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Complexity Estimation of Automata Models Combinational Parts

Anton Epifanov

Institute of precision mechanics and control science of RAS, Rabochaya 24, Saratov, Russia
epifanovas@list.ru

Abstract In the paper are analyzed the processes of functioning of discrete dynamic systems. Discrete deterministic automata with large finite or countably infinite sets of states are considered as mathematical models of systems. In view of the removal of the limitation on the size of the memory of automata due to the use of the apparatus of geometric images of automata, the analysis of automaton models is reduced to the analysis of the combination parts of the automata, namely, to the development of estimates of the complexity of the combination parts. As a mathematical tool for analyzing the combinational parts assumed by the realized Boolean functions, a special spectrum of dynamic parameters for the recurrent determination of numerical sequences is used. The analysis of the complexity of the Boolean functions is carried out for various methods of ordering of the domain of definition of such functions.

Keywords discrete dynamical system, mathematical model of system, automata model of system, geometrical image of automaton, combinational part, complexity estimation

Introduction

The problem of developing models and methods for analyzing the processes of functioning of complex dynamical systems is the subject of research both by thousands of individual scientists and by a significant number of research teams of various sizes: from small laboratories to conglomerations of dozens of laboratories and Institutes around the world. In the set of tasks that make up this scientific problem, a number of tasks can be distinguished, without the solution of which it is impossible: ensuring and maintaining the basic safety of the functioning of systems (we conventionally use the "Safety" tag) (which has an ever-increasing relevance when the dimension of systems increases and their practical use); development of both architectural and software infrastructures that provide greater energy efficiency at a given performance and speed (or a different formulation is possible - an increase in productivity and speed while maintaining a given level of energy supply) - we are talking primarily about space and transport systems of new generations, data transmission systems, etc. (this direction was formed under the tag "Green" and is now more and more widespread). These tasks include the tasks of control and diagnosing the processes of functioning of dynamic systems, the tasks of the complexity estimation of both specific processes of functioning and the laws of functioning of the system as a whole, the task of determining effective (in the context of "Green" - including the most energy efficient, in the context of "Safety "- ensuring a given minimum level of safety) means of countering defects and functional failures, the tasks of ensuring the fault tolerance of systems, the tasks of complete or partial restoration of the system's operability (or, if impossible, the task of decommissioning the system), etc. In this paper, we consider the problems of developing methods for assessing the complexity of processes of functioning of dynamic systems. One of the basic classical mathematical models of dynamical systems using the paradigm of determinism is a mathematical model in the form of a discrete deterministic automaton. It should be noted that the classical ways of defining of automata models of systems in the form of tables, matrices, graphs, systems of logical equations, etc. are a significant limitation for the use of automata as mathematical models of real systems due to the fundamental dimensionality barrier.

However, in the works of Professor Tverdokhlebov V.A., starting, for the first time since 1993, a new approach to specifying automata models of systems is proposed and further developed, namely, specifying automata using geometric structures. This approach allows through the use of mathematical idealizations of continuity, actual infinity, summation of infinite series, etc. formally represent the laws of functioning of automata with large finite or countably infinite sets of states as geometric structures. In some cases, this makes it possible to effectively overcome the dimensionality barrier. In this work, studies are carried out within the framework of this approach, i.e. the sets of states of the considered automata models of systems are assumed to be sufficiently large, finite or countably infinite.

The fundamental concepts of complexity - the class of P-problems solvable in polynomial time by deterministic Turing machines, and the class of NP-problems solvable in polynomial time by non-deterministic Turing machines, are related to automatons (Turing machines). Along with the general "algorithmic" understanding of complexity in the theory of automata, particular variants of complexity indicators were investigated: the number of states of the automaton, the lengths of the input and output sequences, the number of connectivity components in the structure of the automaton, the complexity of the structure of the structural automaton, etc.

The problem of estimating the complexity of automata was investigated by many authors immediately after the introduction of automata models. John von Neumann devoted a number of sections of his work "The Theory of Self-Reproducing Automata" to problems of estimating the complexity of automata: “The role of high and very high complexity (including the role of complexity and the need for appropriate theoretical justification, etc.); Re-evalution of the problems of complicated automata - problems of hierarchy and evolution."

The estimation of the complexity of structural automata by the number of their constituent elements has become widespread. Such estimates change significantly with changes in the element base and synthesis method. The paper considers the classical decomposition of an automaton into a combinational part and memory and research, in particular, the combinational parts, the models of which are considered functions of logic algebra (Boolean functions or logic functions).

Number of variants of concept of complexity is sufficiently great and continues to increase in works of many researchers. For example, estimations of algorithms on their belong to NP and P classes (detail review see, for example, in work [2] and one of contemporary papers [3], in which Radoslaw Hofman show, that P not equal NP), Kolmogorov complexity [4], complexity from below, from above, complexity on the average, bit complexity (one of basic work in this area is [5]), multiplicate complexity, algebraic complexity, there are very large amount of works on asymptotical estimations of complexity (see, for example works [6,7,8,9]), etc.

In this work with use of the apparatus of geometrical images of automatons [1], is offered and investigated new concept of estimation of complexity of laws of functioning of the discrete determined dynamic systems (automatons) on the basis of the estimation of the complexity of the automata by indicators characterizing the behavior of the combination parts, i.e. based on indicators that depend only on the laws of functioning. These indicators were developed and systematized by V.A. Tverdokhlebov in the spectrum of indicators in [1, 14]. The spectrum Ω is applied to sequences and has a hierarchical structure of indicators, where indicators of each next level contain new data on the structure of the sequence: the smallest order of the recurrent form that determines the sequence; the number of changes of recurrent forms of a fixed order, when determining the sequence; lengths of subsequences defined by recurrent forms of fixed orders, etc. In this work, such a range of indicators is used as a means for assessing the complexity of the laws of the functioning of automata in general and for assessing specific functioning processes.

This work is devoted to the study of the properties and classification of Boolean functions based on the spectrum of dynamic parameters. The specificity of the Boolean function $f(x_1,x_2,\ldots,x_n)$ of n variables with a fixed domain of definition can be specified by a column of 2^n values (a sequence of values $\xi = \left\langle a(1), a(2), \ldots, a(2^n) \right\rangle$ of length 2^n). The number of Boolean functions of n variables is 2^{2^m}. The study of the properties of Boolean functions in this work is reduced to the study of the properties of numerical sequences based on the spectrum of dynamic parameters. For this, each of the considered classes of Boolean functions is associated with an equally powerful class of sequences. This assignment of the Boolean function class is valid for a fixed domain of definition. Classes of Boolean functions of 3 and of 4 variables are considered for two ways of defining the domain of definition: the classical way, when binary vectors are ordered lexicographically, and the method based on the use of a compact sequence (see [10] Gill A. Introduction to the theory of finite automata (Lippel, Epstein)).

Methods: Recurrent definition dynamic parameters as a means of sequence structure complexity estimation

In [1, 14] the concept of a spectrum was introduced and developed, which characterizes the structure of a sequence, which can be considered as a way of definition of a geometric image of the laws of functioning.

Let be $U=\{u_1,u_2,\dots,u_k\}$ a finite set and ξ a sequence of elements from the set U: $\xi=\langle u(1),u(2),\dots,u(t),\dots\rangle$. The sets of all finite sequences, all finite sequences of length v and infinite sequences of elements from the set U will be denoted by U^*, U^ν, U^∞, respectively. The spectrum $\Omega(\xi)$ of dynamic characteristics of the sequence $\xi\in U^*\cup U^\infty$ has a hierarchical structure consisting of the levels $\Omega(\xi)=(\Omega_0(\xi), \Omega_1(\xi), \Omega_2(\xi), \Omega_3(\xi), \Omega_4(\xi))$. Each specific implementation option (represented by parameter values) of any level $\Omega_i(\xi)$ defines a partition of each of the sets U^*, U^ν, U^∞ into subsets according to the coincidence properties of the characteristics corresponding to the level. We will consider subsets of such a partition as equivalence classes of sequences.

Definition 1. For any sequence $\overline{\xi}\in U^\nu$, the least order of the recurrent form that defines the sequence $\overline{\xi}$ will be denoted $m_0(\overline{\xi})$.

Definition 2. For any sequence $\overline{\xi}\in U^\nu$ and $m\in N^+$, where $1\le m\le m_0(\overline{\xi})$, the longest length of the initial segment of the sequence $\overline{\xi}$ determined by the recurrence form of order m will be denoted $d^m(\overline{\xi})$.

Definition 3. For any sequence $\overline{\xi}\in U^\nu$ and $m\in N^+$, where, $1\le m\le|\overline{\xi}|-1$, the number of shifts of recurrent forms of order m required in determining the sequence $\overline{\xi}$ will be denoted $r^m(\overline{\xi})$.

Definition 4. For any sequence $\overline{\xi}\in U^\nu$ and $m\in N^+$, where $1\le m\le m_0(\overline{\xi})$ and j, where $1\le j\le r^m(\overline{\xi})$, the length of the j-th segment in the definition of a sequence $\overline{\xi}$, will be denoted $d_j^m(\overline{\xi})$.

Using the introduced notation, we define the spectrum of parameters characterizing the sequence as the following structure:

$\Omega_0(\overline{\xi})=\langle m_0(\overline{\xi})\rangle$; $\Omega_1(\overline{\xi})=\langle d^1(\overline{\xi}),d^2(\overline{\xi}),\dots,d^\alpha(\overline{\xi})\rangle$; $\Omega_2(\overline{\xi})=\langle r^1(\overline{\xi}),r^2(\overline{\xi}),\dots,r^\alpha(\overline{\xi})\rangle$; $\Omega_3(\overline{\xi})=\langle \Omega_3^1(\overline{\xi}),\Omega_3^2(\overline{\xi}),\dots,\Omega_3^\alpha(\overline{\xi})\rangle$ where $\alpha=m_0(\overline{\xi})$ and $\Omega_3^j(\overline{\xi})=\langle d_1^j(\overline{\xi}),d_2^j(\overline{\xi}),\dots,d_{n_j}^j(\overline{\xi})\rangle$ (n_j – the number of the last segment in the definition of a sequence $\overline{\xi}$ as a sequence of segments defined by individual recurrent forms of order j);

$\Omega_4(\overline{\xi})=\Theta(\Omega_3(\overline{\xi}))$, where Θ is the operator of replacing in $\Omega_3(\overline{\xi})$ the lengths of the segments by the weights of the used recurrent forms for determining the segments.

The fourth level $\Omega_4(\overline{\xi})$ of the spectrum $\Omega(\overline{\xi})$ adds to the characteristics in the previous levels an assessment of the complexity of the rules and the options for using the rules.

The construction of the spectrum on the basis of the hierarchical structure was made with the aim of successively deepening and expanding the characteristics of the properties of the sequence when moving from the previous level to the next. Such structure allows one to reduce the set of parameters used, if the goal of analysis is achieved, and to be limited to the corresponding initial fragments of the spectrum. Spectrum levels are hierarchically constructed by adding new parameters to the spectrum. In this regard, the equivalence of sequences is represented through t - equivalences determined by the level Ω_t, $0\le t\le 3$.

Analysis of classes of functions of the algebra of logic in the classical ordering of binary vectors of the domain of definition and when using compact sequences.

One of the classical ways to completely define of a Boolean function depending on *n* variables is the tabular way, when the values of the function are explicitly set on each of 2^n sets of arguments. With this method of defining a function, in order to achieve uniqueness, it is necessary to fix the order of the binary vectors of the domain of the function. The order on the set of 2^n binary argument sets can be chosen according to a number of criteria, depending, for example, on the application area. This part of the paper is devoted to the study of the properties of Boolean functions in the classical lexicographic ordering of binary sets of arguments. Thus, the study of Boolean logic functions of *n* variables is reduced to the study of the properties of a sequence, the first element of which is the value of the function on the set of arguments 0000...00, the second element is the value on the set 0000...01, etc., the last 2^n -th element of the sequence is the function value on the set 1111...11.

To research class of Boolean functions depenping from 3 arguments is associated with set of sequences Ξ_3, consisting of 256 binary sequences of length 8. For each sequence $\xi \in \Xi_3$, the quantitative values of the spectrum indices at the levels Ω_0 - Ω_3 are calculated and, based on the coincidence of the spectrum indices, partitions P_0, P_1, P_2, P_3 into classes of equivalent sequences of the set of sequences Ξ_3 are constructed. Information about partitions P_0, P_1, P_2, P_3 is presented in table 1.

Table 1: Information about partitions P_0, P_1, P_2, P_3 of set Ξ_3 by spectrum indicators .

Characteristic	**P_0**	**P_1**	**P_2**	**P_3**
Number of subclasses in partition	7	26	70	81
Max cardinality of subclass	106	26	16	8
Min cardinality of subclass	2	2	2	2

Also, for the elements of the set Ξ_3, the values of the characteristic $\theta = \frac{m_0 \cdot k}{n^{m_0 - 1}}$ are calculated, where *k* is the number of characters in the sequence generated by the use of the recurrent form *F*. By coincidence of the value of the characteristic θ, a partition P_θ of the set Ξ_3 into classes of sequences equivalent in complexity is constructed. Under the assumption that $|W| = 1$ only for two sequences (00000000 and 11111111), the partition P_θ of the set Ξ_3 contains 6 classes of equivalent sequences, the minimum cardinality is 2, the maximum is 106. Under the assumption that | W | = 2 for all sequences (including 00000000 and 11111111), the partition P_θ of the set Ξ_3 does not change its structure due to the fact that the value of the characteristic θ for m_0=1 does not depend from the value n = | W | since for m_0=1, the denominator in the expression becomes 1 for any *n*.

The investigated class of Boolean functions of 4 variables is associated with the class of sequences Ξ_4, consisting of 65536 binary sequences of length 16. For each sequence $\xi \in \Xi_4$, the quantitative values of the spectrum indices at the levels Ω_0 - Ω_3 are calculated and, based on the coincidence of the spectrum indices, partitions P_0, P_1, P_2, P_3 of the set of sequences Ξ_4 are constructed into classes of equivalent sequences. Information on the partitions P_0, P_1, P_2, P_3 of the set of sequences Ξ_4 is presented in table 2.

Table 2: Information about partitions P_0, P_1, P_2, P_3 of set Ξ_4 by spectrum indicators

Characteristic	**P_0**	**P_1**	**P_2**	**P_3**
Number of subclasses in partition	15	1634	12568	19086
Max cardinality of subclass	22152	1188	132	132
Min cardinality of subclass	2	2	2	2

Analysis of the obtained results shows that a significant increase (by 2 orders of magnitude) in the number of classes of equivalent sequences is observed when the partition P_1 is constructed on the basis of the partition P_0 (the number of classes increases from 15 to 1634), while the value of the maximum cardinality of the class in the partition (with 22152 elements per class down to 1188 elements). A similar jump in the number of classes is observed when the partition P_2 is constructed on the basis of the partition P_1 (the number of classes increases by one order of magnitude, and the value of the maximum cardinality of the class in the partition decreases by an order of magnitude).

For each sequence $\xi \in \Xi_4$, the value of the characteristic θ is calculated. By coincidence of the value of the characteristic θ, a partition P_θ of the set Ξ_4 into classes of sequences equivalent in complexity is constructed. The partition P_θ of the set Ξ_4 contains 14 classes of equivalent sequences, the minimum cardinality is 2, the maximum is 22152. For a class with a maximum cardinality of 22152, the value of the characteristic θ = 3. The minimum value of the characteristic θ_{min} = 0.0146484 (for a class with number 2, consisting of 2 elements), the maximum value θ_{max} =16 (for class 1, consisting of 26 elements). Below are some of the properties of Boolean functions noted as a result of this study.

Lemma 1. The minimum value of the cardinality of the class of equivalent sequences in the partitions P_0, P_1, P_2, P_3 of the set of sequences Ξ_n (corresponding to the class of Boolean functions of n variables) cannot be less than 2.

Proof. This statement is true in view of the fact that in the set Ξ_n for any sequence $\xi = \xi_1, \xi_2, ..., \xi_{2^n}$, where $\xi_i \in \{0,1\}, 1 \le i \le 2^n$, there necessarily exists a sequence $\overline{\xi} = \overline{\xi}_1, \overline{\xi}_2, ..., \overline{\xi}_{2^n}$ that is a bitwise negation of the sequence ξ. The set Ξ_n contains exactly $2^{2^n - 1}$ pairs of sequences, each component of which can be obtained from the other by a one-to-one renaming of elements (replacing 0 by 1 everywhere and 1 by 0). The spectrum of dynamic parameters is invariant with respect to one-to-one redesignation of sequence elements. Therefore, the two sequences $\xi = \xi_1, \xi_2, ..., \xi_{2^n}$ and $\overline{\xi} = \overline{\xi}_1, \overline{\xi}_2, ..., \overline{\xi}_{2^n}$ are equivalent in complexity from the point of view of the spectrum. This means that in any of the partitions P_0, P_1, P_2, P_3 of the set Ξ_n, where n = 1,2,3,… the minimum possible value of the cardinality of the class of equivalent sequences cannot be less than 2.

The analysis of the classes of Boolean functions as a result of the study using the spectrum Ω shows that in the partition P_3 of the set Ξ_n, where n = 1,2,3,… the minimum value of the cardinality of the class of equivalent sequences is 2.

Consequence. The value of the cardinality of an arbitrary class of equivalent sequences in partitions P_0, P_1, P_2, P_3 of the set of sequences Ξ_n, where n = 1,2,3,… can only be an even number $k \ge 2$.

The spectrum of dynamic parameters of the recurrent determination of sequences characterizes the structure of the mutual arrangement of elements in the sequence. Spectrum can be used to assess complexity and classify sequences by complexity. To estimate the complexity in the paper we use the characteristic $\theta = \frac{m_0 \cdot k}{n^{m_0 - 1}}$, where k is the number of characters in the sequence generated by the use of the recurrent form F, m_0 is the smallest order of the recurrent form necessary to determine the entire sequence, and n = | W |, where W is the set of values of the variables ξ_i.

Boolean function can be determined by a formula. One of the classical estimates of the complexity of a Boolean function is the number of occurrences of variables in the minimal formula that defines the function. To test the assumption that the spectrum Ω can fairly be used to classify functions of the algebra of logic, a study was carried out, which included the following stages:

1. extraction of classes from partitions P_0, P_1, P_2, P_3 of the set of sequences Ξ_4;
2. construction formulas (in the form of a perfect disjunctive normal form - PDNF) for elements of the extracted classes;
3. minimization of PDNF by the Quine - McCluskey tabular method;
4. analysis of functions that are equivalent in terms of the spectrum for the coincidence or difference in the number of variables in the minimum formulas that define these functions.

The type of PDNF function depends on the selected order in the domain of the function definition. A significant advantage of using a compact sequence to define the scope, compared to the classic tabular method, is a significant reduction in the amount of memory required for storage.

In this paper, the study of Boolean functions was carried out with both methods of defining the domain of definition of the function. Table 3 lists some classes of spectrum-equivalent functions and minimal formulas defining such functions for the classical lexicographic ordering of binary sets of the domain of the function.

Table 3: Classes of equivalent sequences in the partition P_3 of the set of sequences Ξ_4 (in the classical way of defining the domain)

Class	Number of function	Boolean function value string	The minimum formula that defines a function	The number of occurrences of variables in the minimum formula
K_2	f_{21846}	0101010101010101	$f_{21846} = x_4$	1
	f_{43691}	1010101010101010	$f_{43691} = \bar{x}_4$	1
K_7	f_{32767}	0111111111111110	$f_{32767} = (\bar{x}_1 x_4) \vee (\bar{x}_2 x_3) \vee (x_2 \bar{x}_4) \vee (x_1 \bar{x}_3)$	8
	f_{32770}	1000000000000001	$f_{32770} = (\bar{x}_1 \& \bar{x}_2 \& \bar{x}_3 \& \bar{x}_4) \vee (x_1 x_2 x_3 x_4)$	8

Table 4 shows some classes of spectrum-equivalent functions and the number of occurrences of variables in the minimal formulas defining such functions when defining the domain using a compact sequence.

Table 4: Classes of equivalent sequences in the partition P_3 of the set of sequences Ξ_4 (when defining the domain using a compact sequence)

Class	Number of function	Boolean function value string	The minimum formula that defines a function
K_7	f_{32767}	0111111111111110	3
	f_{32770}	1000000000000001	3
K_{17}	f_9	0000000000001000	4
	f_{10}	0000000000001001	8
	f_{15}	0000000000001110	7
	f_{16}	0000000000001111	6
	f_{65521}	1111111111110000	8
	f_{65522}	1111111111110001	8
	f_{65527}	1111111111110110	7
	f_{65528}	1111111111110111	4
K_{117}	f_{26213}	0110011001100100	12
	f_{26214}	0110011001100101	7
	f_{39323}	1001100110011010	7
	f_{39324}	1001100110011011	12

Conclusions

The results presented in the paper show the possibility of practical use of the spectrum of dynamic parameters to determine the properties and classification of Boolean functions presented in the form of sequences. Sequences equivalent in terms of spectrum indices have been determined, i.e. equivalent Boolean functions by complexity and, accordingly, based on the analysis of the combinational parts of automata, a classification of automata models of systems is constructed.

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