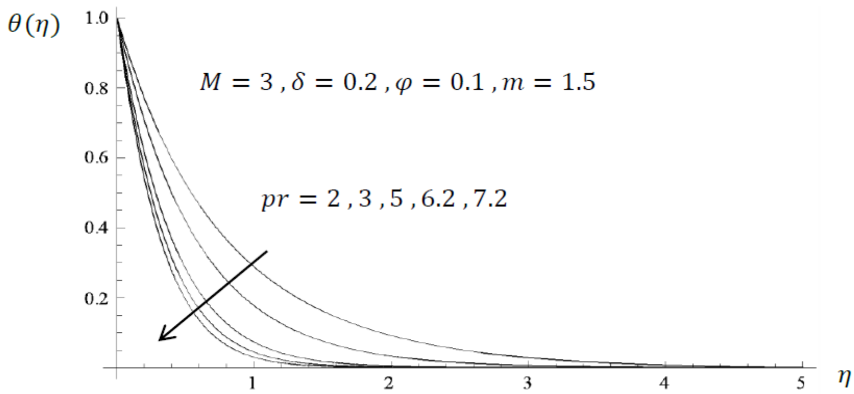

Figure 16: The temperature profile with increasing of prandtl number

4. Conclusion

In this article, we presented a boundary layer flow over continuous moving surface with Hall current effects, The study based on three types of nanoparticle which are the most used types (cu, Ag, AL_2O_3). Here we studied three types only of nanoparticles, according to this study, the best type used to improve the mechanical properties of the surface is cu-nanofluid. In general, using a nanofluid in the cooling process is more active to improve the mechanical properties of the surface, such that using nanofluid increasing the rate of heat transfer by (10-40 %) more than in the case of pure water that leads to accelerate the cooling of the surface which increases the surface hardness and strength. The governing partial differential equations are transformed into ordinary differential equations using a similarity transformation, before solved numerically by a Rung-Kutta method with shooting technique. The impact of physical parameters on the velocity and transvers velocity and temperature. This study and following results are obtained:

1) An analytical solution for boundary layer problem over a moving surface in a nanofluid in the presence of magnetic field, suction/injection and Hall current was obtained.
2) The velocity within the boundary layer increases with increase of nanoparticles volume fraction and decreases with magnetic parameter, Hall parameter and slip parameter.
3) The transverse velocity within the boundary layer increases with increase of magnetic parameter, Hall parameter and nanoparticles volume fraction and deceases with slip parameter.
4) The temperature within the boundary layer increases with increase of magnetic parameter, Hall parameter and slip parameter and decreases with nanoparticles volume fraction and Prandtl number.

Prandtl number: defined as the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity, i.e. Pr = viscous diffusion rate/thermal diffusion rate.

$$\mathrm{pr} = \frac{\nu}{\alpha}$$

Where $\nu = \mu/\rho$ is the kinematic viscosity in which ρ is density and μ dynamic viscosity, α is the thermal diffusivity for liquid metal, $\mathrm{pr} < 0.01$ for air and gases $\mathrm{pr} \approx 1$ for water $\mathrm{pr} \approx 10$ and for heavy oil and grease $\mathrm{pr} > 10^5$ This number assumes significance when both momentum and energy are propagated through the system. pr provides a measure of relative effective of momentum and energy transport by diffusion in velocity and thermal boundary layer respectively. Higher pr means higher nusselt number Nu and so it causes higher heat transfer it is a physical parameter depending upon the medium properties.

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