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Technical method in using generalized theory of thermoelastic diffusion with double porosity under one relaxation time

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Abstract Most of authors, in solving two-dimensional thermoelastic medium, based on thermoelasticity-generalized theorems, have discussed the behavior of the solution only. This discussion has been taken two ways. The first is using normal mode method and the second is using Laplace and Fourier transformation method. The two methods are failed completely to explain the behavior of the time. Here, we use a new technique to give and discuss, directly the time function and its behavior. In addition, the analytical solution, of the problem is obtained directly. Moreover, the expressions of many physical quantities with respect to time are explained. The general solutions, under specific boundary conditions of the problem were found in some details. In addition, numerical results are computed.

Keywords L-S theory- Time function- thermoelastic diffusion - Double porosity

1. Introduction

Lord-Shulman [1] discussed a thermoelasticity generalized theory, (L-S) theory, which includes unique relaxation time for a thermoelastic process. The basis of the model introduce a new physical concept, called a relaxation time, to modify Fourier's law of the heat conduction equation. In addition; the theory suggests a wave type for the heat equations to ensure finite velocities of propagation of the elastic and heat waves. In this head, the double porosity functions added into the heat equation to yield [2].

$$K^{*}\nabla^{2}T = (1+\tau_{0}\frac{\partial}{\partial t})(\rho C^{*}\dot{T} + \beta T_{0}\dot{e} + \gamma_{1}T_{0}\dot{\Phi} + \gamma_{2}T_{0}\dot{\Psi}).$$

Most of authors have studied the behavior of the solution through two groups.The first is concerned with studying the behavior of the solution using a method called "normal mode". This method is based on the general form that the authors propose to visualize the solution. More, some users of this road assumed that time were imaginary, which led to these problems giving me precise physical meanings.

The second team linked the study of the behavior of the solution using the "Laplace Integral Method". This method is given a good solution to solve when it is easy to find the inverse of Laplace transform. The solution is to be studied in this way if each Laplace transform is difficult to generate.Add to that the function of time was not able to get one of the researchers directly.

In this paper the work is focused on the linear theory of elastic materials with double porosity [3, 4, 5, 6, 7, 8, 9, 10, 11, 12], and discussed the linear theory of diffusion [13, 14]. Here, the authors devised analytical solutions for this type of problem by looking for a time function that corresponds to all changes in the presented problem. In addition, the authors discussed the equations of generalized thermoelastic diffusion material with double porosity structure with one relaxation time using a separation of variable methods.

2. Formulation of the problem and basic equations

We consider a homogenous isotropic elastic solid with generalized thermoelastic diffusion with double porosity in a half space using the L-S theory. We are studied this problem in a plane strain of x-y with displacement components u, v such that $u = u(x, y, t), v = v(x, y, t)$

Stress-strain equation

$$\sigma_{ij} = \lambda\delta_{ij}e_{rr} + 2\mu e_{ij} + b\delta_{ij}\Phi + d\,\delta_{ij}\Psi - \beta\delta_{ij}(T - T_0) - \gamma^{*}C\,\delta_{ij};\ (i, j = 1, 2). \tag{1}$$

Equations of double porosity

$$\sigma_i = \alpha\,\Phi_{,i} + b_1\,\Psi_{,i}\,, \tag{2}$$

$$\tau_i = b_1\,\Phi_{,i} + \gamma\,\Psi_{,i}\,. \tag{3}$$

Equations of motion

$$\mu\nabla^2 u + (\lambda+\mu)\frac{\partial e}{\partial x} + b\frac{\partial \Phi}{\partial x} + d\frac{\partial \Psi}{\partial x} - \beta\frac{\partial T}{\partial x} - \gamma^{*}\frac{\partial C}{\partial x} + \rho g\frac{\partial v}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{4}$$

$$\mu\nabla^2 v + (\lambda+\mu)\frac{\partial e}{\partial y} + b\frac{\partial \Phi}{\partial y} + d\frac{\partial \Psi}{\partial y} - \beta\frac{\partial T}{\partial y} - \gamma^{*}\frac{\partial C}{\partial y} - \rho g\frac{\partial u}{\partial x} = \rho\frac{\partial^2 v}{\partial t^2}. \tag{5}$$

Equations of heat

$$K\nabla^2 T = (1+\tau_0\frac{\partial}{\partial t})(\rho c^{*}\dot{T} + aT_0\dot{C} + \beta T_0\dot{e} + \gamma_1 T_0\dot{\Phi} + \gamma_2 T_0\dot{\Psi}). \tag{6}$$

Double porosity equations

$$\alpha\nabla^2\Phi + b_1\nabla^2\Psi - be - \alpha_1\Phi - \alpha_3\Psi + \gamma_1 T + \vartheta C = K_1\ddot{\Phi}, \tag{7}$$

$$b_1\nabla^2\Phi + \gamma\nabla^2\Psi - de - \alpha_3\Phi - \alpha_2\Psi + \gamma_2 T + mC = K_2\ddot{\Psi}. \tag{8}$$

Diffusion equation

$$\beta_1\gamma^{*}\nabla^2 e_{kk} + \beta_1 a\nabla^2 T + \dot{C} + \tau^0\ddot{C} - \beta_1\beta_2\nabla^2 C + \beta_1\vartheta\,\nabla^2\Phi + \beta_1 m\,\nabla^2\Psi = 0. \tag{9}$$

Potential equation for diffusion

$$p = -\gamma^{*}e_{kk} + \beta_2 C - a(T - T_0) - \vartheta\,\Phi - m\,\Psi. \tag{10}$$

Where σ_{ij} are the stress components; λ, μ are elastic constants, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t is the thermal expansion coefficient, δ_{ij} is Kronecker delta. In addition, T is the temperature above the reference temperature T_0, K is the thermal conductivity, u_i the vector of displacement, ρ the mass density, c_e the specific heat at constant strain, Here, Ψ, Φ the volume fraction fields. Here, k_1 and k_2 the coefficients of equilibrated inertia , τ_0 the relaxation time, $b, d, b_1, \gamma, \gamma_1, \gamma_2, m, \vartheta$ the constitutive coefficients of double porosity, p the chemical potential, C concentration distribution, a coefficient describing the measure of thermoelastic diffusion effect, γ^{*} coefficient of diffusion thermal expansion, β_1 thermoelastic diffusion constant, β_2 coefficient describing the measure of diffusive effect, τ^0 diffusion relaxation time, σ_i , τ_i the stresses of equilibrium, and β is the coefficient of linear thermal expansion $\beta = (3\lambda + 2\mu)\alpha_t$, $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$.

For the aim of numerical evaluation, we illustrate dimensionless variables

$$(x', y') = \frac{w_1}{c_1}(x, y),\ (u', v') = \frac{w_1}{c_1}(u, v),\ (\sigma_1', \tau_1') = \frac{c_1}{\alpha w_1}(\sigma_1, \tau_1),\ (t', \tau_0') = w_1(t, \tau_0),\ c_1^2 = \frac{\lambda + 2\mu}{\rho}, w_1 = \frac{\rho c_e c_1^2}{K},$$

$$\nabla^2 = \frac{w_1^2}{c_1^2}\nabla'^2, (\sigma_{ij}') = (\frac{1}{\beta T_0})(\sigma_{ij}),\ T' = \frac{T}{T_0}, g' = \frac{g}{c_1 w_1}, P' = \frac{P}{\rho\beta_2}, C' = \frac{C}{\rho}, (\Phi', \Psi') = \frac{K_1 w_1^2}{\alpha_1}(\Phi, \Psi), \tag{G.A}$$

Using the above dimensionless quantities (G.A), equations (4) - (10) become:
The equation of motion

$$a_1\nabla^2 u + a_2\frac{\partial e}{\partial x} + a_3\frac{\partial \Phi}{\partial x} + a_4\frac{\partial \Psi}{\partial x} - a_5\frac{\partial T}{\partial x} - a_6\frac{\partial C}{\partial x} + a_7\frac{\partial v}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{11}$$

$$a_1\nabla^2 v + a_2\frac{\partial e}{\partial y} + a_3\frac{\partial \Phi}{\partial y} + a_4\frac{\partial \Psi}{\partial y} - a_5\frac{\partial T}{\partial y} - a_6\frac{\partial C}{\partial y} - a_7\frac{\partial u}{\partial x} = \frac{\partial^2 v}{\partial t^2}. \tag{12}$$

$$\nabla^2 T = (1+\tau_0\frac{\partial}{\partial t})(\dot{T} + a_8\dot{C} + a_9\dot{e} + a_{10}\dot{\Phi} + a_{11}\dot{\Psi}). \tag{13}$$

$$a_{12}\nabla^2\Phi + a_{13}\nabla^2\Psi - \frac{b}{\alpha_1}e - a_{14}\Phi - a_{15}\Psi + a_{16}T + a_{17}C = \frac{d^2\Phi}{dt^2}, \tag{14}$$

$$a_{18}\nabla^2\Phi + a_{19}\nabla^2\Psi - a_{20}e - a_{21}\Phi - a_{22}\Psi + a_{23}T + a_{24}C = \frac{d^2\Psi}{dt^2}. \tag{15}$$

$$a_{29}\nabla^2 e_{kk} + a_{30}\nabla^2 T + a_{31}\dot{C} + a_{32}\tau^0\ddot{C} - a_{33}\nabla^2 C + a_{34}\nabla^2\Phi + a_{35}\nabla^2\Psi = 0. \tag{16}$$

$$p = -a_{25}e_{kk} + C - a_{26}T - a_{27}\Phi - a_{28}\Psi. \tag{17}$$

Where, $a_1 = \frac{\mu}{\rho c_1^2},\ a_2 = \frac{(\mu+\lambda)}{\rho c_1^2},\ a_3 = \frac{b\alpha_1}{\rho c_1^2 K_1 w_1^2},\ a_4 = \frac{d\alpha_1}{\rho c_1^2 K_1 w_1^2},\ a_5 = \frac{\beta T_0}{\rho c_1^2},\ a_6 = \frac{\gamma^*}{c_1^2},\ a_7 = g,\ a_8 = \frac{a}{c_e},\ a_9 = \frac{\beta}{\rho c_e},$

$a_{10} = \frac{\gamma_1\alpha_1}{\rho c_e K_1 w_1^2},\ a_{11} = \frac{\gamma_2\alpha_1}{\rho c_e K_1 w_1^2}\ a_{12} = \frac{\alpha}{c_1^2 K_1},\ a_{13} = \frac{b}{c_1^2 K_1},\ a_{14} = \frac{\alpha_1}{K_1 w_1^2},\ a_{15} = \frac{\alpha_3}{K_1 w_1^2},\ a_{16} = \frac{\gamma_1 T_0}{\alpha_1},\ a_{17} = \frac{\vartheta\rho}{\alpha_1},\ a_{18} = \frac{b_1}{c_1^2 K_2},$

$a_{19} = \frac{\gamma}{c_1^2 K_2},\ a_{20} = \frac{dK_1}{\alpha_1 K_2},\ a_{21} = \frac{\alpha_3}{w_1^2 K_2},\ a_{22} = \frac{\alpha_2}{w_1^2 K_2},\ a_{23} = \frac{\gamma_2 T_0 K_1}{\alpha_1 K_2},\ a_{24} = \frac{m\rho K_1}{\alpha_1 K_2},\ a_{25} = \frac{\gamma^*}{\beta_2\rho},\ a_{26} = \frac{aT_0}{\beta_2\rho},\ a_{27} = \frac{\vartheta\alpha_1}{\rho\beta_2 K_1 w_1^2},$

$a_{28} = \frac{m\alpha_1}{\rho\beta_2 K_1 w_1^2},\ a_{29} = \beta_1\gamma^*,\ a_{30} = \beta_1 aT_0,\ a_{31} = \frac{\rho c_1^2}{w_1},\ a_{32} = \rho c_1^2,\ a_{33} = \beta_1\beta_2\rho,\ a_{34} = \frac{\beta_1\alpha_1\vartheta}{K_1 w_1^2},\ a_{35} = \frac{\beta_1\alpha_1 m}{K_1 w_1^2}$

$(G.B)$

The dimensionless variables of the stress components take the form,

$$\sigma_{xx} = (\frac{\lambda}{\beta T_0})\frac{\partial v}{\partial y} + (\frac{2\mu+\lambda}{\beta T_0})\frac{\partial u}{\partial x} - T - (\frac{\gamma^*\rho}{\beta T_0})C + (\frac{b\alpha_1}{K_1 w_1^2\beta T_0})\Phi + (\frac{d\alpha_1}{K_1 w_1^2\beta T_0})\Psi, \tag{18}$$

$$\sigma_{yy} = (\frac{\lambda}{\beta T_0})\frac{\partial u}{\partial x} + (\frac{2\mu+\lambda}{\beta T_0})\frac{\partial v}{\partial y} - T - (\frac{\gamma^*\rho}{\beta T_0})C + (\frac{b\alpha_1}{K_1 w_1^2\beta T_0})\Phi + (\frac{d\alpha_1}{K_1 w_1^2\beta T_0})\Psi, \tag{19}$$

$$\sigma_{xy} = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}). \tag{20}$$

3. Method of Solution

In this section, we go to discuss the solution in the separable form. For this, let us separate the displacement and the heat function variables as:

$$\begin{aligned} &u(x,y,t) = U(x,y) + F(t);\quad v(x,y,t) = V(x,y) + F(t),\\ &T(x,y,t) = \theta(x,y) + F(t);\ \Phi(x,y,t) = \phi(x,y) + F(t),\\ &\Psi(x,y,t) = \psi(x,y) + F(t);\quad e(x,y,t) = e(x,y) + F(t),\\ &C(x,y,t) = C(x,y) + F(t), \end{aligned} \tag{21}$$

Where, $F(t)$ is a function of time.

Using (21) in (11)-(17) we get

$$a_1\nabla^2 U + a_2\frac{\partial e}{\partial x} + a_3\frac{\partial \phi}{\partial x} + a_4\frac{\partial \psi}{\partial x} - a_5\frac{\partial \theta}{\partial x} - a_6\frac{\partial C}{\partial x} + a_7\frac{\partial V}{\partial x} = \frac{\partial^2 F}{\partial t^2}, \tag{22}$$

$$a_1\nabla^2 V + a_2\frac{\partial e}{\partial y} + a_3\frac{\partial \phi}{\partial y} + a_4\frac{\partial \psi}{\partial y} - a_5\frac{\partial \theta}{\partial y} - a_6\frac{\partial C}{\partial y} - a_7\frac{\partial U}{\partial x} = \frac{\partial^2 F}{\partial t^2}. \quad (23)$$

$$\nabla^2\theta = (1 + a_8 + a_9 + a_{10} + a_{11})\frac{\partial F}{\partial t} + (1 + a_8 + a_9 + a_{10} + a_{11})\tau_0\frac{\partial^2 F}{\partial t^2}. \quad (24)$$

$$a_{12}\nabla^2\phi + a_{13}\nabla^2\psi - \frac{b}{\alpha_1}e - a_{14}\phi - a_{15}\psi + a_{16}\theta + +a_{17}C = (\frac{b}{\alpha_1} + a_{14} + a_{15} - a_{16} - a_{17})F(t) + \frac{d^2F}{dt^2}, \quad (25)$$

$$a_{18}\nabla^2\phi + a_{19}\nabla^2\psi - a_{20}e - a_{21}\phi - a_{22}\psi + a_{23}\theta + +a_{24}C = (a_{20} + a_{21} + a_{22} - a_{23} - a_{24})F(t) + \frac{d^2F}{dt^2}. \quad (26)$$

$$a_{29}\nabla^2 e_{kk} + a_{30}\nabla^2\theta - a_{33}\nabla^2 C + a_{34}\nabla^2\phi + a_{35}\nabla^2\psi = -a_{31}\dot{F}(t) - a_{32}\tau^0\ddot{F}(t). \quad (27)$$

$$p = -a_{25}e_{kk} + C - a_{26}T - a_{27}\phi - a_{28}\psi. \quad (28)$$

Here, the general form of time function depending on the value of the primary temperature and the type of constants of the properties of the material. Therefore, the material chosen for the purpose of numerical computation is copper.

4. A suitable time function:-

In this section, we discuss the different forms of time function, and then we choose a suitable function that satisfies the conditions of the problem

4.1. The first formula: From (22) and (23), we get

$$\frac{d^2F}{dt^2} = 0 \quad (29)$$

The above equation, under the conditions $F(0) = T_0;\ F(\infty) \to 0$ has the solution:

$$F(t) = T_0 \quad (30)$$

The time function (30) is constant for at any time t, for this it refused

4.2. The second formula: From (24), after using the same initial condition, we have

$$F(t) = T_0 e^{-\frac{t}{\tau_0}} \quad (31)$$

4.3. The third formula: From (25)

$$\frac{d^2F}{dt^2} = -(\frac{b}{\alpha_1} + a_{14} + a_{15} - a_{16} - a_{17})F \quad (32)$$

This formula depends on the values of the constants.

Therefore, we have three cases

(i) When $(\frac{b}{\alpha_1} + a_{14} + a_{15} - a_{16} - a_{17}) = 0,$ and then after solving the result and using the values of the constants from (G.A) and (G.B), we get

$$T_0 = (\frac{b}{\gamma_1} + \frac{\alpha_1^2}{K_1 w_1^2\gamma_1} + \frac{\alpha_3\alpha_1}{K_1 w_1^2\gamma_1} - \frac{\upsilon\rho}{\gamma_1}) = (2\times 10^8)$$ (Refused because it is too high temperature) (33)

(ii) When $(\frac{b}{\alpha_1} + a_{14} + a_{15} - a_{16} - a_{17}) > 0,$ the time function takes the form

$$F(t) = T_0 \cos\sqrt{(\frac{b}{\alpha_1} + a_{14} + a_{15} - a_{16} - a_{17})}\,t;$$ $(T_0 < 2\times 10^8)$ (Refused because it is not specific and unknown temperature) (34)

(iii) When $(\frac{b}{\alpha_1}+a_{14}+a_{15}-a_{16}-a_{17})<0,$ the time function takes the form

$$F(t)=T_0 e^{-\sqrt{(\frac{b}{\gamma_1}+a_{14}+a_{15}-a_{16}-a_{17})}\,t};\ (T_0>2\times10^8)\ \text{(Refused because it is too high temperature)} \quad (35)$$

4.4. The fourth formula: From (26)

$$(a_{20}+a_{21}+a_{22}-a_{23}-a_{24})F(t)+\frac{d^2F}{dt^2}=0 \quad (36)$$

This formula depends on the values of the constants.
Therefore, we have three cases

(i)When $(a_{20}+a_{21}+a_{22}-a_{23}-a_{24})=0$.Using the values of the above constants from (G.A) and (G.B), we get

$$(\frac{d}{\gamma_2}+\frac{\alpha_3\alpha_1}{w_1^2\gamma_2K_1}+\frac{\alpha_2\alpha_1}{w_1^2\gamma_2K_1}-\frac{m\rho}{\gamma_2})=T_0=(6\times10^{10})\ \text{(Refused because it is too high temperature)} \quad (37)$$

(ii) When $(a_{20}+a_{21}+a_{22}-a_{23}-a_{24})>0$ the time function takes the form

$F(t)=T_0\cos\ (\sqrt{(a_{20}+a_{21}+a_{22}-a_{23}-a_{24})}\,t;$ $(T_0<6\times10^{10})$ (Refused because it is not specific and unknown temperature as it should be known) (38)

(iii) When $(a_{20}+a_{21}+a_{22}-a_{23}-a_{24})<0$ the time function takes the form

$$F(t)=T_0\ e^{(-\sqrt{(a_{20}+a_{21}+a_{22}-a_{23}-a_{24})}\,t};\quad (T_0>6\times10^{10})\ \text{(Refused because it is too high temperature)} \quad (39)$$

So the reference temperature refused in all above cases as it is very high temperature where, the reference temperature should be known. We will review the reference temperature in the fifth formula as $273^\circ c$ to get relaxation time.

4.5. The fifth formula: From (27)

$$a_{31}\dot{F}(t)+a_{32}\tau^0\ddot{F}(t)=0. \quad (40)$$

The above equation has the solution $F(t)=T_0 e^{-\frac{t}{\omega_1\tau^0}}$ (41)

From (31),(35),(39) and (41), we found that the relaxation time and diffusion relaxation time after equating the exponential function become

$$(\frac{b}{\gamma_1}+a_{14}+a_{15}-a_{16}-a_{17})=(a_{20}+a_{21}+a_{22}-a_{23}-a_{24})=\frac{1}{(\omega_1\tau^0)^2}=\frac{1}{(\tau_0)^2}=5.7\times10^4.$$

$$\tau_0=0.0042,\ \tau^0=2.71\times10^{-13}. \quad (42)$$

These results depend on the constants of material, which means that every material has a different relaxation time.

5. The functions of position: After obtaining analytically the time function, wego to discuss, analytically the functions of position. For this, the general solution of the total heat, under the conditions

$$\theta(x,0)=0,\quad \theta(x,b)=\varepsilon_0,$$

Takes the general form

$$\theta(x,y)=\sum_{n=1}^{\infty}\frac{4\varepsilon_0}{n\pi\sinh(\frac{n\pi b}{a})}\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a}) \quad (43)$$

$$T(x,y,t)=\sum_{n=1}^{\infty}\frac{4\varepsilon_0}{n\pi\sinh(\frac{n\pi b}{a})}\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})+T_0e^{-t(\frac{1}{\tau_0}+\frac{1}{\omega_1\tau^0})} \tag{44}$$

Define displacement potentials q_1 and P_1 which relate to displacement components u and v as,

$$\nabla^2 q_1=u_{,x}+v_{,y},\qquad \nabla^2 P_1=u_{,y}-v_{,x} \tag{45}$$

Equations (24)-(27), after using (45) can be reformulated

$$(a_1+a_2)\nabla^2 q_1+a_3\phi+a_4\psi-a_6C-a_7P_{1,x}=0, \tag{46}$$

$$a_1\nabla^2 P_1+a_7\, q_{1,x}=0, \tag{47}$$

$$(a_{12}\nabla^2-a_{14})\phi+(a_{13}\nabla^2-a_{15})\psi-\frac{b}{\alpha_1}\nabla^2 q_1+a_{17}C=-a_{16}\sum_{n=1}^{\infty}\frac{4\varepsilon_0}{n\pi\sinh(\frac{n\pi b}{a})}\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a}), \tag{48}$$

$$(a_{18}\nabla^2-a_{21})\phi+(a_{19}\nabla^2-a_{22})\psi-a_{20}\nabla^2 q_1+a_{24}C=-a_{23}\sum_{n=1}^{\infty}\frac{4\varepsilon_0}{n\pi\sinh(\frac{n\pi b}{a})}\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a}), \tag{49}$$

$$a_{29}\nabla^2 q_1+a_{34}\phi+a_{35}\psi-a_{33}C=0, \tag{50}$$

Let us, in general, consider the solution in the form:

$$\begin{aligned}&\phi=A\sin(\lambda_n y)e^{-j_m x};\quad \psi=B\sin(\lambda_n y)e^{-j_m x};\quad C=D\sin(\lambda_n y)e^{-j_m x}\\ &q_1=E\sin(\lambda_n y)e^{-j_m x};\qquad P_1=N\sin(\lambda_n y)e^{-j_m x}\end{aligned} \tag{51}$$

Where, A, B, D, E andNare functions;

Using (51) in (46-50), will be determined. For this, we follow

$$a_3A+a_4B-a_6D+(a_1+a_2)\,\mathrm{I}\,E-a_7j_mN=0, \tag{52}$$

$$-a_7j_mE+a_1\,\mathrm{I}\,N=0, \tag{53}$$

$$(a_{12}\mathrm{I}-a_{14})A+(a_{13}\mathrm{I}-a_{15})B-\frac{b}{\alpha_1}\mathrm{I}\,E+a_{17}D=a_{16}\sum_{n=1}^{\infty}\frac{4\varepsilon_0}{n\pi\sinh(\frac{n\pi b}{a})\sin(\lambda_n y)e^{-j_m x}}\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a}), \tag{54}$$

$$(a_{18}\,\mathrm{I}-a_{21})A+(a_{19}\,\mathrm{I}-a_{22})B-a_{20}\,\mathrm{I}\,E+a_{24}D=-a_{23}\sum_{n=1}^{\infty}\frac{4\varepsilon_0}{n\pi\sinh(\frac{n\pi b}{a})\sin(\lambda_n y)e^{-j_m x}}\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a}), \tag{55}$$

$$a_{34}A+a_{35}B+a_{29}\,\mathrm{I}\,E-a_{33}D=0. \tag{56}$$

Solving (52-56) in a matrix to get A, B, D, E and N

$$\begin{bmatrix} a_3 & a_4 & -a_6 & (a_1+a_2)\,\mathrm{I} & a_7j_m\\ 0 & 0 & 0 & -a_7j_m & a_1\mathrm{I}\\ (a_{12}\mathrm{I}-a_{14}) & (a_{13}\mathrm{I}-a_{15}) & a_{17} & -\frac{b}{\alpha_1}\mathrm{I} & 0\\ (a_{18}\mathrm{I}-a_{21}) & (a_{19}\mathrm{I}-a_{22}) & a_{24} & -a_{20}\mathrm{I} & 0\\ a_{34} & a_{35} & -a_{33} & a_{29}\mathrm{I} & 0\end{bmatrix}\begin{bmatrix}A\\B\\D\\E\\N\end{bmatrix}=\begin{bmatrix}0\\0\\a_{16}\mathrm{H}\\-a_{23}\mathrm{H}\\0\end{bmatrix}$$

$$A=(-a_4a_{24}a_1a_{29}\mathrm{I}^2+a_4a_{33}a_{20}a_1\mathrm{I}^2-a_{19}a_6a_1a_{29}\mathrm{I}^3+a_{19}a_{33}a_1^2\mathrm{I}^3+a_{19}a_{33}a_1a_2\mathrm{I}^3+a_7^2j^2a_{33}a_{19}+a_{22}a_6a_1a_{29}\mathrm{I}^2-a_{33}a_{22}a_1^2j^2-a_{22}a_{33}a_1a_2\mathrm{I}^2$$

$$-a_{22}a_{33}a_7^2j^2-a_{35}a_6a_{20}a_1\mathrm{I}^2+a_{35}a_{24}a_1^2\mathrm{I}^2+a_{35}a_{24}a_2a_1\mathrm{I}^2+a_7^2j^2a_{35}a_{24}-a_4\mathrm{I}^2a_1a_{17}a_{29}\alpha+a_4\mathrm{I}^2a_1a_{33}b-a_7\mathrm{I}^3a_6a_{13}a_{29}\alpha+\mathrm{I}^3a_1^2a_{13}a_{33}\alpha+a_{33}\mathrm{I}^3a_1a_{13}a_2\alpha$$

$$+a_7^2j^2\mathrm{I}a_{13}a_{33}\alpha+a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2-\alpha a_{15}a_{33}a_1^2\mathrm{I}^2-\alpha a_{15}a_{33}a_1a_2\mathrm{I}^2-\alpha a_{15}a_{33}a_7^2j^2-a_{35}\mathrm{I}^2a_1a_6b+\alpha a_{35}a_1^2a_{17}\mathrm{I}^2+\alpha a_{35}a_1a_{17}a_2\mathrm{I}^2+\alpha a_{35}a_{17}a_7^2j^2)\,/$$

$$(a_3a_1a_{13}a_{24}a_{29}\mathrm{I}^3-a_3a_1a_{13}a_{33}a_{20}\mathrm{I}^3-a_3a_1a_{15}a_{24}a_{29}\mathrm{I}^2+a_3a_1a_{15}a_{33}a_{20}\alpha\mathrm{I}^2-a_3a_1a_{19}a_{17}a_{29}\alpha+a_3a_1a_{19}a_{33}a_{20}\mathrm{I}^3b+a_3a_1a_{22}a_{29}a_{17}\mathrm{I}^2\alpha-a_3a_1a_{22}a_{33}\mathrm{I}^2b$$

$$+a_3a_1a_{17}a_{35}a_{20}\alpha\mathrm{I}^2+a_3a_1a_{24}a_{35}b\,\mathrm{I}^2-a_4a_{24}a_{12}a_{29}a_1\alpha\mathrm{I}^3+a_{12}a_4a_{33}a_1a_{20}\alpha\mathrm{I}^3-a_{12}a_6a_{19}a_1a_{29}\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1^2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1a_2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_7^2j^2\alpha\mathrm{I}^2$$

$$+a_{14}a_4a_{24}a_1a_{29}\alpha\mathrm{I}^2-a_{14}a_{33}a_4a_1a_{20}\alpha\mathrm{I}^2+a_{14}a_{19}a_6a_1a_{29}\alpha\mathrm{I}^3-\alpha a_{14}a_{19}a_{33}\mathrm{I}^3a_1^2-\alpha a_{14}a_{33}a_{19}a_1a_2\mathrm{I}^3+a_{12}a_{24}a_{35}a_1^2\alpha\mathrm{I}^3+a_{12}a_{24}a_{35}a_1a_2\alpha\mathrm{I}^3+a_{12}a_{35}a_{24}\alpha a_7^2j^2\mathrm{I}$$

$$+a_{12}a_{22}a_6a_1a_{29}\alpha\mathrm{I}-a_{14}a_{19}a_{33}a_7^2j^2\alpha\mathrm{I}-\alpha a_{14}a_{22}a_{29}\mathrm{I}^2a_1a_6+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1^2+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1a_2+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_7^2j^2+\alpha a_{14}a_{35}a_{20}\mathrm{I}^2a_1-\alpha a_{24}a_{35}a_{14}\mathrm{I}^2a_1^2$$

$$-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_1a_2-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_7^2j^2+a_{18}a_{17}a_4a_1a_{29}\alpha\mathrm{I}^3-a_{18}a_4a_1a_{33}b\,\mathrm{I}^3+a_{13}a_6a_1a_{29}b\,\mathrm{I}^4-a_{18}a_1^2a_{13}a_{33}\alpha\,\mathrm{I}^4-a_{18}a_{13}a_{33}a_1a_2\alpha\,\mathrm{I}^4-\alpha a_{18}a_{13}a_{33}\mathrm{I}^2a_7^2j^2$$

$$-\alpha a_{18}a_{15}a_6\mathrm{I}^3a_1a_{29}+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1^2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1a_2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_7^2j^2+b\,a_{18}a_6a_1a_{35}\mathrm{I}^3-\alpha a_{17}a_{18}a_{35}\mathrm{I}^3a_1^2-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_1a_2-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_7^2j^2$$

$$-\alpha a_{21}a_1a_4\mathrm{I}^2a_7^2a_{29}j^2+b\,a_1a_4a_{21}\mathrm{I}^2a_{33}j^2-a_{21}a_{13}a_6a_1a_{29}\alpha\mathrm{I}^3+a_{21}a_{15}a_{33}a_{13}a_1^2\alpha\mathrm{I}^3+a_{21}a_{13}a_{33}a_1a_2\alpha\mathrm{I}^3-a_{21}a_{13}a_{33}\alpha\mathrm{I}a_7^2j^2+a_{21}a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1^2$$

$$-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1a_2-\alpha a_{21}a_{15}a_{33}a_7^2j^2-b\,a_{21}a_{35}a_6\mathrm{I}^2a_1+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1^2+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1a_2+\alpha a_{21}a_{35}a_{17}a_7^2j^2+\alpha a_{34}a_4a_1a_{17}a_{20}\mathrm{I}^2-b\,a_{34}a_4a_1a_{24}\mathrm{I}^2$$

$$+\alpha a_{34}a_6a_{13}a_1a_{20}\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}a_1^2\mathrm{I}^3-\alpha a_{34}a_{24}a_{13}a_1a_2\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}\mathrm{I}a_7^2j^2-\alpha a_{34}a_{15}a_6a_1a_{20}\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1^2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1a_2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_7^2j^2$$

$$-b\,a_{34}a_{19}a_6a_1\mathrm{I}^3+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1^2+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1a_2+\alpha a_{34}a_{19}a_{17}\mathrm{I}a_7^2j^2+b\,a_{34}a_{22}a_1a_6\mathrm{I}-\alpha a_{34}a_{17}a_{22}a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_2a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_1a_2\mathrm{I}^2)(\alpha a_{34}+a_{23})\mathrm{H}$$

$$B=(-a_3a_{24}a_1a_{29}\mathrm{I}^2+a_3a_{33}a_{20}a_1\mathrm{I}^2-a_{18}a_6a_1a_{29}\mathrm{I}^3+a_{18}a_{33}a_1^2\mathrm{I}^3+a_{18}a_{33}a_1a_2\mathrm{I}^3+a_7^2j^2a_{33}a_{18}+a_{21}a_6a_1a_{29}\mathrm{I}^2-a_{33}a_{21}a_1^2j^2-a_{21}a_{33}a_1a_2\mathrm{I}^2$$

$$-a_{21}a_{33}a_7^2j^2-a_{34}a_6a_{20}a_1\mathrm{I}^2+a_{34}a_{24}a_1^2\mathrm{I}^2+a_{34}a_{24}a_2a_1\mathrm{I}^2+a_7^2j^2a_{34}a_{24}-a_3\mathrm{I}^2a_1a_{17}a_{29}\alpha+a_3\mathrm{I}^2a_1a_{33}b-a_1\mathrm{I}^3a_6a_{12}a_{29}\alpha+\mathrm{I}^3a_1^2a_{12}a_{33}\alpha+a_{33}\mathrm{I}^3a_1a_{12}a_2\alpha$$

$$+a_7^2j^2\mathrm{I}a_{12}a_{33}\alpha+a_{14}a_6a_1a_{29}\alpha\mathrm{I}^2-\alpha a_{14}a_{33}a_1^2\mathrm{I}^2-\alpha a_{14}a_{33}a_1a_2\mathrm{I}^2-\alpha a_{14}a_{33}a_7^2j^2-a_{34}\mathrm{I}^2a_1a_6b+\alpha a_{34}a_1^2a_{17}\mathrm{I}^2+\alpha a_{34}a_1a_{17}a_2\mathrm{I}^2+\alpha a_{34}a_{17}a_7^2j^2)\,/$$

$$(a_3a_1a_{13}a_{24}a_{29}\mathrm{I}^3-a_3a_1a_{13}a_{33}a_{20}\mathrm{I}^3-a_3a_1a_{15}a_{24}a_{29}\mathrm{I}^2+a_3a_1a_{15}a_{33}a_{20}\alpha\mathrm{I}^2-a_3a_1a_{19}a_{17}a_{29}\alpha+a_3a_1a_{19}a_{33}a_{20}\mathrm{I}^3b+a_3a_1a_{22}a_{29}a_{17}\mathrm{I}^2\alpha-a_3a_1a_{22}a_{33}\mathrm{I}^2b$$

$$+a_3a_1a_{17}a_{35}a_{20}\alpha\mathrm{I}^2+a_3a_1a_{24}a_{35}b\,\mathrm{I}^2-a_4a_{24}a_{12}a_{29}a_1\alpha\mathrm{I}^3+a_{12}a_4a_{33}a_1a_{20}\alpha\mathrm{I}^3-a_{12}a_6a_{19}a_1a_{29}\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1^2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1a_2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_7^2j^2\alpha\mathrm{I}^2$$

$$+a_{14}a_4a_{24}a_1a_{29}\alpha\mathrm{I}^2-a_{14}a_{33}a_4a_1a_{20}\alpha\mathrm{I}^2+a_{14}a_{19}a_6a_1a_{29}\alpha\mathrm{I}^3-\alpha a_{14}a_{19}a_{33}\mathrm{I}^3a_1^2-\alpha a_{14}a_{33}a_{19}a_1a_2\mathrm{I}^3+a_{12}a_{24}a_{35}a_1^2\alpha\mathrm{I}^3+a_{12}a_{24}a_{35}a_1a_2\alpha\mathrm{I}^3+a_{12}a_{35}a_{24}\alpha a_7^2j^2\mathrm{I}$$

$$+a_{12}a_{22}a_6a_1a_{29}\alpha\mathrm{I}-a_{14}a_{19}a_{33}a_7^2j^2\alpha\mathrm{I}-\alpha a_{14}a_{22}a_{29}\mathrm{I}^2a_1a_6+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1^2+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1a_2+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_7^2j^2+\alpha a_{14}a_{35}a_{20}\mathrm{I}^2a_1-\alpha a_{24}a_{35}a_{14}\mathrm{I}^2a_1^2$$

$$-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_1a_2-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_7^2j^2+a_{18}a_{17}a_4a_1a_{29}\alpha\mathrm{I}^3-a_{18}a_4a_1a_{33}b\,\mathrm{I}^3+a_{13}a_6a_1a_{29}b\,\mathrm{I}^4-a_{18}a_1^2a_{13}a_{33}\alpha\,\mathrm{I}^4-a_{18}a_{13}a_{33}a_1a_2\alpha\,\mathrm{I}^4-\alpha a_{18}a_{13}a_{33}\mathrm{I}^2a_7^2j^2$$

$$-\alpha a_{18}a_{15}a_6\mathrm{I}^3a_1a_{29}+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1^2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1a_2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_7^2j^2+b\,a_{18}a_6a_1a_{35}\mathrm{I}^3-\alpha a_{17}a_{18}a_{35}\mathrm{I}^3a_1^2-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_1a_2-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_7^2j^2$$

$$-\alpha a_{21}a_1a_4\mathrm{I}^2a_7^2a_{29}j^2+b\,a_1a_4a_{21}\mathrm{I}^2a_{33}j^2-a_{21}a_{13}a_6a_1a_{29}\alpha\mathrm{I}^3+a_{21}a_{15}a_{33}a_{13}a_1^2\alpha\mathrm{I}^3+a_{21}a_{13}a_{33}a_1a_2\alpha\mathrm{I}^3-a_{21}a_{13}a_{33}\alpha\mathrm{I}a_7^2j^2+a_{21}a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1^2$$

$$-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1a_2-\alpha a_{21}a_{15}a_{33}a_7^2j^2-b\,a_{21}a_{35}a_6\mathrm{I}^2a_1+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1^2+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1a_2+\alpha a_{21}a_{35}a_{17}a_7^2j^2+\alpha a_{34}a_4a_1a_{17}a_{20}\mathrm{I}^2-b\,a_{34}a_4a_1a_{24}\mathrm{I}^2$$

$$+\alpha a_{34}a_6a_{13}a_1a_{20}\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}a_1^2\mathrm{I}^3-\alpha a_{34}a_{24}a_{13}a_1a_2\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}\mathrm{I}a_7^2j^2-\alpha a_{34}a_{15}a_6a_1a_{20}\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1^2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1a_2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_7^2j^2$$

$$-b\,a_{34}a_{19}a_6a_1\mathrm{I}^3+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1^2+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1a_2+\alpha a_{34}a_{19}a_{17}\mathrm{I}a_7^2j^2+b\,a_{34}a_{22}a_1a_6\mathrm{I}-\alpha a_{34}a_{17}a_{22}a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_2a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_1a_2\mathrm{I}^2)(\alpha a_{34}+a_{23})\mathrm{H}$$

$$D=-(a_3a_{19}a_1a_{29}\mathrm{I}^3+a_3a_{22}a_{29}a_1\mathrm{I}^2-a_3a_{35}a_1a_{20}\mathrm{I}^2+a_{18}a_4a_1a_{29}\mathrm{I}^3+a_{18}a_{35}a_1^2\mathrm{I}^3+a_7^2j^2a_{35}a_{18}+a_{21}a_4a_1a_{29}\mathrm{I}^2-a_{21}a_{35}a_1^2\mathrm{I}^2-a_{21}a_{35}a_1a_2\mathrm{I}^2$$

$$-a_{22}a_{35}a_7^2j^2-a_{34}a_4a_{20}a_1\mathrm{I}^2+a_{34}a_{19}a_1^2\mathrm{I}^3+a_{34}a_{19}a_2a_1\mathrm{I}^3+a_7^2j^2\mathrm{I}a_{34}a_{19}-a_{34}\mathrm{I}^2a_{22}a_1^2+a_{34}\mathrm{I}^2a_{22}a_1a_2-a_1\mathrm{I}^3a_3a_{13}a_{29}\alpha+\mathrm{I}^2a_1a_3\alpha a_{15}a_{29}+a_{35}\mathrm{I}^2a_1a_3b$$

$$+a_{12}a_4a_{13}a_{29}\alpha\mathrm{I}^3-\alpha a_{12}a_{35}a_1^2\mathrm{I}^3-\alpha a_{12}a_{35}a_1a_2\mathrm{I}^3-\alpha a_{12}a_{35}\mathrm{I}a_7^2j^2-\alpha a_{14}a_4a_1a_{29}\mathrm{I}^2-\alpha a_{14}a_{35}a_1^2\mathrm{I}^2+\alpha a_{35}a_{14}a_7^2j^2+\mathrm{I}^2a_1a_2\alpha a_{35}a_{14}+\mathrm{I}^2a_1b\,a_{34}a_4+\alpha a_{13}a_{34}a_1^2\mathrm{I}^3$$

$$+a_{13}a_1a_2a_{34}\alpha\mathrm{I}^3+\alpha a_{13}a_{34}\mathrm{I}a_7^2j^2-\alpha a_{15}a_{34}a_1^2\mathrm{I}^2+a_{34}a_{15}a_2a_1\alpha\mathrm{I}^2-\alpha a_{15}a_{34}a_7^2j^2)\,/\,(a_3a_1a_{13}a_{24}a_{29}\mathrm{I}^3-a_3a_1a_{13}a_{33}a_{20}\mathrm{I}^3-a_3a_1a_{15}a_{24}a_{29}\mathrm{I}^2+a_3a_1a_{15}a_{33}a_{20}\alpha\mathrm{I}^2$$

$$-a_3a_1a_{19}a_{17}a_{29}\alpha+a_3a_1a_{19}a_{33}a_{20}\mathrm{I}^3b+a_3a_1a_{22}a_{29}a_{17}\mathrm{I}^2\alpha-a_3a_1a_{22}a_{33}\mathrm{I}^2b+a_3a_1a_{17}a_{35}a_{20}\alpha\mathrm{I}^2+a_3a_1a_{24}a_{35}b\,\mathrm{I}^2-a_4a_{24}a_{12}a_{29}a_1\alpha\mathrm{I}^3+a_{12}a_4a_{33}a_1a_{20}\alpha\mathrm{I}^3$$

$$-a_{12}a_6a_{19}a_1a_{29}\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1^2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1a_2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_7^2j^2\alpha\mathrm{I}^2+a_{14}a_4a_{24}a_1a_{29}\alpha\mathrm{I}^2-a_{14}a_{33}a_4a_1a_{20}\alpha\mathrm{I}^2+a_{14}a_{19}a_6a_1a_{29}\alpha\mathrm{I}^3-\alpha a_{14}a_{19}a_{33}\mathrm{I}^3a_1^2$$

$$-\alpha a_{14}a_{33}a_{19}a_1a_2\mathrm{I}^3+a_{12}a_{24}a_{35}a_1^2\alpha\mathrm{I}^3+a_{12}a_{24}a_{35}a_1a_2\alpha\mathrm{I}^3+a_{12}a_{35}a_{24}\alpha a_7^2j^2\mathrm{I}+a_{12}a_{22}a_6a_1a_{29}\alpha\mathrm{I}-a_{14}a_{19}a_{33}a_7^2j^2\alpha\mathrm{I}-\alpha a_{14}a_{22}a_{29}\mathrm{I}^2a_1a_6+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1^2+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1a_2$$

$$+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_7^2j^2+\alpha a_{14}a_{35}a_{20}\mathrm{I}^2a_1-\alpha a_{24}a_{35}a_{14}\mathrm{I}^2a_1^2-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_1a_2-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_7^2j^2+a_{18}a_{17}a_4a_1a_{29}\alpha\mathrm{I}^3-a_{18}a_4a_1a_{33}b\,\mathrm{I}^3+a_{13}a_6a_1a_{29}b\,\mathrm{I}^4-a_{18}a_1^2a_{13}a_{33}\alpha\,\mathrm{I}^4$$

$$-a_{18}a_{13}a_{33}a_1a_2\alpha\,\mathrm{I}^4-\alpha a_{18}a_{13}a_{33}\mathrm{I}^2a_7^2j^2-\alpha a_{18}a_{15}a_6\mathrm{I}^3a_1a_{29}+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1^2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1a_2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_7^2j^2+b\,a_{18}a_6a_1a_{35}\mathrm{I}^3-\alpha a_{17}a_{18}a_{35}\mathrm{I}^3a_1^2-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_1a_2$$

$$-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_7^2j^2-\alpha a_{21}a_1a_4\mathrm{I}^2a_7^2a_{29}j^2+b\,a_1a_4a_{21}\mathrm{I}^2a_{33}j^2-a_{21}a_{13}a_6a_1a_{29}\alpha\mathrm{I}^3+a_{21}a_{15}a_{33}a_{13}a_1^2\alpha\mathrm{I}^3+a_{21}a_{13}a_{33}a_1a_2\alpha\mathrm{I}^3-a_{21}a_{13}a_{33}\alpha\mathrm{I}a_7^2j^2+a_{21}a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2$$

$$-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1^2-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1a_2-\alpha a_{21}a_{15}a_{33}a_7^2j^2-b\,a_{21}a_{35}a_6\mathrm{I}^2a_1+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1^2+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1a_2+\alpha a_{21}a_{35}a_{17}a_7^2j^2+\alpha a_{34}a_4a_1a_{17}a_{20}\mathrm{I}^2-b\,a_{34}a_4a_1a_{24}\mathrm{I}^2$$

$$+\alpha a_{34}a_6a_{13}a_1a_{20}\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}a_1^2\mathrm{I}^3-\alpha a_{34}a_{24}a_{13}a_1a_2\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}\mathrm{I}a_7^2j^2-\alpha a_{34}a_{15}a_6a_1a_{20}\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1^2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1a_2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_7^2j^2$$

$$-b\,a_{34}a_{19}a_6a_1\mathrm{I}^3+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1^2+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1a_2+\alpha a_{34}a_{19}a_{17}\mathrm{I}a_7^2j^2+b\,a_{34}a_{22}a_1a_6\mathrm{I}-\alpha a_{34}a_{17}a_{22}a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_2a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_1a_2\mathrm{I}^2)(\alpha a_{34}+a_{23})\mathrm{H}$$

$$E=a_1\alpha\mathrm{I}(a_3a_{33}a_{19}-a_3a_{33}a_{22}+a_3a_{35}a_{24}-a_{18}a_{14}a_{33}\mathrm{I}+a_{18}a_{35}a_6\mathrm{I}+a_{33}a_4a_{21}-a_{21}a_{35}a_6-a_{34}a_4a_{24}-a_{35}a_6a_{19}\mathrm{I}+a_{34}a_6a_{22}+a_3a_{13}a_{33}\mathrm{I}-a_3a_{33}a_{15}+a_3a_{35}a_{17}$$

$$-a_{33}a_{12}a_4\mathrm{I}+a_{12}\mathrm{I}a_{35}a_6+a_{33}a_{14}a_4-a_{14}a_{35}a_6-a_{34}a_4a_{17}-a_{34}a_{13}a_6I+a_{34}a_6a_{15})\,/\,(a_3a_1a_{13}a_{24}a_{29}\mathrm{I}^3-a_3a_1a_{13}a_{33}a_{20}\mathrm{I}^3-a_3a_1a_{15}a_{24}a_{29}\mathrm{I}^2+a_3a_1a_{15}a_{33}a_{20}\alpha\mathrm{I}^2$$

$$-a_3a_1a_{19}a_{17}a_{29}\alpha+a_3a_1a_{19}a_{33}a_{20}\mathrm{I}^3b+a_3a_1a_{22}a_{29}a_{17}\mathrm{I}^2\alpha-a_3a_1a_{22}a_{33}\mathrm{I}^2b+a_3a_1a_{17}a_{35}a_{20}\alpha\mathrm{I}^2+a_3a_1a_{24}a_{35}b\,\mathrm{I}^2-a_4a_{24}a_{12}a_{29}a_1\alpha\mathrm{I}^3+a_{12}a_4a_{33}a_1a_{20}\alpha\mathrm{I}^3$$

$$-a_{12}a_6a_{19}a_1a_{29}\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1^2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1a_2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_7^2j^2\alpha\mathrm{I}^2+a_{14}a_4a_{24}a_1a_{29}\alpha\mathrm{I}^2-a_{14}a_{33}a_4a_1a_{20}\alpha\mathrm{I}^2+a_{14}a_{19}a_6a_1a_{29}\alpha\mathrm{I}^3-\alpha a_{14}a_{19}a_{33}\mathrm{I}^3a_1^2$$

$$-\alpha a_{14}a_{33}a_{19}a_1a_2\mathrm{I}^3+a_{12}a_{24}a_{35}a_1^2\alpha\mathrm{I}^3+a_{12}a_{24}a_{35}a_1a_2\alpha\mathrm{I}^3+a_{12}a_{35}a_{24}\alpha a_7^2j^2\mathrm{I}+a_{12}a_{22}a_6a_1a_{29}\alpha\mathrm{I}-a_{14}a_{19}a_{33}a_7^2j^2\alpha\mathrm{I}-\alpha a_{14}a_{22}a_{29}\mathrm{I}^2a_1a_6+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1^2+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1a_2$$

$$+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_7^2j^2+\alpha a_{14}a_{35}a_{20}\mathrm{I}^2a_1-\alpha a_{24}a_{35}a_{14}\mathrm{I}^2a_1^2-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_1a_2-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_7^2j^2+a_{18}a_{17}a_4a_1a_{29}\alpha\mathrm{I}^3-a_{18}a_4a_1a_{33}b\,\mathrm{I}^3+a_{13}a_6a_1a_{29}b\,\mathrm{I}^4-a_{18}a_1^2a_{13}a_{33}\alpha\,\mathrm{I}^4$$

$$-a_{18}a_{13}a_{33}a_1a_2\alpha\,\mathrm{I}^4-\alpha a_{18}a_{13}a_{33}\mathrm{I}^2a_7^2j^2-\alpha a_{18}a_{15}a_6\mathrm{I}^3a_1a_{29}+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1^2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1a_2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_7^2j^2+b\,a_{18}a_6a_1a_{35}\mathrm{I}^3-\alpha a_{17}a_{18}a_{35}\mathrm{I}^3a_1^2-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_1a_2$$

$$-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_7^2j^2-\alpha a_{21}a_1a_4\mathrm{I}^2a_7^2a_{29}j^2+b\,a_1a_4a_{21}\mathrm{I}^2a_{33}j^2-a_{21}a_{13}a_6a_1a_{29}\alpha\mathrm{I}^3+a_{21}a_{15}a_{33}a_{13}a_1^2\alpha\mathrm{I}^3+a_{21}a_{13}a_{33}a_1a_2\alpha\mathrm{I}^3-a_{21}a_{13}a_{33}\alpha\mathrm{I}a_7^2j^2+a_{21}a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2$$

$$-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1^2-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1a_2-\alpha a_{21}a_{15}a_{33}a_7^2j^2-b\,a_{21}a_{35}a_6\mathrm{I}^2a_1+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1^2+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1a_2+\alpha a_{21}a_{35}a_{17}a_7^2j^2+\alpha a_{34}a_4a_1a_{17}a_{20}\mathrm{I}^2-b\,a_{34}a_4a_1a_{24}\mathrm{I}^2$$

$$+\alpha a_{34}a_6a_{13}a_1a_{20}\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}a_1^2\mathrm{I}^3-\alpha a_{34}a_{24}a_{13}a_1a_2\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}\mathrm{I}a_7^2j^2-\alpha a_{34}a_{15}a_6a_1a_{20}\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1^2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1a_2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_7^2j^2$$

$$-b\,a_{34}a_{19}a_6a_1\mathrm{I}^3+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1^2+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1a_2+\alpha a_{34}a_{19}a_{17}\mathrm{I}a_7^2j^2+b\,a_{34}a_{22}a_1a_6\mathrm{I}-\alpha a_{34}a_{17}a_{22}a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_2a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_1a_2\mathrm{I}^2)(\alpha a_{34}+a_{23})\mathrm{H}$$

$$\begin{aligned}
N &= a_7\alpha j\,(a_3a_{33}a_{19}\mathrm{I}-a_3a_{33}a_{22}+a_3a_{35}a_{24}-a_{18}a_4a_{33}\mathrm{I}+a_{18}a_{35}a_6\mathrm{I}+a_{33}a_4a_{21}-a_{21}a_{35}a_6-a_{34}a_4a_{24}-a_{34}a_6a_{19}\mathrm{I}+a_{34}a_6a_{22}+a_3a_{13}a_{33}-a_3a_{33}a_{15}+a_3a_{35}a_{17}\\
&-a_{33}a_{12}a_4\mathrm{I}+a_{12}\mathrm{I}a_{35}a_6+a_{33}a_{14}a_4-a_{14}a_{35}a_6-a_{34}a_4a_{17}-a_{34}a_{13}a_6I+a_{34}a_6a_{15})/(a_3a_1a_{13}a_{24}a_{29}\mathrm{I}^3-a_3a_1a_{13}a_{33}a_{20}\mathrm{I}^3-a_3a_1a_{15}a_{24}a_{29}\mathrm{I}^2+a_3a_1a_{15}a_{33}a_{20}\alpha\mathrm{I}^2\\
&-a_3a_1a_{19}a_{17}a_{29}\alpha+a_3a_1a_{19}a_{33}a_{20}\mathrm{I}^3b+a_3a_1a_{22}a_{29}a_{17}\mathrm{I}^2\alpha-a_3a_1a_{22}a_{33}\mathrm{I}^2b+a_3a_1a_{17}a_{35}a_{20}\alpha\mathrm{I}^2+a_3a_1a_{24}a_{35}b\ \mathrm{I}^2-a_4a_{24}a_{12}a_{29}a_1\alpha\mathrm{I}^3+a_{12}a_4a_{33}a_1a_{20}\alpha\mathrm{I}^3\\
&-a_{12}a_6a_{19}a_1a_{29}\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1^2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1a_2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_7^2j^2\alpha\mathrm{I}^2+a_{14}a_4a_{24}a_1a_{29}\alpha\mathrm{I}^2-a_{14}a_{33}a_4a_1a_{20}\alpha\mathrm{I}^2+a_{14}a_{19}a_6a_1a_{29}\alpha\mathrm{I}^3-\alpha a_{14}a_{19}a_{33}\mathrm{I}^3a_1^2\\
&-\alpha a_{14}a_{33}a_{19}a_1a_2\mathrm{I}^3+a_{12}a_{24}a_{35}a_1^2\alpha\mathrm{I}^3+a_{12}a_{24}a_{35}a_1a_2\alpha\mathrm{I}^3+a_{12}a_{35}a_{24}\alpha a_7^2j^2\mathrm{I}+a_{12}a_{22}a_6a_1a_{29}\alpha\mathrm{I}-a_{14}a_{19}a_{33}a_7^2j^2\alpha\mathrm{I}-\alpha a_{14}a_{22}a_{29}\mathrm{I}^2a_1a_6+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1^2+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1a_2\\
&+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_7^2j^2+\alpha a_{14}a_{35}a_{20}\mathrm{I}^2a_1-\alpha a_{24}a_{35}a_{14}\mathrm{I}^2a_1^2-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_1a_2-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_7^2j^2+a_{18}a_{17}a_4a_1a_{29}\alpha\mathrm{I}^3-a_{18}a_4a_1a_{33}b\ \mathrm{I}^3+a_{13}a_6a_1a_{29}b\ \mathrm{I}^4-a_{18}a_1^2a_{13}a_{33}\alpha\ \mathrm{I}^4\\
&-a_{18}a_{13}a_{33}a_1a_2\alpha\ \mathrm{I}^4-\alpha a_{18}a_{13}a_{33}\mathrm{I}^2a_7^2j^2-\alpha a_{18}a_{15}a_6\mathrm{I}^3a_1a_{29}+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1^2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1a_2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_7^2j^2+b\ a_{18}a_6a_1a_{35}\mathrm{I}^3-\alpha a_{17}a_{18}a_{35}\mathrm{I}^3a_1^2-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_1a_2\\
&-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_7^2j^2-\alpha a_{21}a_1a_4\mathrm{I}^2a_7^2a_{29}j^2+b\ a_1a_4a_{21}\mathrm{I}^2a_{33}j^2-a_{21}a_{13}a_6a_1a_{29}\alpha\mathrm{I}^3+a_{21}a_{15}a_{33}a_{13}a_1^2\alpha\mathrm{I}^3+a_{21}a_{13}a_{33}a_1a_2\alpha\mathrm{I}^3-a_{21}a_{13}a_{33}\alpha\mathrm{I}a_7^2j^2+a_{21}a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2\\
&-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1^2-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1a_2-\alpha a_{21}a_{15}a_{33}a_7^2j^2-b\ a_{21}a_{35}a_6\mathrm{I}^2a_1+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1^2+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1a_2+\alpha a_{21}a_{35}a_{17}a_7^2j^2+\alpha a_{34}a_4a_1a_{17}a_{20}\mathrm{I}^2-b\ a_{34}a_4a_1a_{24}\mathrm{I}^2\\
&+\alpha a_{34}a_6a_{13}a_1a_{20}\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}a_1^2\mathrm{I}^3-\alpha a_{34}a_{24}a_{13}a_1a_2\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}\mathrm{I}a_7^2j^2-\alpha a_{34}a_{15}a_6a_1a_{20}\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1^2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1a_2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_7^2j^2\\
&-b\ a_{34}a_{19}a_6\mathrm{a}_1\mathrm{I}^3+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1^2+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1a_2+\alpha a_{34}a_{19}a_{17}\mathrm{I}a_7^2j^2+b\ a_{34}a_{22}a_1a_6\mathrm{I}-\alpha a_{34}a_{17}a_{22}a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_2a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_1\mathrm{a}_2\mathrm{I}^2)(\alpha a_{34}+a_{23})\mathrm{H}
\end{aligned}$$

$$\mathrm{I}=(j_m^2-\lambda_n^2),\ \mathrm{H}=\sum_{n=1}^{\infty}\frac{4\varepsilon_0}{n\pi\sinh(\frac{n\pi b}{a})\sin(\lambda_n y)e^{-j_m x}}\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a}).$$

Hence, Eq. (51) can be rewritten as

$$\begin{aligned}
\phi &= A^*\sum_{n=1}^{\infty}\frac{n\pi\sin^2(\lambda_n y)e^{-2j_m x}\ \sinh(\frac{n\pi b}{a})}{4\varepsilon_0\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})};\\
\psi &= B^*\sum_{n=1}^{\infty}\frac{n\pi\sin^2(\lambda_n y)e^{-2j_m x}\ \sinh(\frac{n\pi b}{a})}{4\varepsilon_0\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})};\\
C &= D^*\sum_{n=1}^{\infty}\frac{n\pi\sin^2(\lambda_n y)e^{-2j_m x}\ \sinh(\frac{n\pi b}{a})}{4\varepsilon_0\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})};\\
\mathrm{q}_1 &= E^*\sum_{n=1}^{\infty}\frac{n\pi\sin^2(\lambda_n y)e^{-2j_m x}\ \sinh(\frac{n\pi b}{a})}{4\varepsilon_0\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})};\\
P_1 &= N^*\sum_{n=1}^{\infty}\frac{n\pi\sin^2(\lambda_n y)e^{-2j_m x}\ \sinh(\frac{n\pi b}{a})}{4\varepsilon_0\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})};
\end{aligned}\tag{57}$$

Where,

$$\begin{aligned}
A^* &= (-a_4a_{24}a_1a_{29}\mathrm{I}^2+a_4a_{33}a_{20}a_1\mathrm{I}^2-a_{19}a_6a_1a_{29}\mathrm{I}^3+a_{19}a_{33}a_1^2\mathrm{I}^3+a_{19}a_{33}a_1a_2\mathrm{I}^3+a_7^2j^2a_{33}a_{19}+a_{22}a_6a_1a_{29}\mathrm{I}^2-a_{33}a_{22}a_1^2j^2-a_{22}a_{33}a_1a_2\mathrm{I}^2\\
&-a_{22}a_{33}a_7^2j^2-a_{35}a_6a_{20}a_1\mathrm{I}^2+a_{35}a_{24}a_1^2\mathrm{I}^2+a_{35}a_{24}a_2a_1\mathrm{I}^2+a_7^2j^2a_{35}a_{24}-a_4\mathrm{I}^2a_1a_{17}a_{29}\alpha+a_4\mathrm{I}^2a_1a_{33}b-a_7\mathrm{I}^3a_6a_{13}a_{29}\alpha+\mathrm{I}^3a_1^2a_{13}a_{33}\alpha+a_{33}\mathrm{I}^3a_1a_{13}a_2\alpha\\
&+a_7^2j^2\mathrm{I}a_{13}a_{33}\alpha+a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2-\alpha a_{15}a_{33}a_1^2\mathrm{I}^2-\alpha a_{15}a_{33}a_1a_2\mathrm{I}^2-\alpha a_{15}a_{33}a_7^2j^2-a_{35}\mathrm{I}^2a_1a_6b+\alpha a_{35}a_1^2a_{17}\mathrm{I}^2+\alpha a_{35}a_1a_{17}a_2\mathrm{I}^2+\alpha a_{35}a_{17}a_7^2j^2)/\\
&(a_3a_1a_{13}a_{24}a_{29}\mathrm{I}^3-a_3a_1a_{13}a_{33}a_{20}\mathrm{I}^3-a_3a_1a_{15}a_{24}a_{29}\mathrm{I}^2+a_3a_1a_{15}a_{33}a_{20}\alpha\mathrm{I}^2-a_3a_1a_{19}a_{17}a_{29}\alpha+a_3a_1a_{19}a_{33}a_{20}\mathrm{I}^3b+a_3a_1a_{22}a_{29}a_{17}\mathrm{I}^2\alpha-a_3a_1a_{22}a_{33}\mathrm{I}^2b\\
&+a_3a_1a_{17}a_{35}a_{20}\alpha\mathrm{I}^2+a_3a_1a_{24}a_{35}b\ \mathrm{I}^2-a_4a_{24}a_{12}a_{29}a_1\alpha\mathrm{I}^3+a_{12}a_4a_{33}a_1a_{20}\alpha\mathrm{I}^3-a_{12}a_6a_{19}a_1a_{29}\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1^2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_1a_2\alpha\mathrm{I}^4+a_{12}a_{33}a_{19}a_7^2j^2\alpha\mathrm{I}^2\\
&+a_{14}a_4a_{24}a_1a_{29}\alpha\mathrm{I}^2-a_{14}a_{33}a_4a_1a_{20}\alpha\mathrm{I}^2+a_{14}a_{19}a_6a_1a_{29}\alpha\mathrm{I}^3-\alpha a_{14}a_{19}a_{33}\mathrm{I}^3a_1^2-\alpha a_{14}a_{33}a_{19}a_1a_2\mathrm{I}^3+a_{12}a_{24}a_{35}a_1^2\alpha\mathrm{I}^3+a_{12}a_{24}a_{35}a_1a_2\alpha\mathrm{I}^3+a_{12}a_{35}a_{24}\alpha a_7^2j^2\mathrm{I}\\
&+a_{12}a_{22}a_6a_1a_{29}\alpha\mathrm{I}-a_{14}a_{19}a_{33}a_7^2j^2\alpha\mathrm{I}-\alpha a_{14}a_{22}a_{29}\mathrm{I}^2a_1a_6+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1^2+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1a_2+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_7^2j^2+\alpha a_{14}a_{35}a_{20}\mathrm{I}^2a_1-\alpha a_{24}a_{35}a_{14}\mathrm{I}^2a_1^2\\
&-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_1a_2-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_7^2j^2+a_{18}a_{17}a_4a_1a_{29}\alpha\mathrm{I}^3-a_{18}a_4a_1a_{33}b\ \mathrm{I}^3+a_{13}a_6a_1a_{29}b\ \mathrm{I}^4-a_{18}a_1^2a_{13}a_{33}\alpha\ \mathrm{I}^4-a_{18}a_{13}a_{33}a_1a_2\alpha\ \mathrm{I}^4-\alpha a_{18}a_{13}a_{33}\mathrm{I}^2a_7^2j^2\\
&-\alpha a_{18}a_{15}a_6\mathrm{I}^3a_1a_{29}+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1^2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1a_2+\alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_7^2j^2+b\ a_{18}a_6a_1a_{35}\mathrm{I}^3-\alpha a_{17}a_{18}a_{35}\mathrm{I}^3a_1^2-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_1a_2-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_7^2j^2\\
&-\alpha a_{21}a_1a_4\mathrm{I}^2a_7^2a_{29}j^2+b\ a_1a_4a_{21}\mathrm{I}^2a_{33}j^2-a_{21}a_{13}a_6a_1a_{29}\alpha\mathrm{I}^3+a_{21}a_{15}a_{33}a_{13}a_1^2\alpha\mathrm{I}^3+a_{21}a_{13}a_{33}a_1a_2\alpha\mathrm{I}^3-a_{21}a_{13}a_{33}\alpha\mathrm{I}a_7^2j^2+a_{21}a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1^2\\
&-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1a_2-\alpha a_{21}a_{15}a_{33}a_7^2j^2-b\ a_{21}a_{35}a_6\mathrm{I}^2a_1+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1^2+\alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1a_2+\alpha a_{21}a_{35}a_{17}a_7^2j^2+\alpha a_{34}a_4a_1a_{17}a_{20}\mathrm{I}^2-b\ a_{34}a_4a_1a_{24}\mathrm{I}^2\\
&+\alpha a_{34}a_6a_{13}a_1a_{20}\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}a_1^2\mathrm{I}^3-\alpha a_{34}a_{24}a_{13}a_1a_2\mathrm{I}^3-\alpha a_{34}a_{13}a_{24}\mathrm{I}a_7^2j^2-\alpha a_{34}a_{15}a_6a_1a_{20}\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1^2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_1a_2\mathrm{I}^2+\alpha a_{34}a_{15}a_{24}a_7^2j^2\\
&-b\ a_{34}a_{19}a_6\mathrm{a}_1\mathrm{I}^3+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1^2+\alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1a_2+\alpha a_{34}a_{19}a_{17}\mathrm{I}a_7^2j^2+b\ a_{34}a_{22}a_1a_6\mathrm{I}-\alpha a_{34}a_{17}a_{22}a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_2a_1^2\mathrm{I}^2-\alpha a_{34}a_{17}a_{22}a_1\mathrm{a}_2\mathrm{I}^2)(\alpha a_{34}+a_{23})
\end{aligned}$$

$$B^{*} = (-a_3a_{24}a_1a_{29}\mathrm{I}^2 + a_3a_{33}a_{20}a_1\mathrm{I}^2 - a_{18}a_6a_1a_{29}\mathrm{I}^3 + a_{18}a_{33}a_1^2\mathrm{I}^3 + a_{18}a_{33}a_1a_2\mathrm{I}^3 + a_7^2 j^2 a_{33}a_{18} + a_{21}a_6a_1a_{29}\mathrm{I}^2 - a_{33}a_{21}a_1^2 j^2 - a_{21}a_{33}a_1a_2\mathrm{I}^2$$
$$-a_{21}a_{33}a_7^2 j^2 - a_{34}a_6a_{20}a_1\mathrm{I}^2 + a_{34}a_{24}a_1^2\mathrm{I}^2 + a_{34}a_{24}a_2a_1\mathrm{I}^2 + a_7^2 j^2 a_{34}a_{24} - a_3\mathrm{I}^2a_1a_{17}a_{29}\alpha + a_3\mathrm{I}^2a_1a_{33}b - a_1\mathrm{I}^3a_6a_{12}a_{29}\alpha + \mathrm{I}^3a_1^2a_{12}a_{33}\alpha + a_{33}\mathrm{I}^3a_1a_{12}a_2\alpha$$
$$+a_7^2 j^2\mathrm{I}a_{12}a_{33}\alpha + a_{14}a_6a_1a_{29}\alpha\mathrm{I}^2 - \alpha a_{14}a_{33}a_1^2\mathrm{I}^2 - \alpha a_{14}a_{33}a_1a_2\mathrm{I}^2 - \alpha a_{14}a_{33}a_7^2 j^2 - a_{34}\mathrm{I}^2a_1a_6b + \alpha a_{34}a_1^2a_{17}\mathrm{I}^2 + \alpha a_{34}a_1a_{17}a_2\mathrm{I}^2 + \alpha a_{34}a_{17}a_7^2 j^2) /$$
$$(a_3a_1a_{13}a_{24}a_{29}\mathrm{I}^3 - a_3a_1a_{13}a_{33}a_{20}\mathrm{I}^3 - a_3a_1a_{15}a_{24}a_{29}\mathrm{I}^2 + a_3a_1a_{15}a_{33}a_{20}\alpha\mathrm{I}^2 - a_3a_1a_{19}a_{17}a_{29}\alpha + a_3a_1a_{19}a_{33}a_{20}\mathrm{I}^3b + a_3a_1a_{22}a_{29}a_{17}\mathrm{I}^2\alpha - a_3a_1a_{22}a_{33}\mathrm{I}^2b$$
$$+a_3a_1a_{17}a_{35}a_{20}\alpha\mathrm{I}^2 + a_3a_1a_{24}a_{35}b\ \mathrm{I}^2 - a_4a_{24}a_{12}a_{29}a_1\alpha\mathrm{I}^3 + a_{12}a_4a_{33}a_1a_{20}\alpha\mathrm{I}^3 - a_{12}a_6a_{19}a_1a_{29}\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_1^2\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_1a_2\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_7^2 j^2\alpha\mathrm{I}^2$$
$$+a_{14}a_4a_{24}a_1a_{29}\alpha\mathrm{I}^2 - a_{14}a_{33}a_4a_1a_{20}\alpha\mathrm{I}^2 + a_{14}a_{19}a_6a_1a_{29}\alpha\mathrm{I}^3 - \alpha a_{14}a_{19}a_{33}\mathrm{I}^3a_1^2 - \alpha a_{14}a_{33}a_{19}a_1a_2\mathrm{I}^3 + a_{12}a_{24}a_{35}a_1^2\alpha\mathrm{I}^3 + a_{12}a_{24}a_{35}a_1a_2\alpha\mathrm{I}^3 + a_{12}a_{35}a_{24}\alpha a_7^2 j^2\mathrm{I}$$
$$+a_{12}a_{22}a_6a_1a_{29}\alpha\mathrm{I} - a_{14}a_{19}a_{33}a_7^2 j^2\alpha\mathrm{I} - \alpha a_{14}a_{22}a_{29}\mathrm{I}^2a_1a_6 + \alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1^2 + \alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1a_2 + \alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_7^2 j^2 + \alpha a_{14}a_{35}a_{20}\mathrm{I}^2a_1 - \alpha a_{24}a_{35}a_{14}\mathrm{I}^2a_1^2$$
$$-\alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_1a_2 - \alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_7^2 j^2 + a_{18}a_{17}a_4a_1a_{29}\alpha\mathrm{I}^3 - a_{18}a_4a_1a_{33}b\ \mathrm{I}^3 + a_{13}a_6a_1a_{29}b\ \mathrm{I}^4 - a_{18}a_1^2a_{13}a_{33}\alpha\ \mathrm{I}^4 - a_{18}a_{13}a_{33}a_1a_2\alpha\ \mathrm{I}^4 - \alpha a_{18}a_{13}a_{33}\mathrm{I}^2a_7^2 j^2$$
$$-\alpha a_{18}a_{15}a_6\mathrm{I}^3a_1a_{29} + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1^2 + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1a_2 + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_7^2 j^2 + b\ a_{18}a_6a_1a_{35}\mathrm{I}^3 - \alpha a_{17}a_{18}a_{35}\mathrm{I}^3a_1^2 - \alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_1a_2 - \alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_7^2 j^2$$
$$-\alpha a_{21}a_1a_4\mathrm{I}^2a_7^2a_{29} j^2 + b\ a_1a_4a_{21}\mathrm{I}^2a_{33} j^2 - a_{21}a_{13}a_6a_1a_{29}\alpha\mathrm{I}^3 + a_{21}a_{15}a_{33}a_{13}a_1^2\alpha\mathrm{I}^3 + a_{21}a_{13}a_{33}a_1a_2\alpha\mathrm{I}^3 - a_{21}a_{13}a_{33}\alpha\mathrm{I}a_7^2 j^2 + a_{21}a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2 - \alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1^2$$
$$-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1a_2 - \alpha a_{21}a_{15}a_{33}a_7^2 j^2 - b\ a_{21}a_{35}a_6\mathrm{I}^2a_1 + \alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1^2 + \alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1a_2 + \alpha a_{21}a_{35}a_{17}a_7^2 j^2 + \alpha a_{34}a_4a_1a_{17}a_{20}\mathrm{I}^2 - b\ a_{34}a_4a_1a_{24}\mathrm{I}^2$$
$$+\alpha a_{34}a_6a_{13}a_1a_{20}\mathrm{I}^3 - \alpha a_{34}a_{13}a_{24}a_1^2\mathrm{I}^3 - \alpha a_{34}a_{24}a_{13}a_1a_2\mathrm{I}^3 - \alpha a_{34}a_{13}a_{24}\mathrm{I}a_7^2 j^2 - \alpha a_{34}a_{15}a_6a_1a_{20}\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_1^2\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_1a_2\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_7^2 j^2$$
$$-b\ a_{34}a_{19}a_6a_1\mathrm{I}^3 + \alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1^2 + \alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1a_2 + \alpha a_{34}a_{19}a_{17}\mathrm{I}a_7^2 j^2 + b\ a_{34}a_{22}a_1a_6\mathrm{I} - \alpha a_{34}a_{17}a_{22}a_1^2\mathrm{I}^2 - \alpha a_{34}a_{17}a_{22}a_2a_1^2\mathrm{I}^2 - \alpha a_{34}a_{17}a_{22}a_1a_2\mathrm{I}^2)(\alpha a_{34} + a_{23})$$

$$D^{*} = -(a_3a_{19}a_1a_{29}\mathrm{I}^3 + a_3a_{22}a_{29}a_1\mathrm{I}^2 - a_3a_{35}a_1a_{20}\mathrm{I}^2 + a_{18}a_4a_1a_{29}\mathrm{I}^3 + a_{18}a_{35}a_1^2\mathrm{I}^3 + a_7^2 j^2 a_{35}a_{18} + a_{21}a_4a_1a_{29}\mathrm{I}^2 - a_{21}a_{35}a_1^2\mathrm{I}^2 - a_{21}a_{35}a_1a_2\mathrm{I}^2$$
$$-a_{22}a_{35}a_7^2 j^2 - a_{34}a_4a_{20}a_1\mathrm{I}^2 + a_{34}a_{19}a_1^2\mathrm{I}^3 + a_{34}a_{19}a_2a_1\mathrm{I}^3 + a_7^2 j^2\mathrm{I}a_{34}a_{19} - a_{34}\mathrm{I}^2a_{22}a_1^2 + a_{34}\mathrm{I}^2a_{22}a_1a_2 - a_1\mathrm{I}^3a_3a_{13}a_{29}\alpha + \mathrm{I}^2a_1a_3\alpha a_{15}a_{29} + a_{35}\mathrm{I}^2a_1a_3b$$
$$+a_{12}a_4a_{13}a_{29}\alpha\mathrm{I}^3 - \alpha a_{12}a_{35}a_1^2\mathrm{I}^3 - \alpha a_{12}a_{35}a_1a_2\mathrm{I}^3 - \alpha a_{12}a_{35}\mathrm{I}a_7^2 j^2 - \alpha a_{14}a_4a_1a_{29}\mathrm{I}^2 - \alpha a_{14}a_{35}a_1^2\mathrm{I}^2 + \alpha a_{35}a_{14}a_7^2 j^2 + \mathrm{I}^2a_1a_2\alpha a_{35}a_{14} + \mathrm{I}^2a_1b\ a_{34}a_4 + \alpha a_{13}a_{34}a_1^2\mathrm{I}^3$$
$$+a_{13}a_1a_2a_{34}\alpha\mathrm{I}^3 + \alpha a_{13}a_{34}\mathrm{I}a_7^2 j^2 - \alpha a_{15}a_{34}a_1^2\mathrm{I}^2 + a_{34}a_{15}a_2a_1\alpha\mathrm{I}^2 - \alpha a_{15}a_{34}a_7^2 j^2) / (a_3a_1a_{13}a_{24}a_{29}\mathrm{I}^3 - a_3a_1a_{13}a_{33}a_{20}\mathrm{I}^3 - a_3a_1a_{15}a_{24}a_{29}\mathrm{I}^2 + a_3a_1a_{15}a_{33}a_{20}\alpha\mathrm{I}^2$$
$$-a_3a_1a_{19}a_{17}a_{29}\alpha + a_3a_1a_{19}a_{33}a_{20}\mathrm{I}^3b + a_3a_1a_{22}a_{29}a_{17}\mathrm{I}^2\alpha - a_3a_1a_{22}a_{33}\mathrm{I}^2b + a_3a_1a_{17}a_{35}a_{20}\alpha\mathrm{I}^2 + a_3a_1a_{24}a_{35}b\ \mathrm{I}^2 - a_4a_{24}a_{12}a_{29}a_1\alpha\mathrm{I}^3 + a_{12}a_4a_{33}a_1a_{20}\alpha\mathrm{I}^3$$
$$-a_{12}a_6a_{19}a_1a_{29}\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_1^2\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_1a_2\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_7^2 j^2\alpha\mathrm{I}^2 + a_{14}a_4a_{24}a_1a_{29}\alpha\mathrm{I}^2 - a_{14}a_{33}a_4a_1a_{20}\alpha\mathrm{I}^2 + a_{14}a_{19}a_6a_1a_{29}\alpha\mathrm{I}^3 - \alpha a_{14}a_{19}a_{33}\mathrm{I}^3a_1^2$$
$$-\alpha a_{14}a_{33}a_{19}a_1a_2\mathrm{I}^3 + a_{12}a_{24}a_{35}a_1^2\alpha\mathrm{I}^3 + a_{12}a_{24}a_{35}a_1a_2\alpha\mathrm{I}^3 + a_{12}a_{35}a_{24}\alpha a_7^2 j^2\mathrm{I} + a_{12}a_{22}a_6a_1a_{29}\alpha\mathrm{I} - a_{14}a_{19}a_{33}a_7^2 j^2\alpha\mathrm{I} - \alpha a_{14}a_{22}a_{29}\mathrm{I}^2a_1a_6 + \alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1^2 + \alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1a_2$$
$$+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_7^2 j^2 + \alpha a_{14}a_{35}a_{20}\mathrm{I}^2a_1 - \alpha a_{24}a_{35}a_{14}\mathrm{I}^2a_1^2 - \alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_1a_2 - \alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_7^2 j^2 + a_{18}a_{17}a_4a_1a_{29}\alpha\mathrm{I}^3 - a_{18}a_4a_1a_{33}b\ \mathrm{I}^3 + a_{13}a_6a_1a_{29}b\ \mathrm{I}^4 - a_{18}a_1^2a_{13}a_{33}\alpha\ \mathrm{I}^4$$
$$-a_{18}a_{13}a_{33}a_1a_2\alpha\ \mathrm{I}^4 - \alpha a_{18}a_{13}a_{33}\mathrm{I}^2a_7^2 j^2 - \alpha a_{18}a_{15}a_6\mathrm{I}^3a_1a_{29} + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1^2 + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1a_2 + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_7^2 j^2 + b\ a_{18}a_6a_1a_{35}\mathrm{I}^3 - \alpha a_{17}a_{18}a_{35}\mathrm{I}^3a_1^2 - \alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_1a_2$$
$$-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_7^2 j^2 - \alpha a_{21}a_1a_4\mathrm{I}^2a_7^2a_{29} j^2 + b\ a_1a_4a_{21}\mathrm{I}^2a_{33} j^2 - a_{21}a_{13}a_6a_1a_{29}\alpha\mathrm{I}^3 + a_{21}a_{15}a_{33}a_{13}a_1^2\alpha\mathrm{I}^3 + a_{21}a_{13}a_{33}a_1a_2\alpha\mathrm{I}^3 - a_{21}a_{13}a_{33}\alpha\mathrm{I}a_7^2 j^2 + a_{21}a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2$$
$$-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1^2 - \alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1a_2 - \alpha a_{21}a_{15}a_{33}a_7^2 j^2 - b\ a_{21}a_{35}a_6\mathrm{I}^2a_1 + \alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1^2 + \alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1a_2 + \alpha a_{21}a_{35}a_{17}a_7^2 j^2 + \alpha a_{34}a_4a_1a_{17}a_{20}\mathrm{I}^2 - b\ a_{34}a_4a_1a_{24}\mathrm{I}^2$$
$$+\alpha a_{34}a_6a_{13}a_1a_{20}\mathrm{I}^3 - \alpha a_{34}a_{13}a_{24}a_1^2\mathrm{I}^3 - \alpha a_{34}a_{24}a_{13}a_1a_2\mathrm{I}^3 - \alpha a_{34}a_{13}a_{24}\mathrm{I}a_7^2 j^2 - \alpha a_{34}a_{15}a_6a_1a_{20}\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_1^2\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_1a_2\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_7^2 j^2$$
$$-b\ a_{34}a_{19}a_6a_1\mathrm{I}^3 + \alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1^2 + \alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1a_2 + \alpha a_{34}a_{19}a_{17}\mathrm{I}a_7^2 j^2 + b\ a_{34}a_{22}a_1a_6\mathrm{I} - \alpha a_{34}a_{17}a_{22}a_1^2\mathrm{I}^2 - \alpha a_{34}a_{17}a_{22}a_2a_1^2\mathrm{I}^2 - \alpha a_{34}a_{17}a_{22}a_1a_2\mathrm{I}^2)(\alpha a_{34} + a_{23})$$

$$E^{*} = a_1\alpha\mathrm{I}(a_3a_{33}a_{19} - a_3a_{33}a_{22} + a_3a_{35}a_{24} - a_{18}a_{14}a_{33}\mathrm{I} + a_{18}a_{35}a_6\mathrm{I} + a_{33}a_4a_{21} - a_{21}a_{35}a_6 - a_{34}a_4a_{24} - a_{35}a_6a_{19}\mathrm{I} + a_{34}a_6a_{22} + a_3a_{13}a_{33}\mathrm{I} - a_3a_{33}a_{15} + a_3a_{35}a_{17}$$
$$-a_{33}a_{12}a_4\mathrm{I} + a_{12}\mathrm{I}a_{35}a_6 + a_{33}a_{14}a_4 - a_{14}a_{35}a_6 - a_{34}a_4a_{17} - a_{34}a_{13}a_6 I + a_{34}a_6a_{15}) / (a_3a_1a_{13}a_{24}a_{29}\mathrm{I}^3 - a_3a_1a_{13}a_{33}a_{20}\mathrm{I}^3 - a_3a_1a_{15}a_{24}a_{29}\mathrm{I}^2 + a_3a_1a_{15}a_{33}a_{20}\alpha\mathrm{I}^2$$
$$-a_3a_1a_{19}a_{17}a_{29}\alpha + a_3a_1a_{19}a_{33}a_{20}\mathrm{I}^3b + a_3a_1a_{22}a_{29}a_{17}\mathrm{I}^2\alpha - a_3a_1a_{22}a_{33}\mathrm{I}^2b + a_3a_1a_{17}a_{35}a_{20}\alpha\mathrm{I}^2 + a_3a_1a_{24}a_{35}b\ \mathrm{I}^2 - a_4a_{24}a_{12}a_{29}a_1\alpha\mathrm{I}^3 + a_{12}a_4a_{33}a_1a_{20}\alpha\mathrm{I}^3$$
$$-a_{12}a_6a_{19}a_1a_{29}\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_1^2\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_1a_2\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_7^2 j^2\alpha\mathrm{I}^2 + a_{14}a_4a_{24}a_1a_{29}\alpha\mathrm{I}^2 - a_{14}a_{33}a_4a_1a_{20}\alpha\mathrm{I}^2 + a_{14}a_{19}a_6a_1a_{29}\alpha\mathrm{I}^3 - \alpha a_{14}a_{19}a_{33}\mathrm{I}^3a_1^2$$
$$-\alpha a_{14}a_{33}a_{19}a_1a_2\mathrm{I}^3 + a_{12}a_{24}a_{35}a_1^2\alpha\mathrm{I}^3 + a_{12}a_{24}a_{35}a_1a_2\alpha\mathrm{I}^3 + a_{12}a_{35}a_{24}\alpha a_7^2 j^2\mathrm{I} + a_{12}a_{22}a_6a_1a_{29}\alpha\mathrm{I} - a_{14}a_{19}a_{33}a_7^2 j^2\alpha\mathrm{I} - \alpha a_{14}a_{22}a_{29}\mathrm{I}^2a_1a_6 + \alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1^2 + \alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1a_2$$
$$+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_7^2 j^2 + \alpha a_{14}a_{35}a_{20}\mathrm{I}^2a_1 - \alpha a_{24}a_{35}a_{14}\mathrm{I}^2a_1^2 - \alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_1a_2 - \alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_7^2 j^2 + a_{18}a_{17}a_4a_1a_{29}\alpha\mathrm{I}^3 - a_{18}a_4a_1a_{33}b\ \mathrm{I}^3 + a_{13}a_6a_1a_{29}b\ \mathrm{I}^4 - a_{18}a_1^2a_{13}a_{33}\alpha\ \mathrm{I}^4$$
$$-a_{18}a_{13}a_{33}a_1a_2\alpha\ \mathrm{I}^4 - \alpha a_{18}a_{13}a_{33}\mathrm{I}^2a_7^2 j^2 - \alpha a_{18}a_{15}a_6\mathrm{I}^3a_1a_{29} + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1^2 + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1a_2 + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_7^2 j^2 + b\ a_{18}a_6a_1a_{35}\mathrm{I}^3 - \alpha a_{17}a_{18}a_{35}\mathrm{I}^3a_1^2 - \alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_1a_2$$
$$-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_7^2 j^2 - \alpha a_{21}a_1a_4\mathrm{I}^2a_7^2a_{29} j^2 + b\ a_1a_4a_{21}\mathrm{I}^2a_{33} j^2 - a_{21}a_{13}a_6a_1a_{29}\alpha\mathrm{I}^3 + a_{21}a_{15}a_{33}a_{13}a_1^2\alpha\mathrm{I}^3 + a_{21}a_{13}a_{33}a_1a_2\alpha\mathrm{I}^3 - a_{21}a_{13}a_{33}\alpha\mathrm{I}a_7^2 j^2 + a_{21}a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2$$
$$-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1^2 - \alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1a_2 - \alpha a_{21}a_{15}a_{33}a_7^2 j^2 - b\ a_{21}a_{35}a_6\mathrm{I}^2a_1 + \alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1^2 + \alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1a_2 + \alpha a_{21}a_{35}a_{17}a_7^2 j^2 + \alpha a_{34}a_4a_1a_{17}a_{20}\mathrm{I}^2 - b\ a_{34}a_4a_1a_{24}\mathrm{I}^2$$
$$+\alpha a_{34}a_6a_{13}a_1a_{20}\mathrm{I}^3 - \alpha a_{34}a_{13}a_{24}a_1^2\mathrm{I}^3 - \alpha a_{34}a_{24}a_{13}a_1a_2\mathrm{I}^3 - \alpha a_{34}a_{13}a_{24}\mathrm{I}a_7^2 j^2 - \alpha a_{34}a_{15}a_6a_1a_{20}\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_1^2\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_1a_2\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_7^2 j^2$$
$$-b\ a_{34}a_{19}a_6a_1\mathrm{I}^3 + \alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1^2 + \alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1a_2 + \alpha a_{34}a_{19}a_{17}\mathrm{I}a_7^2 j^2 + b\ a_{34}a_{22}a_1a_6\mathrm{I} - \alpha a_{34}a_{17}a_{22}a_1^2\mathrm{I}^2 - \alpha a_{34}a_{17}a_{22}a_2a_1^2\mathrm{I}^2 - \alpha a_{34}a_{17}a_{22}a_1a_2\mathrm{I}^2)(\alpha a_{34} + a_{23})$$

$$N^{*} = a_7\alpha j\,(a_3a_{33}a_{19}\mathrm{I} - a_3a_{33}a_{22} + a_3a_{35}a_{24} - a_{18}a_4a_{33}\mathrm{I} + a_{18}a_{35}a_6\mathrm{I} + a_{33}a_4a_{21} - a_{21}a_{35}a_6 - a_{34}a_4a_{24} - a_{34}a_6a_{19}\mathrm{I} + a_{34}a_6a_{22} + a_3a_{13}a_{33} - a_3a_{33}a_{15} + a_3a_{35}a_{17}$$
$$-a_{33}a_{12}a_4\mathrm{I} + a_{12}\mathrm{I}a_{35}a_6 + a_{33}a_{14}a_4 - a_{14}a_{35}a_6 - a_{34}a_4a_{17} - a_{34}a_{13}a_6 I + a_{34}a_6a_{15}) / (a_3a_1a_{13}a_{24}a_{29}\mathrm{I}^3 - a_3a_1a_{13}a_{33}a_{20}\mathrm{I}^3 - a_3a_1a_{15}a_{24}a_{29}\mathrm{I}^2 + a_3a_1a_{15}a_{33}a_{20}\alpha\mathrm{I}^2$$
$$-a_3a_1a_{19}a_{17}a_{29}\alpha + a_3a_1a_{19}a_{33}a_{20}\mathrm{I}^3b + a_3a_1a_{22}a_{29}a_{17}\mathrm{I}^2\alpha - a_3a_1a_{22}a_{33}\mathrm{I}^2b + a_3a_1a_{17}a_{35}a_{20}\alpha\mathrm{I}^2 + a_3a_1a_{24}a_{35}b\ \mathrm{I}^2 - a_4a_{24}a_{12}a_{29}a_1\alpha\mathrm{I}^3 + a_{12}a_4a_{33}a_1a_{20}\alpha\mathrm{I}^3$$
$$-a_{12}a_6a_{19}a_1a_{29}\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_1^2\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_1a_2\alpha\mathrm{I}^4 + a_{12}a_{33}a_{19}a_7^2 j^2\alpha\mathrm{I}^2 + a_{14}a_4a_{24}a_1a_{29}\alpha\mathrm{I}^2 - a_{14}a_{33}a_4a_1a_{20}\alpha\mathrm{I}^2 + a_{14}a_{19}a_6a_1a_{29}\alpha\mathrm{I}^3 - \alpha a_{14}a_{19}a_{33}\mathrm{I}^3a_1^2$$
$$-\alpha a_{14}a_{33}a_{19}a_1a_2\mathrm{I}^3 + a_{12}a_{24}a_{35}a_1^2\alpha\mathrm{I}^3 + a_{12}a_{24}a_{35}a_1a_2\alpha\mathrm{I}^3 + a_{12}a_{35}a_{24}\alpha a_7^2 j^2\mathrm{I} + a_{12}a_{22}a_6a_1a_{29}\alpha\mathrm{I} - a_{14}a_{19}a_{33}a_7^2 j^2\alpha\mathrm{I} - \alpha a_{14}a_{22}a_{29}\mathrm{I}^2a_1a_6 + \alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1^2 + \alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_1a_2$$
$$+\alpha a_{14}a_{22}a_{33}\mathrm{I}^2a_7^2 j^2 + \alpha a_{14}a_{35}a_{20}\mathrm{I}^2a_1 - \alpha a_{24}a_{35}a_{14}\mathrm{I}^2a_1^2 - \alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_1a_2 - \alpha a_{14}a_{24}a_{35}\mathrm{I}^2a_7^2 j^2 + a_{18}a_{17}a_4a_1a_{29}\alpha\mathrm{I}^3 - a_{18}a_4a_1a_{33}b\ \mathrm{I}^3 + a_{13}a_6a_1a_{29}b\ \mathrm{I}^4 - a_{18}a_1^2a_{13}a_{33}\alpha\ \mathrm{I}^4$$
$$-a_{18}a_{13}a_{33}a_1a_2\alpha\ \mathrm{I}^4 - \alpha a_{18}a_{13}a_{33}\mathrm{I}^2a_7^2 j^2 - \alpha a_{18}a_{15}a_6\mathrm{I}^3a_1a_{29} + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1^2 + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_1a_2 + \alpha a_{18}a_{15}a_{33}\mathrm{I}^3a_7^2 j^2 + b\ a_{18}a_6a_1a_{35}\mathrm{I}^3 - \alpha a_{17}a_{18}a_{35}\mathrm{I}^3a_1^2 - \alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_1a_2$$
$$-\alpha a_{18}a_{17}a_{35}\mathrm{I}^3a_7^2 j^2 - \alpha a_{21}a_1a_4\mathrm{I}^2a_7^2a_{29} j^2 + b\ a_1a_4a_{21}\mathrm{I}^2a_{33} j^2 - a_{21}a_{13}a_6a_1a_{29}\alpha\mathrm{I}^3 + a_{21}a_{15}a_{33}a_{13}a_1^2\alpha\mathrm{I}^3 + a_{21}a_{13}a_{33}a_1a_2\alpha\mathrm{I}^3 - a_{21}a_{13}a_{33}\alpha\mathrm{I}a_7^2 j^2 + a_{21}a_{15}a_6a_1a_{29}\alpha\mathrm{I}^2$$
$$-\alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1^2 - \alpha a_{21}a_{15}a_{33}\mathrm{I}^2a_1a_2 - \alpha a_{21}a_{15}a_{33}a_7^2 j^2 - b\ a_{21}a_{35}a_6\mathrm{I}^2a_1 + \alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1^2 + \alpha a_{21}a_{17}a_{35}\mathrm{I}^2a_1a_2 + \alpha a_{21}a_{35}a_{17}a_7^2 j^2 + \alpha a_{34}a_4a_1a_{17}a_{20}\mathrm{I}^2 - b\ a_{34}a_4a_1a_{24}\mathrm{I}^2$$
$$+\alpha a_{34}a_6a_{13}a_1a_{20}\mathrm{I}^3 - \alpha a_{34}a_{13}a_{24}a_1^2\mathrm{I}^3 - \alpha a_{34}a_{24}a_{13}a_1a_2\mathrm{I}^3 - \alpha a_{34}a_{13}a_{24}\mathrm{I}a_7^2 j^2 - \alpha a_{34}a_{15}a_6a_1a_{20}\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_1^2\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_1a_2\mathrm{I}^2 + \alpha a_{34}a_{15}a_{24}a_7^2 j^2$$
$$-b\ a_{34}a_{19}a_6a_1\mathrm{I}^3 + \alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1^2 + \alpha a_{34}a_{19}a_{17}\mathrm{I}^3a_1a_2 + \alpha a_{34}a_{19}a_{17}\mathrm{I}a_7^2 j^2 + b\ a_{34}a_{22}a_1a_6\mathrm{I} - \alpha a_{34}a_{17}a_{22}a_1^2\mathrm{I}^2 - \alpha a_{34}a_{17}a_{22}a_2a_1^2\mathrm{I}^2 - \alpha a_{34}a_{17}a_{22}a_1a_2\mathrm{I}^2)(\alpha a_{34} + a_{23})$$

Substituting from (57) into (45) with adding the solution of time we get

$$u = E^* \frac{\partial}{\partial x}\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})} - N^* \frac{\partial}{\partial y}\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})} + T_0 e^{-t(\frac{1}{\tau_0}+\frac{1}{\omega_1\tau^0})}, \tag{58}$$

$$v = E^* \frac{\partial}{\partial y}\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})} - N^* \frac{\partial}{\partial x}\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})} + T_0 e^{-t(\frac{1}{\tau_0}+\frac{1}{\omega_1\tau^0})}, \tag{59}$$

Substituting from (58), (59), into (18-20) yields

$$\sigma_{xx} = (\frac{\lambda}{\beta T_0})\frac{\partial v}{\partial y} + (\frac{2\mu+\lambda}{\beta T_0})\frac{\partial u}{\partial x} - \sum_{n=1}^{\infty}\frac{4\varepsilon_0}{n\pi\sinh(\frac{n\pi b}{a})}\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a}) - T_0 e^{-t(\frac{1}{\tau_0}+\frac{1}{\omega_1\tau^0})} - (\frac{\gamma^*\rho}{\beta T_0})D^*\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})} + a_3 A^*\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})} + a_4 B^*\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})}, \tag{60}$$

$$\sigma_{yy} = (\frac{\lambda}{\beta T_0})\frac{\partial u}{\partial x} + (\frac{2\mu+\lambda}{\beta T_0})\frac{\partial v}{\partial y} - \sum_{n=1}^{\infty}\frac{4\varepsilon_0}{n\pi\sinh(\frac{n\pi b}{a})}\sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a}) - T_0 e^{-t(\frac{1}{\tau_0}+\frac{1}{\omega_1\tau^0})} - (\frac{\gamma^*\rho}{\beta T_0})D^*\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})} + a_3 A^*\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})} + a_4 B^*\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})}, \tag{61}$$

$$\sigma_{xy} = \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}). \tag{62}$$

Dimensionless variables for the components of σ_i , τ_i are:

$$\sigma_2 = \eta_1\,\phi_{,y} + \eta_2\,\psi_{,y}\,, \tag{63}$$

$$\tau_2 = \eta_2\,\phi_{,y} + \eta_3\,\psi_{,y}\,. \tag{64}$$

Where, $\eta_1 = \frac{\alpha_1}{k_1\omega_1^2}$, $\eta_2 = \frac{b_1\alpha_1}{\alpha k_1\omega_1^2}$, $\eta_3 = \frac{\gamma\alpha_1}{\alpha k_1\omega_1^2}$.

To get the solution of σ_2 and τ_2 substituting from Eq. (57) in (63) and (64)

$$\sigma_2 = \eta_1 A^*\frac{\partial}{\partial y}\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})} + \eta_2 B^*\frac{\partial}{\partial y}\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})}, \tag{65}$$

$$\tau_2 = \eta_2 A^*\frac{\partial}{\partial y}\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})} + \eta_3 B^*\frac{\partial}{\partial y}\sum_{n=1}^{\infty}\frac{n\pi \sin^2(\lambda_n y)e^{-2j_m x}\sinh(\frac{n\pi b}{a})}{4\varepsilon_0 \sin(\frac{n\pi x}{a})\sinh(\frac{n\pi y}{a})}, \tag{66}$$

6. Boundary Conditions

We apply the following three boundary conditions for the present problem at the plane surface $x = 0$ and $y = a$.

$$\sigma_{xx}(0,a) = \sigma_0, \quad \sigma_{xy}(0,a) = 0, \sigma_{yy}(0,a) = \sigma \tag{67}$$

Finally, employing Eq. (67) in (60), (61) and (62) and using Matlab program for clarify the numerical values and graph the curves of temperature.

7. Numerical Results

We now present some numerical results. For this purpose, copper is taken as a material of thermoelastic for which we take the following values of temperature.

$\lambda = 7.7\times10^{10} N.M^{-2}$, $\mu = 3.86\times10^{10} N.m^{-2}$, $K = 3.86\times10^{3} N.s^{-1}.K^{-1}$,
$\alpha_t = 1.78\times10^{-5} K^{-1}$, $\rho = 8954 Kg.m^{-3}$, $C^* = 383.1 J.Kg^{-1}K^{-1}$, $T_0 = 293K$, $\tau^0 = 2.71\times10^{-13}$
$b = 0.1\times10 \wedge(-5)$, $\varepsilon_0 = 10^4$, $a = 10\wedge 4$, $\sigma_0 = 0.0002$, $\sigma = 0.0001$

The double porous parameters are taken as [15],

$\alpha = 1.3\times10^{-5} N$, $b_1 = 0.12\times10^{-5} N$, $\gamma = 1.1\times10^{-5} N.m^{-2}$, $\gamma_1 = 0.16\times10^{5} N.m^{-2}$, $\gamma_2 = 0.219\times10^{5} N.m^{-2}$,
$\alpha_1 = 2.3\times10^{10} N.m^{-2}$, $\alpha_2 = 2.4\times10^{10} N.m^{-2}$, $\alpha_3 = 2.5\times10^{10} N.m^{-2}$, $d = 0.1\times10^{10} N.m^{-2}$,
$b = 0.9\times10^{10} N.m^{-2}$, $K_2 = 0.1546\times10^{-12} N.m^{-2}$, $K_1 = 0.1456\times10^{-12} N.m^{-2}$.

The numerical technique, outlined above, was used for get the real part of the temperature T at different time. **Figs. 1, 2** describe the variation of the horizontal temperature T against y at $t = 0.1, 0.01$ with different value of τ_0 in the framework of the L–S theory. It is clear that the function of temperature T began to decreases until it decay to zero.

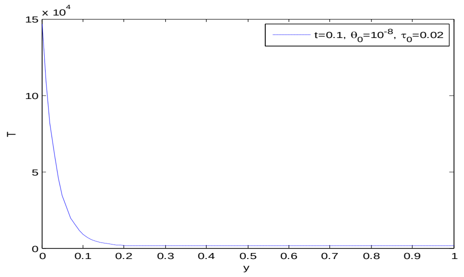

Figure 1: Distribution of the temperature T against y at $t = 0.1, \varepsilon_0 = 10\wedge(-8), \tau_0 = 0.02$.

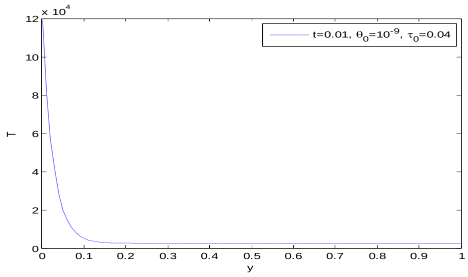

Figure 2: Distribution of temperature T against y at $t = 0.01, \varepsilon_0 = 10\wedge(-9), \tau_0 = 0.04$.

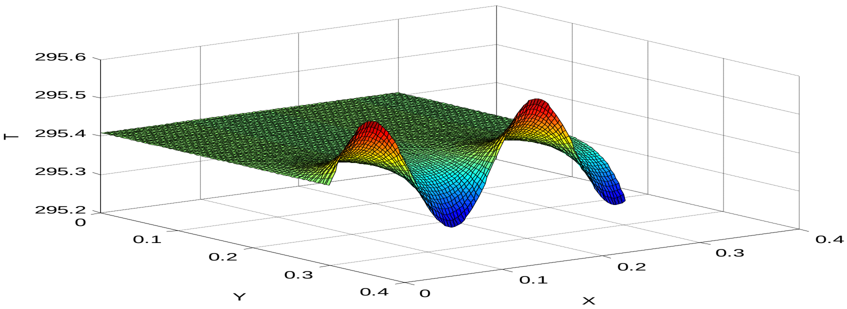

Figure 3: 3D for Distribution of the temperature T at $t = 0.1, \varepsilon_0 = 10\wedge(-8), \tau_0 = 0.02$.

Table 1: The numerical values of temperature T at $t = 0.01, \tau_0 = 0.04$.

T at $t = 0.01, \tau_0 = 0.04$.				
n_1	n_2	n_3	n_4	n_5
0.0000	0.0000	0.0000	0.0000	0.0000
0.0258	0.0261	0.0264	0.0266	0.0269
0.0516	0.0522	0.0527	0.0533	0.0539
0.1032	0.1043	0.1054	0.1066	0.1077
0.1290	0.1304	0.1318	0.1332	0.1346
0.1547	0.1564	0.1582	0.1599	0.1616
0.1805	0.1825	0.1845	0.1865	0.1885

8. Conclusion

There are some important phenomena are observed:

1- General solutions have been found using the separation of variable method, which differ from the normal mode method in that it separates the equation of time from displacement; and makes us see each function separated by different constants; we can see also the behavior of temperature function at certain values using programmingafter applying the boundary conditions.

2- Numerical solutions have been determined by Matlab program.

* Table 1shows the change of the temperature T at every n at certain values.

The problem though theoretical, but it can provide useful information for experimental researchers working in the field of geophysics, earthquake engineering, along with seismologist working in the field of mining tremors and drilling into the crust of the earth. The numerical treatment of the general system of equations and conditions governing the phenomenon may be useful in getting rid of the limitations of the method of separated of variables technique and this task is in progress.

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