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Longitudinal Stability Analysis of Aircrafts

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Abstract Airplane analysis and design is a complex and tedious process. Because of the uncertainties involved in flight, stability analysis bears a clear significance for flight safety. Stability of an aircraft can be analyzed under two directional focus: longitudinal and lateral. Stability is the vital issue to assess the limits of an aircraft. How fast the response of an aircraft is to sequential commands are all limited by the dynamics, and hence the stability of an aircraft. As an example, in this paper, longitudinal stability of A-7A Corsair II and CV-880M are investigated, and juxtaposed. These two aircraft are intentionally selected because of their publicly available aerodynamic parameters. This made it possible to study the models and dynamic behavior of these two above-mentioned planes. Because of their different construction objectives, dynamic stability modes are distinguished fairly clearly and distinctly.

Keywords Aerodynamic, Dynamic Stability Modes, Longitudinal Motion, Flight Dynamics

1. Introduction

Stability is an important subject for designing, controlling and providing safe flight for an aircraft or any flying vehicle. Stability is crucial to each kind of aircraft. Therefore, it must be handled carefully regarding to the type of aircraft which will be investigated. Because, some type of aircrafts as commercial aircrafts have higher stability than warplanes. The reason of that, warplanes must perform maneuver fast regarding to the purpose of these aircrafts [1].

Stability of an aircraft can be investigated under trimmed condition. Trimmed condition is a condition that the aircraft is in straight level flight at a constant velocity and all forces that acting on the aircraft are in equilibrium. Stability refers to the tendency of the aircraft to return to the trim condition when the aircraft is disturbed when it is in trimmed condition according to the general definition. However, stability of an aircraft is separated into parts for accurate investigation [1-2].

Stability of an aircraft is investigated under two topics, static stability and dynamic stability. Static stability can be defined as the initial tendency of aircraft to return to trimmed condition [2]. Static stability has three subtopics as positive, neutral and negative static stability. Positive static stability refers to initial tendency to return the original position or attitude. Neutral static stability refers to staying in the new position. Negative static stability refers to diverging from the initial position.

The dynamic stability of the aircraft involves time to react to its static stability after it is disturbed from its trimmed condition. Dynamic stability involves oscillations that occur when the aircraft tries to return to its original position or attitude. Dynamic stability of an aircraft is investigated under three subtopics as positive, neutral and negative same as static stability [2].

If an aircraft is disturbed from trimmed condition has positive static stability and it returns to straight level flight fast, it has positive dynamic stability. However, the aircraft may pass through level flight and remain pitched down and then continue the recovery process by pitching back up. This pitching up and down is known as an oscillation. If the oscillations decrease over time, that means the aircraft has positive dynamic stability. If the oscillations increase over time, that means the aircraft has negative dynamic stability. If the oscillations remain the same over time, that means the aircraft has neutral dynamic stability.

Dynamic stability of aircrafts can be separated into longitudinal and lateral motion [1-2]. Longitudinal motion describes axial force X, normal force Z and pitching moment M of an aircraft. On the other hand, lateral motion describes side force Y, rolling moment L and yawing moment N of aircraft. Longitudinal and lateral motion of an aircraft can be modeled separately even though there is minor interaction between them.

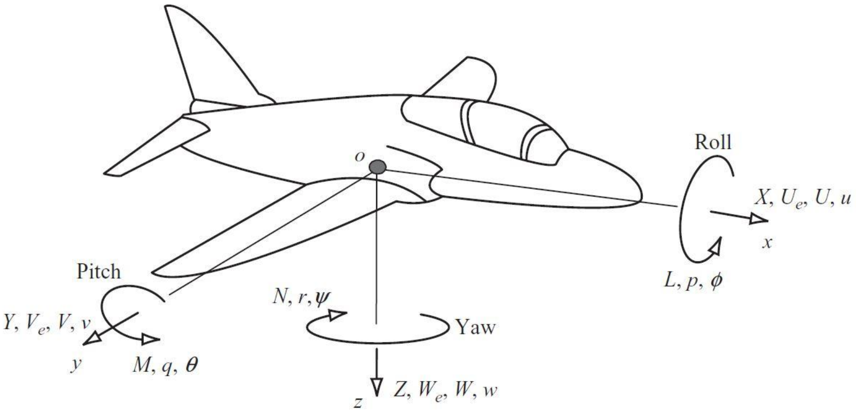

Figure 1: Aircraft motion variables [1]

Longitudinal motion has two stability modes, short period and phugoid mode. Lateral motion has three stability modes, roll subsidence mode, spiral mode, and Dutch roll mode [1-2]. In this paper, longitudinal motion modes are considered to investigate.

Theoretical and experimental studies or investigations about stability of aircrafts are being performed over the years with the development of the new techniques. Stability of aircrafts is studied to design aircraft for specific areas, to develop new controlling techniques or to compare different aircrafts to represent the differences among them.

Jafarov M. E., Ozdemir U. and Kavsaoglu M. investigated longitudinal motion of aircraft which is approximately same as Boeing 737-400. Longitudinal stability analysis of the aircraft has been performed by obtaining transfer functions. The phugoid and short period's modes and frequency and transient responses are analyzed in the study [3].

N. Siepenkötter and W. Alles investigated nonlinear dynamic stability analysis of flexible aircraft. A different approach was used in the paper regarding to the nonlinearity of the system. The eigenvalues of the stability modes are obtained and stability of the modes are interpreted by using bifurcation diagram [4].

T. Goetzendorf-Grabowski, D. Mieszalski and E. Marcinkiewicz studied stability analysis of two aircrafts EADS Ranger 2000 and PW-6 Glider by using Simulation and Dynamic Stability Analysis (SDSA) tool. The project focused on obtaining the stability characteristics of the modes of the aircrafts. A comparison between computational and experimental results is performed and mode characteristics are interpreted for both aircraft [5].

In this paper, longitudinal stability of A-7A Corsair II and CV-880M will be investigated. The dynamic stability of both aircrafts will be analyzed under two topics. Firstly, the roots of the characteristic equations will be found. Secondly, the frequency domain analysis will be performed and interpreted by using Bode plots. In the final part of the paper, both aircraft will be compared.

Aerodynamic parameters of aircrafts are obtained from technical documents [6-7].

2. Longitudinal Dynamic Stability

Longitudinal stability of an aircraft can be defined as the tendency of the aircraft nose to pitch up or pitch down under disturbance. If an airplane is longitudinally stable, it will return to a properly trimmed angle of attack after the disturbance is removed.

Longitudinal dynamic stability modes are excited whenever the aircraft is disturbed from its equilibrium trim state. A disturbance can be created by pilot control inputs, a change in power setting, airframe configuration changes such as flap deployment (rudder, aileron and elevator) and by external atmospheric influences such as gusts and turbulence [2]. Longitudinal response of aircrafts to disturbances can be examined under two dynamic stability modes which are named as short period and phugoid modes [1-2].

2.1. Short Period Mode

Short period mode of longitudinal motion occurs in the form of oscillation. Thus, the mode is also called as the short period oscillation. The short period oscillation has a period no more than a few seconds or less than 1 second. Besides, short period oscillation can be only studied during the early stage of the disturbed flight [2].

The short period mode oscillation is a damped oscillation in pitch about the "oy" axis. The oscillation that occurs in this mode is a classical second order oscillation. In the short period oscillation, pitch angle perturbation (θ) and normal velocity (w) in the same phase and axial velocity (u) and pitch rate (q) responses are very small [1], [2]. An important characteristic of the mode is that the speed remains approximately constant ($u \approx 0$) during the disturbance. As the period of the mode is short, inertia and momentum effects ensure that speed response in the time scale of the mode is negligible [1].

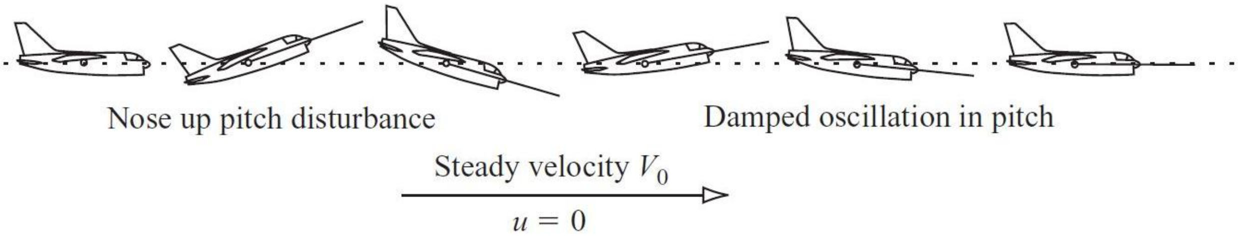

Figure 2: Short Period Mode [1]

2.2. Phugoid Mode

Phugoid mode is also consists of oscillations. Phugoid mode's oscillation is a lightly damped and a low frequency oscillation of axial speed (u), pitch angle (θ) and altitude (h) of aircraft. For example, if speed of aircraft (u) reduces a small amount suddenly, lift force starts to decrease and inherently altitude of aircraft also decreases until lift force equals to weight. However, while aircraft is going down it gains speed and that is also increases lift force and aircraft climbs. This motion continues until the effects of drag gradually damps out all motion. This long period motion of an aircraft is named as phugoid mode [1-2].

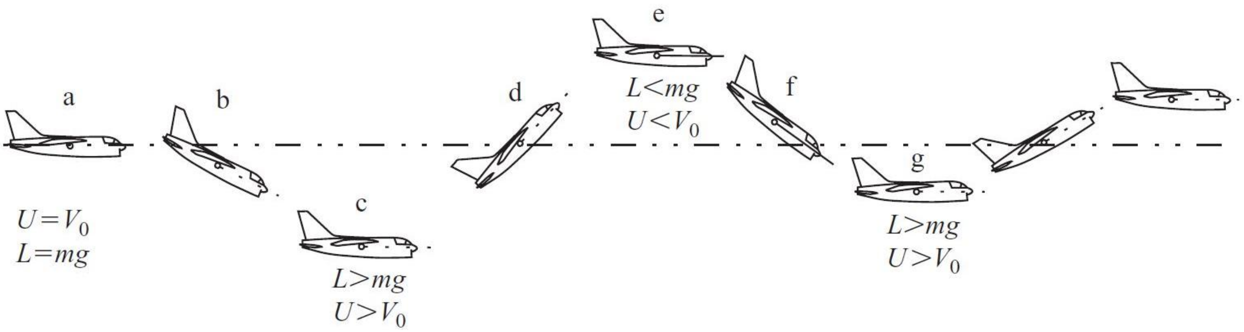

Figure 3: Phugoid Mode [1]

3. Equations of Motion for Longitudinal Motion

By the reason effects of longitudinal and lateral motion to each other is minor, two motion can be accepted as decoupled and equations can be simplify by eliminating small effect variables [1-2]. Hence, longitudinal motion of an aircraft can be described only by the axial force X, normal force Z and pitching moment M variables as below.

$$
\begin{aligned}
& m\dot{u} - \mathring{X}_{\dot{w}}\dot{w} = \mathring{X}_u u - \mathring{X}_w w - (\mathring{X}_q - \mathrm{mW}_e)q - mg\theta\cos\theta_e + \mathring{X}_\eta \eta + \mathring{X}_\tau \tau \\
& \mathrm{m}\dot{\mathrm{w}} - \mathring{Z}_{\dot{w}}\dot{\mathrm{w}} = \mathring{Z}_u u - \mathring{Z}_w w - (\mathring{Z}_q + \mathrm{mU}_e)\mathrm{q} - mg\theta\sin\theta_e + \mathring{Z}_\eta \eta + \mathring{Z}_\tau \tau \\
& I_y\dot{q} - \mathring{M}_{\dot{w}}\dot{w} = \mathring{M}_u u + \mathring{M}_w w + \mathring{M}_q q + \mathring{M}_\eta \eta + \mathring{M}_\tau \tau \\
& \dot{\theta} = q
\end{aligned}
\tag{1}
$$

Characteristic function of longitudinal motion of aircraft can be written in form of,

$$\Delta(\mathrm{s}) = \mathrm{as}^4 + b\mathrm{s}^3 + c\,\mathrm{s}^2 + \mathrm{ds} + \mathrm{e} \tag{2}$$

where a, b, c, d and e are the coefficients which are composed of aerodynamic stability derivatives, mass and moment of inertia, etc. [2].

$$a = mI_y\left(m - \mathring{Z}_{\dot{w}}\right) \tag{3}$$

$$b = I_y\left(\mathring{X}_u\mathring{Z}_{\dot{w}} - \mathring{X}_{\dot{w}}\mathring{Z}_u\right) - mI_y\left(\mathring{X}_u + \mathring{Z}_{\dot{w}}\right) - mM_w\left(\mathring{Z}_q + mU_e\right) - m\mathring{M}_q\left(m - \mathring{Z}_{\dot{w}}\right) \tag{4}$$

$$
\begin{aligned}
c = {} & I_y\left(\mathring{X}_u\mathring{Z}_w - \mathring{X}_w\mathring{Z}_u\right) + \left(\mathring{X}_u\mathring{M}_{\dot{w}} - \mathring{X}_{\dot{w}}\mathring{M}_u\right)\left(\mathring{Z}_q + mU_e\right) \\
& + \mathring{Z}_u\left(\mathring{X}_{\dot{w}}\mathring{M}_q - \mathring{X}_q\mathring{M}_{\dot{w}}\right) + m\left(\mathring{M}_q\mathring{Z}_w - \mathring{M}_w\mathring{Z}_q\right) \\
& + mW_e\left(\mathring{M}_{\dot{w}}\mathring{Z}_u - \mathring{M}_u\mathring{Z}_{\dot{w}}\right) + m^2\left(\mathring{M}_{\dot{w}}\,g\sin\theta_e + W_e\mathring{M}_u - U_e\mathring{M}_w\right)
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
d = {} & \left(\mathring{X}_u\mathring{M}_w - \mathring{X}_w\mathring{M}_u\right)\left(\mathring{Z}_q + mU_e\right) + \left(\mathring{M}_u\mathring{Z}_w - \mathring{M}_w\mathring{Z}_u\right)\left(\mathring{X}_q - mW_e\right) \\
& + \mathring{M}_q\left(\mathring{X}_w\mathring{Z}_u - \mathring{X}_u\mathring{Z}_w\right) + mg\cos\theta_e\left(\mathring{M}_{\dot{w}}\mathring{Z}_u + \mathring{M}_u\left(m - \mathring{Z}_{\dot{w}}\right)\right) \\
& + mg\sin\theta_e\left(\mathring{X}_{\dot{w}}\mathring{M}_u - \mathring{X}_u\mathring{M}_{\dot{w}} + m\mathring{M}_w\right)
\end{aligned}
\tag{6}
$$

$$e = mg\sin\theta_e\left(\mathring{X}_w\mathring{M}_u - \mathring{X}_u\mathring{M}_w\right) + mg\cos\theta_e\left(\mathring{M}_w\mathring{Z}_u - \mathring{M}_u\mathring{Z}_w\right) \tag{7}$$

In this paper longitudinal stability analysis of A-7A Corsair II and CV-880M are investigated. These two aircrafts are intentionally selected because of their already available aerodynamic properties [6-7]. In addition to that aircraft characteristics will distinguish clearly because of their different aerodynamic properties.

Characteristic function of longitudinal motion of A-7A Corsair II is obtained by using aerodynamic parameter values at 0.25 Mach speed and altitude of sea level from the technical document[6] as follows,

$$\Delta s = s^4 + 1.31073728\mathrm{s}^3 + 3.1455877412\mathrm{s}^2 + 0.088845419s + 0.075383193 \tag{8}$$

Characteristic function of longitudinal motion of CV-880M is obtained by using aerodynamic parameter values at 0.25 Mach speed and altitude of sea level from the technical document [7] as follows,

$$\Delta s = s^4 + 1.5626272s^3 + 1.7094611351s^2 + 0.057640488s + 0.031233492 \tag{9}$$

4. Stability Comparison of A-7A Corsair II and CV-880M

Stability features of two aircraft are inherently different, because A-7A Corsair II is a warplane and CV-880M is a commercial airliner. As described before phugoid mode is a long term, oscillatory response and short period mode is short term and non-oscillatory response of aircraft to disturbances. This phenomenon will be clearer if roots of characteristic equation are plotted. As seen in Figure 5 and Table 1. responses of A-7A Corsair II are more rapid than CV-880M. Frequency of short period mode and phugoid mode of A-7A Corsair II is higher than CV-880M and damping ratio of short period mode and phugoid mode of A-7A Corsair II less than CV-880M. It means that maneuverability capability of A-7A Corsair II is better than CV-880M as expected.

Table 1: Comparison of Longitudinal Motion Characteristics

Aircraft	**Phugoid Mode**		**Short Period Mode**	
	Frequency (rad/s)	**Damping Ratio**	**Frequency (rad/s)**	**Damping Ratio**
A-7A Corsair II	0.156	0.0594	1.76	0.367
CV-880M	0.137	0.0628	1.29	0.599

Frequency response of systems is another way to examine the features and limits of the systems. Dynamic analysis of aircrafts in frequency domain whom have advanced control systems gives additional information about the dynamic modes comparing to time domain analysis. Mostly used way to analyze the frequency response is bode plots of the transfer functions.

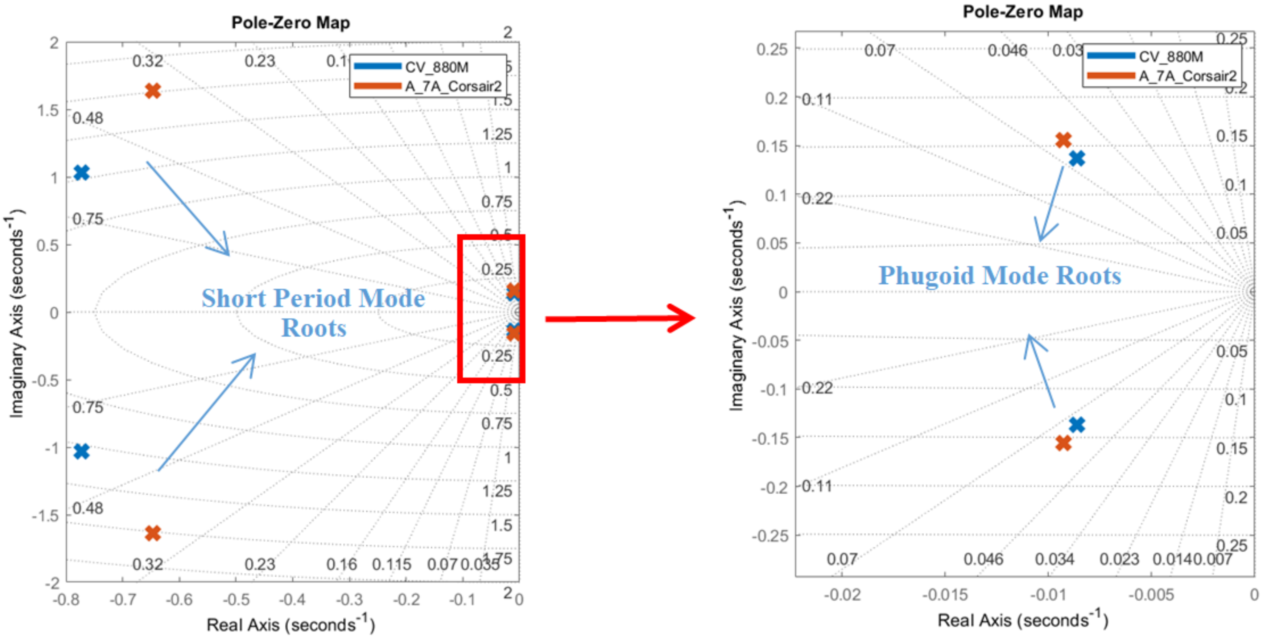

Figure 4: Roots of Longitudinal Motion

Therefore, elevator longitudinal transfer function of A-7A Corsair II at 0.25 Mach speed and sea level altitude conditions obtained from [6] as below.

$$\frac{N_{\delta_e}^{\theta}}{\Delta(s)} = \frac{-5.43s^2 - 3.853128s + 0.084943662}{s^4 + 1.3103728s^3 + 3.145877412s^2 + 0.08845419s + 0.075383193} \qquad (10)$$

and elevator longitudinal transfer function of CV-880M at 0.25 Mach speed and sea level altitude conditions obtained from [7] as below.

$$\frac{N_{\delta_e}^{\theta}}{\Delta(s)} = \frac{-0.642s^2 - 0.415485s - 0.019193232}{s^4 + 1.5626272s^3 + 1.7094611351s^2 + 0.057640488s + 0.031233492} \qquad (11)$$

Aircraft control systems can be considered as a low pass system [1]. Because, as the input frequency increases, magnitude of the output decreases after a certain frequency limit which is named as bandwidth of the system. The actual -3dB magnitude point is very close to short periodfrequency point (ω_s). Hence for practical evaluation short period frequency can be considered as bandwidth of the aircraft systems [6].

As seen in the bode plot of elevator input response of A-7A Corsair II at low frequencies amplify until input frequency reaches phugoid frequency. On the other hand, CV-880M attenuates the input and starts to amplify input signal as approaches to phugoid frequency. Besides, at lower frequencies A-7A Corsair II responses only with a little phase lag, CV-880M has 180 degree phase shift in the output. At higher frequencies, for both A-7A Corsair II and CV-880 magnitude decreases dramatically. Therefore, it can be said that if control frequency exceeds short period mode frequency, aircraft will not maneuver well. However, as clearly seen in the Figure 5, even after short period frequency, magnitude of A-7A Corsair II remains positive. On the other hand, magnitude of CV-880M starts to decrease far before short period frequency. This means that pilots who control A-7A Corsair II can give sharper control inputs as expected.

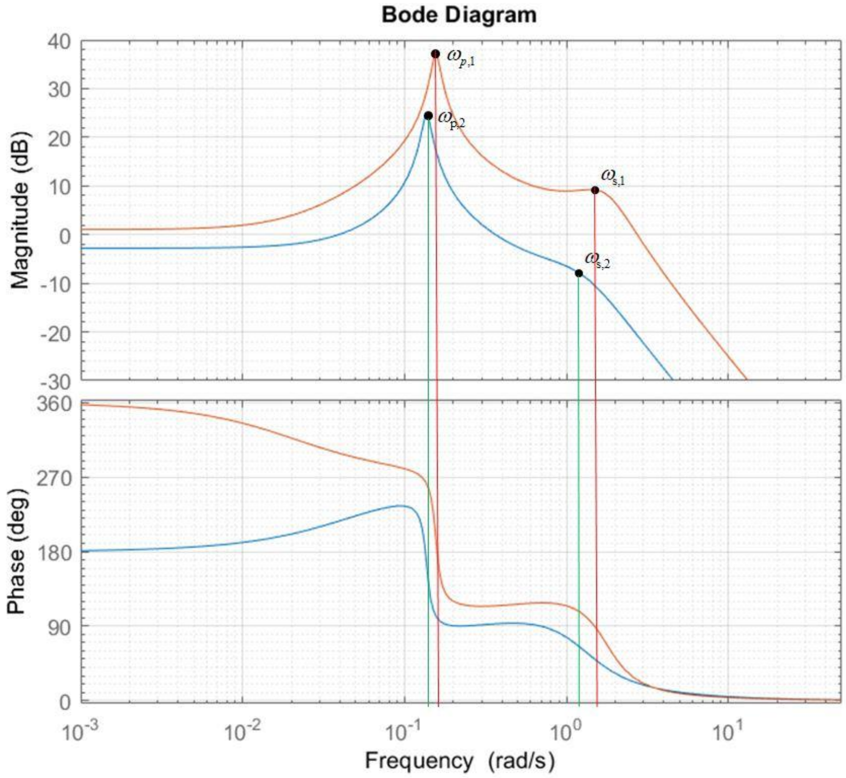

Figure 5: Breaking Frequencies

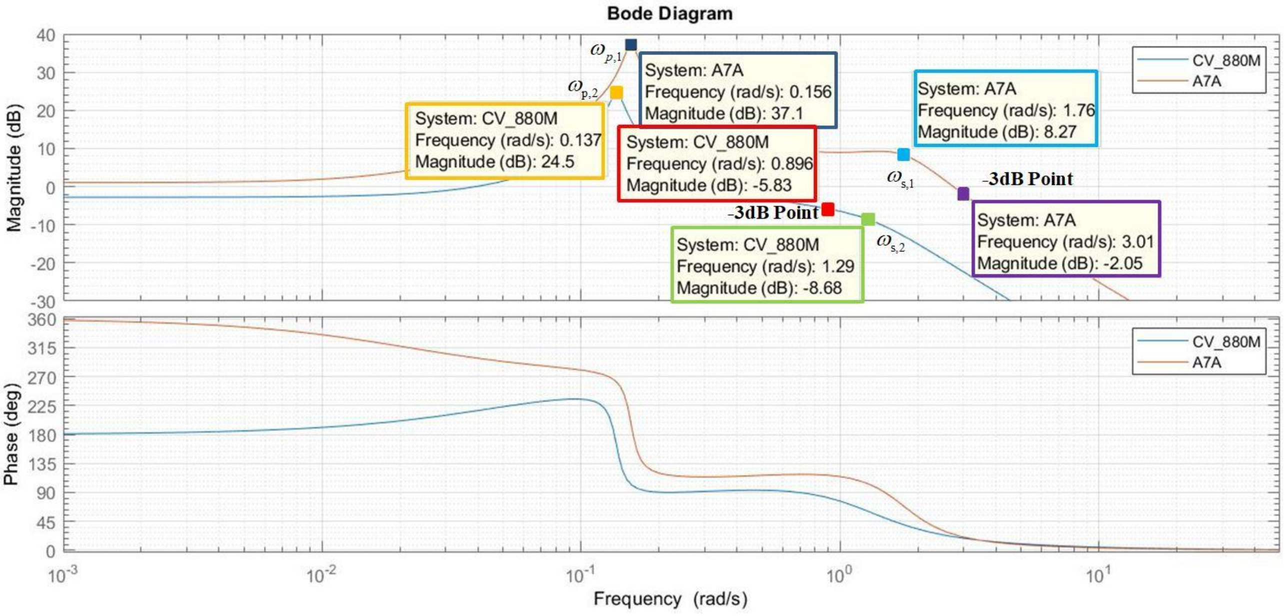

Figure 6: Bandwidth of Systems

5. Conclusion

The current study had aimed to indicate the significant differences of different aircraft types in dynamic stability analysis. Therefore, a warplane A-7A Corsair and a commercial airliner CV-880M are selected to perform dynamic stability analysis.

In this study, dynamic stability analysis of A-7A Corsair II and CV-880M are performed by using characteristic equations and transfer functions of each aircraft. Aerodynamic parameters of both aircrafts are obtained from technical documents [6-7] and used to carry out the analysis. Firstly, roots of the characteristic equations of both aircrafts are found. The roots are shown in the Figure 4 by Pole-Zero Map. Then, by using the roots of the characteristic equations damping ratio and frequency of the dynamic stability modes are computed. According to the tabulated results in Table 1, frequency of short period mode and phugoid mode of A-7A Corsair II is higher than 14 CV-880M and damping ratio of short period mode and phugoid mode of A-7A Corsair II less than CV-880M. These results showed that warplane A-7A Corsair responses to control inputs faster and has better maneuvering capability than commercial airplane CV-880M as expected.

After calculating the dynamic stability mode frequencies and damping ratios, the frequency domain analysis of both aircrafts is also performed. Frequency domain analysis is one of the beneficial analysis methods for obtaining more information about aircrafts that have advanced control systems. To perform this part, bode plots of both aircrafts are plotted as Magnitude (dB) vs Frequency (rad/s) and Phase (deg) vs Frequency (rad/s) in Figure 5. Detailed explanation has given in the Figure 6. It can be seen phugoid mode dominates in frequency response of both aircrafts. According to the results in the Figure 5 and Figure 6, aircraft will not able to maneuver as desired when the control frequency exceeds short period mode frequency. It can be clearly seen in Figure 6, the magnitude of A-7A Corsair II remains positive after short period frequency while the magnitude of CV-880M decreases before short period frequency. This means that pilots who control A-7A Corsair II can able to give sharper control inputs as expected.

Dynamic stability analysis of A-7A Corsair II and CV-880M has performed although the studied aircrafts do not in the same category. Because of their different construction objectives, dynamic stability modes are distinguished fairly clearly and distinctly. For this reason, the researchers aimed this study to be an example and to be used in advanced research.

References

[1]. Cook MV. Flight Dynamics Principles. Burlington, MA, USA: Butterworth-Heinemann, 2007.

[2]. Nelson RC. Flight Stabilty and Automatic Control. Singapore, McGraw-Hill, 1998.

[3]. Jafarov EM, Özdemir U, Kavşaoğlu M. Calculation of the Longitudinal Stability Derivatives of a Transport Aircraft and Analysis of Longitudinal Modes. In: WSEAS International Conference on Automatic Control, Modeling \& Simulation; İstanbul, Turkey; 2007. pp. 106-111.

[4]. Siepenkötter N, Alles W. Stability analysis of the nonlinear dynamics of flexible aircraft. Aerospace Science and Technology 2005; 9 (2); 135-141. doi.org/10.1016/j.ast.2004.10.005.

[5]. Goetzendorf-Grabowski T, Mieszalski D, Marcinkiewicz E. Stability Analysis Using SDSA Tool. Progress in Aerospace Sciences 2011; 47 (8); 636-646. doi.org/10.1016/j.paerosci.2011.08.007

[6]. Teper GL. Aircraft Stability and Control Data. Hawthorne, CA, USA; Systems Technology Inc., 1969.

[7]. Heffley RK, Jewell WF, Aircraft Handling Qualities Data. Hawthorne, CA, USA; Systems Technology Inc., 1972.