Available online www.jsaer.com

Journal of Scientific and Engineering Research, 2020, 7(10):139-145

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

Multi-attribute Decision Making Methods Based on Picture Fuzzy Distance-Cosine Similarity Measures

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Abstract In this paper, we investigate the picture fuzzy multiple attribute decision making problems where the attribute values are expressed in picture fuzzy numbers. By analyzing the limitations of the original normalized Euclidean distance measure on picture fuzzy sets, we present another form of normalized Euclidean distance measures between picture fuzzy sets (PFSs) by considering the degree of positive membership, degree of neutral membership, degree of negative membership, degree of refusal membership and score function in PFSs. We also construct the distance-cosine similarity by combining the cosine similarity and the standard Euclidean distance measure, and verify its rationality. Furthermore, we develop a decision making method using the weighted distance-cosine similarity measure. Finally, a concrete instance of appraising a financial investment risk is given to demonstrate the applications and efficiency of the proposed decision-making method.

Keywords Picture fuzzy set, Distance-cosine similarity measure, Multi-attribute decision making

1. Introduction

The concept of intuitionistic fuzzy sets proposed by Atanassov [1] is an effective extension of fuzzy set theory. It is more helpful than fuzzy sets to deal with fuzzy and uncertain information. Since its proposal, intuitionistic fuzzy sets have been successfully applied in different practical fields such as fuzzy decision-making, pattern recognition, and medical diagnosis. While there are some situations in real life that cannot be described by intuitionistic fuzzy sets. For example, in the voting model, voters' opinions on candidates include many types: yes, abstain, no, refusal, but the intuitionistic fuzzy set only focuses on those who vote for or against it, and the neutrals and abstainers are equally considered. In order to deal with this scenario, Cuong extended the intuitionistic fuzzy set to the picture fuzzy set in 2014 [2]. The picture fuzzy set is characterized by three functions expressing the degree of membership, the degree of neutral membership and the degree of negative membership.

Similarity measure and distance measure are important and useful tools to determine the degree of similarity or dispersion between two objects. They are widely used in the fields of pattern recognition, decision making and clustering. Inspired by the distance on the intuitionistic fuzzy set, Cuong gives the normalized Euclidean distance measure on the picture fuzzy set [2]. Singh gave a geometrical interpretation of picture fuzzy sets and proposed correlation coefficients for picture fuzzy sets, and he also applied the correlation coefficient to clustering analysis under picture fuzzy environments [3]. Wei presented another form of eight similarity measures between PFSs based on the cosine function between PFSs by considering the degree of positive membership, degree of neutral membership, degree of negative membership and degree of refusal membership in PFSs.Then, he applied these weighted cosine function similarity measures between PFSs to strategic decision making [4, 5]. In [6], the picture fuzzy cosine similarity is further extended to the interval value scene fuzzy cosine similarity, and their use in decision-making problems is given. However, the picture fuzzy cosine similarity is essentially the cosine value of the angle between the picture fuzzy vectors. From the geometric point of view, it mainly measures the proximity of the picture fuzzy vector in the direction. For example, if the similarity degree of the picture fuzzy sets A={(x,0.4,0.2,0.2)} with B_1={(x,0.1,0.2,0.3)}, B_2={(x,0.1,0.2,0.4)} and B_3={(x,0.1,0.4,0.3)} is calculated using the picture fuzzy cosine similarity formula in the [4], then the similarity degree of A and B_i (i=1, 2, 3) is equal to 0.7591. From a geometric point of view, their included angles are the same, so only considering the difference in direction, it is impossible to determine that which one of B_i (i=1, 2, 3) is closer to A.

In order to solve the limitation of the picture fuzzy cosine similarity, this paper improves the original normalizedEuclidean distance measure on the picture fuzzy set, and gives an improved normalized Euclidean distance measure, which consider the comprehensive effects of positive membership, neutral membership, negative membership, refusal membership and score function. Since the picture fuzzy vector is a quantity with both magnitude and direction, the difference between the picture fuzzy vectors should fully consider its influence in the magnitude and direction. Based on the idea, this paper combines cosine similarity and the improved normalized Euclidean distance measure to construct the distance-cosine similarity. Then, a picture fuzzy multi-attribute decision-making method based on weighted distance-cosine similarity is established, and an example is used to verify that the decision-making method proposed in this paper is reasonable and effective.

2. Preliminaries

Voting is common in daily life, human voters can be divided into four groups: vote for, abstain, vote against, refusal of the voting. In order to describe this situation, Cuong *et al* generalized the concept of intuitionistic fuzzy sets to the concept of picture fuzzy sets as follows.

Definition 2.1 [2] Let X be a universe of discourse. A picture fuzzy set (PFS) A on the universe X is an object of the form

$$A=\{(x,\mu_A(x),\eta_A(x),\gamma_A(x))\,|\,x\in X\},$$

where $\mu_A(x)\ (\in[0,1])$ is called the "degree of positive membership of A", $\eta_A(x)\ (\in[0,1])$ is called the "degree of neutral membership of A" and $\gamma_A(x)\ (\in[0,1])$ is called the "degree of negative membership", and μ_A,η_A,γ_A satisfy: $\mu_A(x)+\eta_A(x)+\gamma_A(x)\le 1,\ \forall x\in X$. For any $x\in X$, $\nu_A(x)=1-(\mu_A(x)+\eta_A(x)+\gamma_A(x))$ could be called the degree of refusal membership of x in A. In particularly, if X only has one element, then $A(x)=(\mu_A(x),\eta_A(x),\gamma_A(x))$ is called a picture fuzzy number (PFN). For convenience, a PFN is denoted as (μ_A,η_A,γ_A).

Definition 2.2 [2] Given two PFSs represented by A and B on universe X, the inclusion, union, intersection and complement operations are defined as follows:

(1) $A\bigcap B=\{(x,\min\{\mu_A(x),\mu_B(x)\},\min\{\eta_A(x),\eta_B(x)\},\max\{\gamma_A(x),\gamma_B(x)\})\,|\,x\in X\}$;

(2) $A\bigcup B=\{(x,\max\{\mu_A(x),\mu_B(x)\},\min\{\eta_A(x),\eta_B(x)\},\min\{\gamma_A(x),\gamma_B(x)\})\,|\,x\in X\}$;

(3) $A\subseteq B\Leftrightarrow\forall x\in X,\ \mu_A(x)\le\mu_B(x),\eta_A(x)\le\eta_B(x),\ \gamma_A(x)\ge\gamma_B(x)$ and $\nu_A(x)\le\nu_B(x)$;

(4) $A=B\Leftrightarrow A\subseteq B$ and $B\subseteq A$.

Definition 2.3 [7,8] Let $\alpha=(\mu_\alpha,\eta_\alpha,\gamma_\alpha)$ be a PFN, then a score function s and an accuracy function h of a PFN can be represented as follows:

$$h(\alpha)=\mu_\alpha+\eta_\alpha+\gamma_\alpha,\ s(\alpha)=\frac{1+\mu_\alpha-\gamma_\alpha}{2},$$

where $h(\alpha)\in[0,1]$, $s(\alpha)\in[0,1]$.

We also use voting as a good example to explain the above definition, where $s(\alpha)$ represents goal difference and $h(\alpha)$ can be interpreted as the effective degree of voting. When S(α) increases, we can know that there are more people who vote for α and people who vote against α become less. When $h(\alpha)$ increases, we can know that there are more people who vote for or against α and people who refuse to vote become less. So, $h(\alpha)$ depicts the effective degree of voting.

Definition 2.4 [7] For any two PFSs A and B on a discrete universe of discourse $X=\{x_1,x_2,\cdots,x_n\}$, the normalized Euclidean distance $d_E(A,B)$ between A and B is defined by

$$d_E(A,B)=\sqrt{\frac{1}{2n}\sum_{i=1}^{n}\left((\mu_A(x_i)-\mu_B(x_i))^2+(\eta_A(x_i)-\eta_B(x_i))^2+(\gamma_A(x_i)-\gamma_B(x_i))^2\right)}.$$

The distance measurement formula $d_E(A,B)$ in Definition 2.4 only considers the difference between two PFSs A and B from the three aspects of positive membership degree, neutral membership degree, and negative membership degree. However, the contribution of refusal membership degree to difference is ignored. Inspired by the idea of intuitionistic fuzzy distance in [9], the distance measure formula between A and B should fully consider the comprehensive influence of five aspects: positive membership, neutral membership, negative membership, refusal membership and score function, we give an improved Euclidean distance measure formula.

Definition 2.5 For any two PFSs A and B on a discrete universe of discourse $X=\{x_1,x_2,\cdots,x_n\}$, the improved normalized Euclidean distance $d_E(A,B)$ between A and B is defined by

$$d_{E_1}(A,B)=\sqrt{\frac{1}{3n}\sum_{i=1}^{n}\begin{pmatrix}(\mu_A(x_i)-\mu_B(x_i))^2+(\eta_A(x_i)-\eta_B(x_i))^2+(\gamma_A(x_i)-\gamma_B(x_i))^2\\+(\nu_A(x_i)-\nu_B(x_i))^2+(s_A(x_i)-s_B(x_i))^2\end{pmatrix}}.$$

In many cases, the weight of the elements $x_i\in X$ should be taken into account. As a result, an improved normalized weighted Euclidean distance between PFSs A and B is also proposed as follows:

$$d_{E_2}(A,B)=\sqrt{\frac{1}{3}\sum_{i=1}^{n}w_i\begin{pmatrix}(\mu_A(x_i)-\mu_B(x_i))^2+(\eta_A(x_i)-\eta_B(x_i))^2+(\gamma_A(x_i)-\gamma_B(x_i))^2\\+(\nu_A(x_i)-\nu_B(x_i))^2+(s_A(x_i)-s_B(x_i))^2\end{pmatrix}},$$

where $s_A(x_i)$ and $s_B(x_i)$ are the score functions of the PFSs A and B with respect to the element x_i, respectively .

In fact,

$$\begin{aligned}(\nu_A(x_i)-\nu_B(x_i))^2&=\left([1-(\mu_A(x_i)+\eta_A(x_i)+\gamma_A(x_i)]+[1-(\mu_B(x_i)+\eta_B(x_i)+\gamma_B(x_i)]\right)^2\\&=(h_A(x_i)-h_B(x_i))^2\end{aligned}$$

where $h_A(x_i)$ and $h_B(x_i)$ are the accuracy functions of the PFSs A and B with respect to the element x_i, respectively. It can be seen that the difference of refusal membership degree in Definition 2.5 also contains the difference of accuracy functions.

Example 2.6 Let $A=\{(x,0.5,0.3,0.1)\}$, $B=\{(x,0.1,0.6,0.1)\}$ and $C=\{(x,0.1,0.3,0.4)\}$ be three PFSs on a discrete universe of discourse $X=\{x\}$. Then

$$d_E(A,B)=0.3606{=}d_E(A,C),\ d_{E_1}(A,B)=0.3162<d_{E_1}(A,C)=0.3571.$$

It can be seen that B and C are similar close to A under the original normalized Euclidean distance. In the decision model, if A is regarded as an ideal solution, the order of schemes B and C can not be given by using the original Euclidean distance formula. However, according to the calculation values by adopting the improved normalized Euclidean distance formula, it can be known that scheme B is closer to the ideal scheme A, so it can be seen that scheme B is better than scheme C.

3. Distance-cosine similarity based on picture fuzzy sets

The idea of cosine similarity (angle distance) comes from the cosine value of the angle between vectors. The cosine similarity of the picture fuzzy set was proposed by treating the positive membership degree, the neutral membership degree, the negative membership degree and the refusal membership degree of the picture fuzzy set as the vector space of the four terms [3].

Definition 3.1 [3,4] Let A and B be two PFSs on a discrete universe of discourse $X=\{x_1,x_2,\cdots,x_n\}$, and $w=\{w_1,w_2,\cdots,w_n\}$ be a weighting vector. Then a cosine similarity measure between A and B is defined by

$$cos_1(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{\mu_A(x_i)\mu_B(x_i)+\eta_A(x_i)\eta_B(x_i)+\gamma_A(x_i)\gamma_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i)+\eta_A^2(x_i)+\gamma_A^2(x_i)+\nu_A^2(x_i)}\sqrt{\mu_B^2(x_i)+\eta_B^2(x_i)+\gamma_B^2(x_i)+\nu_B^2(x_i)}}.$$

And the weighted cosine similarity measure between PFSs A and B is also proposed as follows:

$$cos_2(A,B)=\sum_{i=1}^{n}w_i\frac{\mu_A(x_i)\mu_B(x_i)+\eta_A(x_i)\eta_B(x_i)+\gamma_A(x_i)\gamma_B(x_i)+\nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i)+\eta_A^2(x_i)+\gamma_A^2(x_i)+\nu_A^2(x_i)}\sqrt{\mu_B^2(x_i)+\eta_B^2(x_i)+\gamma_B^2(x_i)+\nu_B^2(x_i)}}.$$

The cosine (weighted) similarity measure between A and B satisfies the following conditions:

(1) $0\le \cos_i(A,B)\le 1$,

(2) $\cos_i(A,B)=\cos_i(B,A)$,

(3) if $A=B$, then $\cos_i(A,B)=1$,

(4) if A, B and C are PFSs on X, and $A\subseteq B\subseteq C$, then $\cos_i(A,C)\le \min\{\cos_i(A,B),\cos_i(B,C)\}$.

Example 3.2 Let $D=\{(x,0.2,0.8,0.0)\}$, $E=\{(x,0.8,0.2,0.0)\}$ and $F=\{(x,0.5,0.5,0)\}$ be three PFSs on $X=\{x\}$. The result calculated according to the formulas of Definition 3.1 and Definition 3.2 shows that, $cos_1(D,F)=cos_1(E,F)=0.8575$, it means that D and E are similar close to F, which is not conform to our intuition.

The results obtained by the cosine similarity measure model of Definition 3.2 are occasionally counter-intuitive, because the cosine similarity is a measure of the approaching degree of the two picture fuzzy vectors in the direction. The larger the value of the cosine similarity is, the closer the two picture fuzzy vectors are in the direction. The distance measure emphasizes the difference of picture fuzzy vector in mode (or length) in vector space. Considering that the picture fuzzy vector is a quantity composed of direction and mode, it is not comprehensive to consider the similarity of vector in direction or mode. In order to effectively measure the similarity of picture fuzzy vectors, we combine cosine similarity with improved normalized weighted Euclidean distance to give the following concept of distance-cosine similarity.

Definition 3.3 Let A and B be two PFSs on $X=\{x_1,x_2,\cdots,x_n\}$, and $w=\{w_1,w_2,\cdots,w_n\}$ be a weighting vector. Then the distance-cosine similarity measure between A and B is defined by

$$\rho_1(A,B)=\frac{\cos_1(A,B)}{\cos_1(A,B)+d_{E_1}(A,B)}.$$

And the weighted cosine similarity measure between PFSs A and B is also proposed as follows:

$$\rho_2(A,B)=\frac{\cos_2(A,B)}{\cos_2(A,B)+d_{E_2}(A,B)}.$$

Theorem 3.4 Let A, B and C be PFSs on X. Then the distance-cosine similarity measure $\rho_1(A,B)$ between A and B satisfies the following conditions:

(1) $0\le\rho_1(A,B)\le 1$,

(2) $\rho_1(A,B)=\rho_1(B,A)$,

(3) A=B if and only if $\rho_1(A,B)=1$,

(4) if $A\subseteq B\subseteq C$, then $\rho_1(A,C)\le\min\{\rho_1(A,B),\rho_1(B,C)\}$.

Proof. (1) and (2) are obviously.

(3) If A=B, it is easy to get that $\rho_1(A,B)=1$. Conversely, if $\rho_1(A,B)=1$, then $\dfrac{\cos_1(A,B)}{\cos_1(A,B)+d_{E_1}(A,B)}=1$,

and so $d_{E_1}(A,B)=0$. And for any $x\in X$, $\mu_A(x)=\mu_B(x),\eta_A(x)=\eta_B(x))^2,\gamma_A(x)=\gamma_B(x),\nu_A(x)=\nu_B(x)$, thus A=B.

(4) if $A\subseteq B\subseteq C$, then $\cos_1(A,C)\le\min\{\cos_1(A,B),\cos_1(B,C)\}$. One can easy get that $d_{E_1}(A,C)\ge\max\{d_{E_1}(A,B),d_{E_1}(B,C)\}$, then

$$\rho_1(A,C)=\frac{\cos_1(A,C)}{\cos_1(A,C)+d_{E_1}(A,C)}\le\frac{\cos_1(A,B)}{\cos_1(A,B)+d_{E_1}(A,B)}=\rho_1(A,B).$$

Similarly, we can get $\rho_1(A,C)\le\rho_1(B,C)$, thus $\rho_1(A,C)\le\min\{\rho_1(A,B),\rho_1(B,C)\}$.

For the weighted cosine similarity measure, it has properties similar to Theorem 3.4.

4. A decision-making approach based the distance-cosine similarity

Let $A=\{A_1,A_2,\cdots,A_m\}$ be a discrete set of alternatives, and $C=\{C_1,C_2,\cdots,C_n\}$ be the set of attributes, $w=\{w_1,w_2,\cdots,w_n\}$ is the weighting vector of the attribute C_j $(j=1,2,\cdots,n)$, where $w_j\in[0,1]$, $\sum_{j=1}^{n}w_j=1$.The specific procedures are shown as following:

Step1: Obtain the picture fuzzy decision matrix $D=(d_{ij})_{mn}$=$(\mu_{ij},\eta_{ij},\gamma_{ij})_{mn}$, where μ_{ij} indicates the degree of positive membership that the alternative A_i satisfies the attribute C_j given by the decision maker, η_{ij} indicates the degree of neutral membership, ν_{ij} indicates the degree of negative membership.

Step2: Determine the positive ideal solution $A^{+}=(r_1^{+},r_2^{+},\cdots,r_n^{+})$, where

$$r_j^{+}=(\mu_j^{+},\eta_j^{+},\gamma_j^{+})=(\max_i \mu_{ij},\min_i \eta_{ij},\min_i \gamma_{ij}).$$

Step3: Calculate the weighted cosine similarity measure $\rho_2(A_i,A^{+})$ between each alternative A_i and the positive ideal solution A^{+}.

Step 4: Rank all the alternatives and select the best one(s).

5. Numerical example

In this section, we utilize a practical multiple attribute decision making problems to illustrate the application of the developed approaches.

Transdisciplinary research of emerging technologies appears on the scene. Evaluating transdisciplinary research of emerging technologies has important theoretical and practical significance. Thus, we shall give a numerical example for potential evaluation of emerging technology commercialization with PFNs. There are five possible emerging technology enterprises (ETES) A_1,A_2,A_3,A_4,A_5 to select. The expert selects four attributes to assess the five possible ETES: (1) C_1 is the human resources and financial conditions; (2) C_2 is the industrialization infrastructure; (3) C_3 is the technical advancement; (4) C_4 is the development of science and technology. The five possible ETES A_i ($i=1,2,\cdots,5$) are to be assessed with PFNs according to four attributes (whose weighting vector $w=\{0.3, 0.2, 0.4, 0.1\}$, and the decision matrix $D=(d_{ij})_{5\times 4}$ is presented in Table 1, where are in the form of PFNs.

Step 1: the picture fuzzy decision matrix $D=(d_{ij})_{5\times 4}$ is shown as Table 1.

Table1: The Picture Fuzzy Decision Matrix

	C_1	C_2	C_3	C_4
A_1	(0.43,0.36,0.19)	(0.79,0.02,0.01)	(0.43,0.45,0.08)	(0.18,0.39,0.04)
A_2	(0.43,0.32,0.18)	(0.73,0.04,0.11)	(0.03,0.62,0.33)	(0.53,0.25,0.18)
A_3	(0.71,0.23,0.01)	(0.87,0.02,0.03)	(0.04,0.55,0.30)	(0.48,0.26,0.16)
A_4	(0.25,0.49,0.15)	(0.64,0.12,0.13)	(0.01,0.69,0.25)	(0.02,0.54,0.26)
A_5	(0.50,0.45,0.03)	(0.78,0.03,0.11)	(0.03,0.57,0.26)	(0.13,0.65,0.19)

Step 2: Determine the positive ideal solution A^{+} and the negative ideal solution A^{-}, where

$$A^{+}=\{(0.71,0.23,0.01), (0.87,0.02,0.01), (0.43,0.45,0.08), (0.53,0.25,0.04)\}.$$

Step 3: Calculate the weight cosine similarity measure $k_w(A_i,A^{+})$ and $k_w(A_i,A^{-})$, then

$$\rho_2(A_1,A^{+})=0.8524,\ \rho_2(A_2,A^{+})=0.7646,\ \rho_2(A_3,A^{+})=0.8123,\ \rho_2(A_4,A^{+})=0.6897,\ \rho_2(A_5,A^{+})=0.7601.$$

Step 4: Rank all the alternatives A_1-A_5in accordance with the values of $\rho_2(A_i)$, since

$$\rho_2(A_1)>\rho_2(A_3)>\rho_2(A_2)>\rho_2(A_5)>\rho_2(A_4),$$

then $A_1 \succ A_3 \succ A_2 \succ A_5 \succ A_4$. Note that "$\succ$" means "preferred to". Thus, the best region is A_1.

Acknowledgment

The works described in this paper are partially supported by Undergraduate Innovation Foundation Project of Anyang Normal University (No. X2020104790140).

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