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Research Article

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Model-Based Predictive Control of Drum Boiler

Zuheir Ahmad

Scientific Services Department, Atomic Energy Commission of Syria, P.O. Box 6091, Damascus, Syria Tel.: +963-11-2132580; fax: +963-11-6112289; e-mail: pscientific5@aec.org.sy

Abstract This paper is concerned with the drum boiler control problem. The study is based on Åström-Bell model of the boiler. A linearized model of the boiler is extracted around medium load operating point. The linearized model is used to design an MPC algorithm, which consists of two main steps: state estimation and MPC optimizer. Kalman filter is used to estimate the state of the model. Some constraints conditions are accounted for in the design of the MPC optimizer. The performance of the controller was tested via simulation, and the results showed that the controller works very well.

Keywords Drum boiler; A-B model; Nonlinear; Non-minimum phase; constraints; Model predictive control; Kalman filter; MPC optimizer; Linearization

1. Introduction

Drum boiler is an important part of most power plants. The cost of the power plant operation comes mainly from fuel, which is used to supply the boiler with heat. In order to minimize the fuel bill and, as a result, to maximize the plant efficiency, an advanced control strategy is needed to follow the changes of the demanded power with keeping the drum pressure and the drum water level under control. The need for an advanced control strategy comes from the complexity of the drum boiler, which arises from the nonlinear and non-minimum phase characteristics of the boiler.

In the boiler area, there are several models ranging from complex knowledge based models to experimental models derived from special plant tests [1]. The control studies described in this paper are based on the model developed by Åström and Bell for a Swedish power station. The A-B model is very attractive due to its simplicity (i.e. it needs only few parameters to characterize) and its compatibility with real plant experimental data (see [2]).

The nonlinear characteristics of the boiler results, mainly, from the fact that the behavior of the boiler relies on the operating power. On the other hand, the non-minimum phase characteristics arise from the shrink and swell effects. There are several approaches used to overcome these difficulties. One of the most commonly used approach in boilers control is the so-called "three-element control". This strategy is based on several cascaded PID controllers used to regulate the drum pressure and the water level.

The main disadvantage of PID controllers, like other linear controllers, is that they can not deal with constraints. The constraints arise from the physical limits of the input and output of the plant actuators. In these circumstances, nonlinear control strategy is necessary.

In this paper, a model-based predictive control is used. The performance of the cont- roller will be evaluated via some nonlinear simulations using Matlab/Simulink tools.

2. Drum Boiler Model

A simplified diagram of the drum boiler and downcomer-riser circulation loop is shown in Figure 1. The drum boiler model used in this study has been developed by Åström and Bell. The model is formed using the basic

thermodynamics mass and energy balance equations [2]. The model is implemented in Matlab. The parameters used in the simulation are based on the P16-G16 power station in Sweden.



Figure 1: Diagram of drum-downcomer-riser circulation loop

3. Factors Leading To Poor Control

Concerning the drum boiler control problem, researchers have to take into account the difficulties listed below when they deal with this problem:

- *Nonlinear plant characteristics*. The plant dynamics are highly nonlinear. This can be shown by Figure 2. The response of the water level in the drum shows significant variation with operating power.
- *Non-minimum phase plant characteristics*. The plant exhibits inverse response behavior (Figure 2), particularly at low operating power due to the so-called "swell and shrink" effects. This phen- omenon can mislead any classical linear controller (e.g. PID controllers) causing wide swings in the response of the system.
- *Constraints*. The feedwater, steam and fuel valves can only deliver limited quantities of water, steam and fuel, respectively. In addition, these valves can't change their states with an infinite speed. This imposes a hard limitation on the available control action. Moreover input constraints can lead to the classical controller windup problem [3], which causes degradation of system performance and sometimes even instability if not accounted for in the controller design.



Figure 2: Responses of the drum water level to a step of 10 kg/s in steam flow rate (left) and to a step corresponding to 10MW in fuel flow rate (right) at low (dotted), medium (solid) and high (dashdot) loads [4]

4. Model Predictive Control

The current developments in computers speed have made the implementation of modern control algorithms, such as MPC, more feasible in industry. The attraction of MPC algorithm arises from the fact that several process models as well as many performances criteria (e.g. constraints) can be handled using MPC.

The basic concept of model predictive control relies on the idea that at each time step k (see Figure 3), a sequence of m control moves u(k+i/k), i=0, 1, ..., m-1 are computed by minimizing a previously selected cost function, using plant information at time k.





Figure 3: Basic concept of MPC

The cost function has the general following form,

$$T(k) = \sum_{i=1}^{m-1} (y^{sp} - y(k+i|k))^T \Gamma_y (y^{sp} - y(k+i|k))$$

+
$$\sum_{i=0}^{m-1} u(k+i|k)^T \Gamma_u u(k+i|k))$$

+
$$\sum_{i=0}^{m-1} \Delta u(k+i|k)^T \Gamma_{\Delta u} \Delta u(k+i|k))$$
(1)

subject to the following constraints:

J

$$u_{\min} \le u(k+i | k) \le u_{\max}, i = 0, ..., m-1$$

$$\Delta u_{\min} \le \Delta u(k+i | k) \le \Delta u_{\max}, i = 0, ..., m-1$$

$$y_{\min} \le y(k+i | k) \le y_{\max}, i = 1, ..., p$$
(2)

where y^{sp} is the set-point for y, p is the prediction horizon, m is the control horizon and $\Gamma_y, \Gamma_u, \Gamma_{\Delta u} \ge 0$ are weighting matrices. It is assumed that the control action is not changed after time k+m-1 which means that u(k+i/k)=u(k+m-1/k) for $i\ge m$.

Although the optimization problem results in more than one optimal control moves, only the first one u(k/k) is implemented. In the next time step k+1 the above optimization is solved again and a new control action u(k+1/k+1) is implemented.

4.1. MPC Algorithm

As can be shown by the form of the cost function, a model of the plant is needed for the future outputs to be predicted. The model is assumed to be linear and time-invariant with controllability. The assumed model has the following discrete state-space form,

$$\overline{x}(k+1) = \overline{A}\overline{x}(k) + \overline{B}u(k) + \overline{B}n_1(k)$$

$$y(k) = \overline{C}\overline{x}(k) + \overline{D}u(k) + n_2(k)$$
(3)

where u(.) is the input, y(.) is the output, $\overline{x}(.)$ is the state vector, $n_1(.)$ is a zero-mean white Gaussian plant noise and $n_2(.)$ is a zero-mean white Gaussian output noise uncorrelated with $n_1(.)$. Eq.(5) Can be rewritten as follows,

$$\begin{bmatrix} \overline{x}(k+1) \\ u(k+1) \end{bmatrix} = \begin{bmatrix} \overline{A} & \overline{B} \\ 0 & I \end{bmatrix} \begin{bmatrix} \overline{x}(k) \\ u(k) \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \Delta u(k) + \begin{bmatrix} \overline{B} \\ 0 \end{bmatrix} n_1(k)$$
$$y(k) = \begin{bmatrix} \overline{C} & \overline{D} \end{bmatrix} \begin{bmatrix} \overline{x}(k) \\ u(k) \end{bmatrix} + n_2(k)$$
(4)



where $\Delta = \frac{1 - z^{-1}}{z^{-1}}$ and z^{-1} is the backward shift operator. Eq.(6) can be represented as follows, $x(k+1) = Ax(k) + B\Delta u(k) + Gn$

$$x(k+1) = Ax(k) + B\Delta u(k) + Gn_1$$

$$y(k) = Cx(k) + n_2$$
(5)

where

$$\mathbf{x}(k) = \begin{bmatrix} \overline{\mathbf{x}}(k) \\ u(k) \end{bmatrix}, A = \begin{bmatrix} \overline{A} & \overline{B} \\ 0 & I \end{bmatrix}, B = \begin{bmatrix} 0 \\ I \end{bmatrix}, G = \begin{bmatrix} \overline{B} \\ 0 \end{bmatrix}, C = \begin{bmatrix} \overline{C} & \overline{D} \end{bmatrix}$$

According to the model represented by Eq. (7) the structure of MPC algorithm will be as shown in Figure 4. At each time step k a plant measurement y(k) is obtained. Y(k) and $\Delta u(k)$ are used by an estimator to obtain an estimate $\tilde{x}(k)$ of the plant state x(k). The MPC optimizer uses the plant model and the estimated state to predict the future outputs of the system over the prediction horizon p. Then, the optimizer solves the optimization problem and, as a result, optimal control actions over the control horizon m are obtained. The first control action $\Delta u(k | k)$ is then applied to the plant.



Figure 4: MPC structure

4.2. State Estimator

Since the exact value of the noise $n_1(k)$ is not known, so the equation Δu used to predict the state of the plant according to previous state is,

$$x(k | k-1) = Ax(k-1) + B\Delta u(k-1)$$
(6)

In addition to the noise uncertainty, there is, in general, uncertainty in the initial state of the plant x(0). In this paper, discrete Kalman filter is used.

The equations of the Kalman filter fall into two groups: time update equations and measure- ment update equations [5]. Time update equations are as follows,

$$x(k | k-1) = Ax(k-1) + B\Delta u(k-1)$$

$$P(k | k-1) = AP(k-1)A^{T} + GWG^{T}$$
(7)

and the measurement update equations,

$$K(k) = P(k \mid k-1)C^{T} (CP(k \mid k-1)C^{T} + V)^{-1}$$

$$\tilde{x}(k) = x(k \mid k-1) + K(k)(y(k) - Cx(k \mid k-1))$$

$$P(k) = (I - K(k)C)P(k \mid k-1)$$
(8)



where *K*(.) the optimal Kalman filter gain, *W* is the covariance matrix of n_1 , *V* is the covariance matrix of n_2 , $P(k) = E[e(k)e(k)^T]$ is the covari- ance of the estimation error $e(k) = x(k) - \tilde{x}(k)$ and P(k|k-1) is the prediction of the estimation error.

If the initial state is known exactly in advance, P(0) can be initialized to zero matrix. In general, this is not the case, so P(0) should be initialized with a suitably large number on its diagonal.

4.2. MPC Optimizer

If we set Γ_{y} to *I*, Γ_{u} to zero and $\Gamma_{\Delta u}$ to λI in Eq.(3), then the cost function will have the form,

$$J(k) = \sum_{i=1}^{p} (y^{sp} - y(k+i \mid k))^{T} (y^{sp} - y(k+i \mid k)) + \lambda \sum_{i=0}^{m-1} \Delta u(k+i \mid k)^{T} \Delta u(k+i \mid k))$$

or,

$$J(k) = (Y_k^{sp} - Y_k)^T (Y_k^{sp} - Y_k) + \lambda \Delta U_k^T \Delta U_k$$
⁽⁹⁾

where

$$Y_k^{sp} = [y^{sp}(k+1)...y^{sp}(k+p)]^T$$

$$Y_k = [y(k+1 | k)...y(k+p | k)]^T$$

$$\Delta U_k = [\Delta u(k | k)...\Delta u(k+m-1 | k)]^T$$

At time k, the equation used to predict the output at time k+1 is,

$$y(k+1|k) = Cx(k+1|k)$$
$$= CA\tilde{x}(k) + CB\Delta u(k)$$
(10)

Then, at time k, the predicted output at time k+j will be,

$$y(k+j|k) = CA^{j}\tilde{x}(k) + \sum_{i=0}^{j-1} CA^{j-i-1} B\Delta u(k+i|k)$$
(11)

So, the output vector over the prediction horizon will be,

$$Y = F\widetilde{x}(k) + H\Delta U_k$$

where

$$F = \begin{bmatrix} CA \\ \vdots \\ CA^{p} \end{bmatrix}, H = \begin{bmatrix} h_{1,1} & \cdots & h_{1,m} \\ \vdots & \ddots & \vdots \\ h_{p,1} & \cdots & h_{p,m} \end{bmatrix}$$
$$h_{i,j} = \begin{cases} CA^{i-j} & \text{if } i \ge j \\ 0 & \text{if } i < j \end{cases}$$

By substituting Eq.(12) into Eq.(9), we get Eq.(13),

$$J(k) = \Delta U_k^T [H^T H + \lambda I] \Delta U_k - 2(Y_k^{sp} - f_k)^T H \Delta U_k$$
$$+ (Y_k^{sp} - f_k)^T (Y_k^{sp} - f_k)$$
(13)

where $f_k = F\tilde{x}(k)$

As mentioned before, our goal is to find the optimal control actions ΔU_k^* by minimizing the cost function,

$$\Delta U_k^* = \arg\min_{\Delta U_k} \left(J(k) \right) \tag{14}$$

In Eq.(13) the term $(Y_k^{sp} - f_k)^T (Y_k^{sp} - f_k)$ is not a function of ΔU_k , so the solution of Eq.(13) can be rewritten as,

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(12)

$$\Delta U_k^* = \arg \min_{\Delta U_k} \left[\frac{1}{2} \Delta U_k^T [H^T H + \lambda I] \Delta U_k - (Y_k^{sp} - f_k)^T H \Delta U_k) \right]$$
(15)

The problem defined in Eq.(17) has a quadratic form, so can be solved by Matlab using quadratic programming tools. The form of the quadratic programming is,

$$x^{*} = \arg\min_{x} \left[\frac{1}{2}x^{T}px + q^{T}x\right]$$

subject to $Ax \le b$ (16)

where *p* and *q* in Eq. correspond to $H^T H + \lambda I$ and $-(Y_k^{sp} - f_k)^T H$, respectively. *A* and *b* can be formed from the constraints.

The second constraint in Eq.(4) can be formed as follows,

$$\begin{bmatrix} I & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I \end{bmatrix} \begin{bmatrix} \Delta u(k \mid k) \\ \Delta u(k+1 \mid k) \\ \vdots \\ \Delta u(k+m-1 \mid k) \end{bmatrix} \leq \begin{bmatrix} \Delta u_{\max} \\ \Delta u_{\max} \\ \vdots \\ \Delta u_{\max} \end{bmatrix}$$
$$-\begin{bmatrix} I & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I \end{bmatrix} \begin{bmatrix} \Delta u(k \mid k) \\ \Delta u(k+1 \mid k) \\ \vdots \\ \Delta u(k+m-1 \mid k) \end{bmatrix} \leq -\begin{bmatrix} \Delta u_{\min} \\ \Delta u_{\min} \\ \vdots \\ \Delta u_{\min} \end{bmatrix}$$

The constraints on the amplitude of the control actions in, can be transformed to constraints on their increments as follows,

$$u_{\min} \le u(k+i \mid k) \le u_{\max}$$
$$u(k+i \mid k) = \sum_{j=0}^{i} \Delta u(k+j \mid k) + u(k-1) \Biggr\} \Rightarrow$$
$$u_{\min} - u(k-1) \le \sum_{j=0}^{i} \Delta u(k+j \mid k) \le u_{\max} - u(k-1)$$

The previous inequalities can be formed as,

$$\begin{bmatrix} I & 0 & \cdots & 0 \\ I & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I & I & \cdots & I \end{bmatrix} \begin{bmatrix} \Delta u(k \mid k) \\ \Delta u(k+1 \mid k) \\ \vdots \\ \Delta u(k+m-1 \mid k) \end{bmatrix} \leq \begin{bmatrix} u_{\max} - u(k-1) \\ u_{\max} - u(k-1) \\ \vdots \\ u_{\max} - u(k-1) \end{bmatrix} \\ -\begin{bmatrix} I & 0 & \cdots & 0 \\ I & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I & I & \cdots & I \end{bmatrix} \begin{bmatrix} \Delta u(k \mid k) \\ \Delta u(k+1 \mid k) \\ \vdots \\ \Delta u(k+m-1 \mid k) \end{bmatrix} \leq \begin{bmatrix} u(k-1) - u_{\min} \\ u(k-1) - u_{\min} \\ \vdots \\ u(k-1) - u_{\min} \end{bmatrix}$$

The constraints on the amplitude of the future control actions and their increments can be represented by the following general form,

$$A.x \leq b$$

$$A = \begin{bmatrix} \Phi \\ -\Phi \\ \Phi \\ -\Phi \end{bmatrix}, b = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_3 \end{bmatrix}, x = \begin{bmatrix} \Delta U_k \\ \Delta U_k \\ \Delta U_k \\ \Delta U_k \end{bmatrix}, \Phi = \begin{bmatrix} I & 0 & \cdots & 0 \\ I & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I & I & \cdots & I \end{bmatrix},$$

$$\varphi_1 = \begin{bmatrix} \Delta u_{\max} \\ \Delta u_{\max} \\ \vdots \\ \Delta u_{\max} \end{bmatrix}, \varphi_2 = -\begin{bmatrix} \Delta u_{\min} \\ \Delta u_{\min} \\ \vdots \\ \Delta u_{\min} \end{bmatrix}, \varphi_3 = \begin{bmatrix} u_{\max} - u(k-1) \\ u_{\max} - u(k-1) \\ \vdots \\ u_{\max} - u(k-1) \end{bmatrix},$$

$$\varphi_4 = -\begin{bmatrix} u_{\min} - u(k-1) \\ u_{\min} - u(k-1) \\ \vdots \\ u_{\min} - u(k-1) \end{bmatrix}$$
(17)

5. Simulation

The simulation is organized as follows; first, a linear state-space model of the plant is extracted by perturbing the inputs around an operating point. Then, MPC algorithm is applied using a suitable set of parameters. Effects of different parameters values are then tested.

5.1. Linearization of the Model

As mentioned before, to obtain a linear model of the plant, perturbations are applied to the inputs (i.e. heat input, feedwater flow rate and steam flow rate). The operating point can be determined by two parameters: drum pressure *p* and heat power supplied to the boiler *Q*. The nominal values of feedwater and steam flow rates can be calculated by solving Eq.(1) in steady-state. In simulation, the values: p=9.35(MP) and Q=80(MW) are considered as nominal values. The corresponding nominal values of the other inputs are: $q_s = q_f = 47$ (kg/s).

These values define a medium load operating point [4]. Figure 5 shows the perturb- ations added to the nominal values of the inputs. The inputs in Figure 5 with the responses of the nonlinear model to them are then used to initialize the identification process. As a result, a five state-space linear model is extracted. A comparison between the model response (ie. Drum pressure and water level) and that of the linear model is shown in Figure 6.

In practice, the steam flow rate is determined by the power demanded from the power plant. It can be considered as an external disturbance, which has to be accounted for in the MPC algorithm, or can be treated as a controlled variable if the controller has an access to the steam actuator. In simulation, the steam flow rate will be considered as an output of the model as well as an input.

The input and the output is considered as follows,



Figure 5: Perturbations applied to the nonlinear model inputs around the medium load operating point.

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Figure 6: Validation of the linear model, the non-linear model responses (solid line) and the five states linear model responses (dashed line)

5.2. Controller Performance

It is assumed that the input actuators have the following constraints,

$$\begin{split} 0 &\leq Q \leq 160 (MW), \ -5 \leq \Delta Q \leq 5 (MW) \\ 0 &\leq q_f \leq 90 (kg \mid s), \ -5 \leq \Delta q_f \leq 5 (kg \mid s) \end{split}$$

 $0 \le q_s \le 90(kg/s), -5 \le \Delta q_s \le 5(kg/s)$

The following values are chosen to be the parameters of the controller in the simulation,

 $m = 2, p = 50, \lambda = 0.00001, T_s = 1(sec),$

$$W = 10^{-8} I, V = 10^{-8} I.$$

The sampling time for discrete system is taken as 1sec. And the pressure reference is a step increase of 0.02MP applied at 500sec. The drum level reference is a step increase of 0.03m at 1500sec. And the steam flow rate reference is a step increase of 2kg/s at 2500sec. Responses of the model outputs are shown in Figures 7, 8, 9. Oscillations shown in the responses of the pressure and the water level at the start of the simulation are due to the fact that the state estimation has not convergent yet. These oscillation disappeared after a while, when the estimator reaches the exact initial state of the plant. Figures 10, 11, 12 show the control inputs before and after the actuator. The identity between inputs and outputs of the actuator means that the MPC optimizer works completely according to the constraints conditions.



Figure 7: Response of the drum pressure (dashed line) to the reference input shown in solid line with parameters (m = 2, p = 50, $\lambda = 0.00001$).



Figure 8: Response of the drum water level (dashed line) to the reference input shown in solid line with parameters (m = 2, p = 50, $\lambda = 0.00001$).



Figure 9: Response of the steam flow rate (dashed line) to the reference input shown in solid line with parameters (m = 2, p = 50, $\lambda = 0.00001$).



Figure 10: Heat control input before (solid line) and after (dashed line) the actuator.



Figure 11: Feedwater flow rate control input before (solid line) and after (dashed line) the actuator.



Figure 12: Steam flow rate control input before (solid line) and after (dashed line) the actuator

5.3. Sensitivity to Tuning Parameters

Figures 13, 14 show the responses of the pressure and the water level to the same reference signals described in the previous section, but with $\lambda = 0.1$. The MPC controller shows instability in the water level response, and poor control action concerning the drum pressure compared to the response shown in Figure 7.

Figure 15 show the response of the drum water level with $\lambda = 0.0001$ and p = 30. In com- parison with the response shown in Figure 8, the response exhibits stronger oscillation in the transient state with a better settlement in the steady state.



Figure 13: Response of the drum water level (dashed line) to the reference input shown in solid line with parameters



Figure 14: Response of the drum pressure (dashed line) to the reference input shown in solid line with parameters (m=2, p=50, $\lambda=0.1$).



Figure 15: Response of the drum water level (dashed line) to the reference input shown in solid line with parameters

 $(m = 2, p = 30, \lambda = 0.00001).$

5.4. Comparison with 3-Element Controller

Figure 16 shows the water level response of a plant controlled by three-element control stra- tegy. The PID controllers are tuned so that the shrink and swell effects are overcome (see [4]). The performance improvement using the MPC controller (shown by the dashed curve) is significant.



Figure 16: Water level response obtained for three-element controller (dotted) and the MPC controller (dashed). The solid curve denotes the water level set-point. The set-points for drum pressure and steam demanded are as shown in Figures 7, 9.

6. Conclusion and Future Work

The drum boiler control problem was viewed as multiple inputs/ multiple outputs (MIMO) control problem with the fuel, feedwater and steam flow rates as the manipul- ated variables, the drum pressure, drum water level and steam flow rate as the controlled variables.

The difficulty of the control tasks arise from several reasons, the most important among them being the nonlinear plant dynamics, the non-minimum plant characteristics and the constraints of the actuators.

Model predictive control with Kalman filter is investigated. The results show that the MPC optimizer takes actuators constraints into consideration explicitly when determining the control action. Due to the nonlinearity of the plant, a linearized model of the plant is extracted in an operating point. The simulations of the system response around the operating point show that the controller works very well. In the future, more works are needed to control the drum boiler along its operating region (i.e. 0 percent - 100 percent load). To achieve this goal, an effective on-line identification is necessary. Our future work will concentrate on fuzzy modeling and identification suing Takagi-Sugeno procedure as a proposed choice.

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