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Modeling Volatility Dynamics of Foreign Exchange Rates in Nigeria: Application of Univariate GARCH Models

David A. Kuhe*, Augustine A. Attah

Department of Mathematics/Statistics/Computer Science, Federal University of Agriculture, Makurdi, Benue State – Nigeria

Abstract This study models the exchange rate volatility of Naira against Euro, Great British Pounds, US Dollar and West African Unit of Account (WAUA) in Nigeria using symmetric and asymmetric Autoregressive Conditional Heteroskedasticity (GARCH) models in the presence of Gaussian and non-Gaussian errors. The study utilizes daily quotations of these exchange rates from 12/11/2001 to 07/04/2017 making a total of 3755 observations each. Symmetric GARCH as well as asymmetric EGARCH and TGARCH specifications were used to model the exchange rates log return series. Results shows that symmetric GARCH (1,1) model captured all volatility clustering with evidence of shock persistence in the four exchange rate return series. The asymmetric EGARCH (1,1) and TGARCH (1,1) models produced substantial evidence for the existence of asymmetric responses and leverage effects in the four exchange rates log return series suggesting that negative shocks produces more volatility in Nigerian foreign exchange market than positive shocks of the same magnitude. All the estimated models were found to be stationary and mean reverting indicating the predictability and stability of the conditional variances. Results also show that the best fitted models were not necessarily the best models selected for forecasting exchange rate volatility in Nigeria.

Keywords Exchange Rate; Leverage Effect; Mean Reversion; Shock Persistence; Volatility

1. Introduction

For a large open economy like Nigeria, exchange rate plays a crucial role in both financial transactions and international trade. Exchange rate volatility is of crucial interest for policy makers in every economy. By definition, volatility refers to the spread of all likely outcomes of an uncertain random variable and is often measured as the sample standard deviation. Volatility and risk are closely related but not exactly the same. While risk is associated with the spread of negative or undesirable outcomes, volatility is associated with both negative and positive spread of an outcome. In financial context, volatility is defined as a measure of the changes in prices over a given period of time. Exchange rate volatility is a measure of fluctuations in local currencies relative to foreign currencies in exchange rate markets. Exchange rate risk or foreign exchange risk is a financial risk associated with an exposure to unanticipated changes in the exchange rate between two currencies in an exchange market. Exchange rate volatility or fluctuation is distributive in the foreign exchange market as both buyers and sellers, traders and investors as well as importers and exporters of goods and services are all expose to the same level of risk and uncertainty. Foreign currency's value fluctuates according to the forces of demand and supply, meaning that if demand for a particular currency decreases and supply increases, it can cause depreciation of the currency's value. On the other hand if supply decreases and demand increases, this can cause appreciation of the currency's value [1]. When exchange rate volatility increases it leads to uncertainty in pricing which hurts traders, investors, buyers or importers who spent more on the same quantity. Exchange rate volatility and price fluctuation have significant implications on the profits and survival of any business enterprises as well as on the volume of international trade [2].

It is generally believed that exchange rate volatility causes serious reduction in the overall level of international trade. In this regard, [3] found significant negative effects of exchange rate volatility on exports volume for some African countries. While exchange rate stability is believed to have a positive impact on economic growth and contributes to more international trade, capital inflows and macroeconomic stability, [4] found a significant negative relationship existing between exchange rate volatility and economic growth.

Foreign exchange rate volatility has been a useful measure of uncertainty about the financial and economic environment of any country including Nigeria. Traders and investors in foreign exchange market, financial institutions as well as Policy makers need accurate estimates about the future values of exchange rates. Since exchange rate volatility or fluctuation increases transaction costs and reduces the gains of international trade, having a good understanding of exchange rate volatility measures and forecasting using accurate volatility measuring tools is paramount and imperative for asset pricing and risk management in exchange markets.

The Autoregressive Conditional Heteroskedasticity (ARCH) and the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models developed by [5-6] respectively have been found useful in modeling and forecasting volatility of financial time series data. While the symmetric ARCH and GARCH model are good at capturing volatility clustering, shocks persistence and other stylized facts, they do not capture asymmetric response and leverage effect in financial returns. The extensions of GARCH models called the asymmetric models such as the exponential GARCH (EGARCH) due to [7], threshold GARCH (TGARCH) due to [8], power GARCH (PGARCH) due to [9] and Glosten, Jaganathan and Runkle GARCH (GJR-GARCH) introduced by [10] among others are very useful in capturing asymmetry and leverage effect in financial return series.

Several researchers have applied GARCH family models in measuring the volatility of exchange rates across developed and developing countries of the world and large volume of empirical evidence are well documented in the literature. For example, [11] applied GARCH modeling approach in fitting Exchange rate volatility in Rwanda. Bošnjak *et al*. [12] examined the performance of different GARCH models for the EUR/HRK and USD/HRK on daily exchange rate data spanning from 1997 to 2015. Results showed that GARCH (2,1) was the best fitted model for the EUR/HRK exchange rate while GARCH (1,1) was the best fitted model for the USD/HRK daily exchange rate return. The study found no empirical evidence for asymmetry and leverage effects for both daily EUR/HRK and USD/HRK exchange rate returns in Croatia. Pelinescu [13] applied different family of ARCH/GARCH models to analyze the volatility of the Romanian Leu/Euro exchange rate using daily time series data for the period 05/01/2000 to 31/08/2013. Results indicate that the volatility of Leu/Euro exchange rate followed an ARCH process with asymmetric property. Marreh *et al.* [14] employed ARMA-GARCH model to examine the daily Euro and US dollars exchange rates against the Gambian Dalasi (GMD) from 2003 to 2013. ARMA(1,1)-GARCH (1,1) and ARMA(2,1)-GARCH(1,1) were found to be the best fitting models for Euro/GMD and USD/GMD exchange rates return series respectively. The empirical findings revealed that the distribution of exchange rate return series was fat-tailed and there was high persistence of volatility shocks in the Gambian foreign exchange market. Ramzan *et al.* [15] compared the ability of ARMA, ARCH, GARCH and EGARCH models in modeling and predicting exchange rates volatility in Pakistan using monthly exchange rates of Pakistan for the period of July 1981 to May 2010. Symmetric GARCH (1,2) model was found to captured volatility better than other models while EGARCH(1,2) successfully captured the leverage effect in the exchange rate returns in Pakistan.

Narsoo [16] Modeled the volatility of US Dollar/Mauritian Rupee (USD/MUR) exchange rate by comparing the predictive ability of symmetric GARCH model with asymmetric EGARCH, TGARCH, PGARCH and GJR-GARCH models. The models results revealed stylized facts such volatility persistence, volatility clustering and leverage effects. The results also found the suitability of asymmetric GARCH models in predicting the volatility of USD/MUR exchange rates over symmetric GARCH models. Petrică and Stancu [17] examined the changes in the volatility of daily returns of EUR/RON exchange rate using symmetric ARCH and GARCH models as well as asymmetric EGARCH, TARCH and PARCH models with varying distributions while taking into account the daily quotations of EUR/RON exchange rate over the period January 4, 1999 to June 13, 2016. Asymmetric EGARCH (2,1) with student-t innovation was found to be the best model for estimating daily EUR/RON exchange rate volatility. Kutu and Ngalawa [18] conducted a study that examined the global shocks and exchange rate volatility of Russian rubble/United States dollar using symmetric GARCH and Asymmetric

Power ARCH (APARCH) models with normal and non-normal errors while employing monthly exchange rate data from January 1994 to December 2013. The symmetric GARCH model was found to fit better under student-t distribution. The APARCH model did not find any evidence for asymmetric response and leverage effect in the exchange rate volatility and global shocks in Russia. Epaphra [19] applied univariate nonlinear time series analysis to the daily TZS/USD exchange rate data spanning from January 4, 2009 to July 27, 2015 to examine the behaviour of exchange rate in Tanzania. He employed both ARCH and GARCH models to capture the symmetric effect in exchange rate data as well as EGARCH model to capture the asymmetry in volatility clustering and the leverage effect in exchange rate. The study revealed that exchange rate series exhibits the empirical regularities such as volatility clustering, non-normality and serial correlation. The estimate for asymmetric volatility showed that positive shocks imply a higher next period conditional variance than negative shocks of the same sign.

In Nigeria, many documented evidence on exchange rate volatility modeling are also found in the literature. For example, [20] found the suitability of ARCH family models for measuring exchange rate volatility in Nigeria. Awogbemi & Alagbe [21] applied symmetric GARCH model in examining volatility of Naira/US Dollar and Naira/UK Pound Sterling exchange rates in Nigeria using data on the monthly exchange rates from 2007-2010. They found evidence of volatility clustering and persistence in Nigerian foreign exchange market. Bala & Asemota [22] employed GARCH family models to examined foreign exchange rate volatility in Nigeria using data on monthly exchange rate return from January 1985 to July 2011 for Naira/US dollar return and from January 2004 to July 2011 for Naira/British Pounds and Naira/Euro returns in the presence of structural breaks. Results revealed the presence of volatility clustering and persistence in the three currencies without leverage effect except for models with volatility break. Incorporation of volatility breaks improved estimation of volatility models and reduced the level of shock persistence in most of the models. The study recommended the incorporation of structural breaks in GARCH models while estimating volatility of key asset prices. Adeoye & Atanda [23] applied ARCH and GARCH models to examine the degree of volatility of USD/Naira exchange rate using monthly exchange rate data from 1986 to 2008. Result showed the presence of volatility clustering and over persistence of volatility shocks. David *et al.* [24] examined the naira exchange rate against four foreign currencies: US Dollar, Euro, British Pound and Japanese Yen. The weekly data on these exchange rates spanned from January 2002 to May 2015. They employed lower symmetric and asymmetric GARCH specifications. Results of the symmetric models showed volatility persistence in all the foreign exchange rate returns. Results of the asymmetric model showed superior forecasting performance over symmetric GARCH with different impacts for both negative and positive volatility shocks. On volatility modeling of financial time series data in Nigeria, see also the empirical works of [25, 26, 27, 28, 29, 30, 31, 32, 33] for more surveys.

From the above reviewed literature, it is interesting to know that GARCH family models have been found useful by many scholars from the world over including Nigeria in measuring exchange rate volatility. This study therefore extends the existing literature by combining symmetric GARCH models as well as asymmetric EGARCH and TGARCH models with normal and non-normal error distributions in measuring the exchange rate risk exposure in the Nigerian foreign exchange market using more current data.

2. Materials and Methods

2.1. Source of Data and Data Integration

The data used in this research work are the daily quotations of Naira/EURO, Naira/GBP, Naira/US Dollar and Naira/WAUA (West African Units of Account) exchange rates from 12/11/2001 to 07/04/2017 making a total of 3755 observations each. The daily returns r_t are calculated as the continuously compounded log returns corresponding to the first differences in logarithms of closing prices of successive days.

$$r_t = \log\left(\frac{R_t}{R_{t-1}}\right) \times 100 = [\log(R_t) - \log(R_{t-1})] \times 100 \quad (1)$$

where R_t denotes the closing market index at the current day (t) and R_{t-1} denotes the closing market index at the previous day $(t-1)$.

2.2. Methods of Data Analysis

The following statistical tools are employed in the analysis of data in this study.

2.2.1. The Phillips-Perron (PP) Unit Root Test

Phillips and Perron [34] propose an alternative non-parametric method of controlling for serial correlation when testing for a unit root. The PP method estimates the non-augmented DF test equation

$$\Delta Y_t = \alpha Y_{t-1} + X_t^{'}\delta + \varepsilon_t \quad (2)$$

and modifies the t −ratio of the α coefficient so that serial correlation does not affect the asymptotic distribution of the test statistic. The PP test is based on the statistic:

$$\tilde{t}_\alpha = t_\alpha \left(\frac{\varphi_0}{\phi_0}\right)^{1/2} - \frac{T(\phi_0 - \varphi_0)(se(\hat{\alpha}))}{2\phi_0^{1/2}s} \quad (3)$$

Where $\hat{\alpha}$ is the estimate of α, and t_α is the t −ratio for $\alpha, se(\hat{\alpha})$ is the standard error of $\hat{\alpha}$, and s is the standard error of the test regression, φ_0 is a consistent estimate of the error variance in equation (3) which is calculated as $(T-k)s^2/T$, where k is the number of regressors and ϕ_0 is an estimator of the residual spectrum at frequency zero.

2.2.2. Measuring Historical Volatility

Volatility is a measure of the spread of outcomes of asset returns. It is associated with the sample standard deviation of returns over a given period of time. It is computed using the following equation:

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_t - \bar{r})} \quad (4)$$

where $\bar{r}$ is the mean return defined by:

$$\bar{r} = \frac{1}{n} \quad (5)$$

Historical volatility is the *annualized* standard deviation of returns. The annualized volatility is given by

$$\sigma_{annualized} = \sqrt{252.\frac{1}{n}\sum_{t=1}^{n} r_t^2} \quad (6)$$

where r_t is the return of an asset over period $t, \bar{r}$ is the average return over t periods, 252 is the annual number of trading days and n is the rate of return over tth time interval.

2.3. Model Specification

2.3.1. Autoregressive Conditional Heteroskedasticity (ARCH) Model

The ARCH model was developed by [5]. For the log return series (r_t), the ARCH (p) model is specified as:

$$r_t = \mu + \varepsilon_t \quad (7)$$

$$(\varepsilon_t | \varepsilon_{t1}, \varepsilon_{t2}, \varepsilon_{t3}, \ldots) \sim N(0, h_t) \quad (8)$$

$$\varepsilon_t = \sqrt{h_t}u_t,\ u_t \sim N(0,1) \quad (9)$$

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 = \omega + \alpha(L)\varepsilon_{t-1}^2 \quad (10)$$

where ε_t is the innovation or shock at day t which follows heteroskedastic error process, r_t is the return series, μ is the conditional mean of (r_t), h_t is the volatility (conditional variance) at day t, L is the lag operator and ε_{t-i}^2 is the square innovation at day $t = 1 - i$. For an ARCH (p) process to be stationary, the sum of ARCH terms must be less than one (i.e., $\sum \alpha_i < 1$).

2.3.2. Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Model

The GARCH model was developed by [6]. Assuming a log return series $r_t = \mu + \varepsilon_t$ where ε_t is the error term at time t. The ε_t follows a GARCH (p, q) model if:

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \quad (11)$$

with constraints $\omega > 0, \alpha_i \geq 0, i = 1, 2, \ldots, q$ and $\beta_j \geq 0, j = 1,2, \ldots, p$; $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$ to ensure conditional variance to be positive as well as stationary. The basic GARCH (1,1) model is given by:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \quad (12)$$

The stationarity condition of a basic GARCH (1,1) is that the sum of ARCH and GARCH terms are strictly less than one (i.e., $\alpha_1 + \beta_1 < 1$). Rewriting equation (11) in terms of the lag operator (L), we have the following ARMA (p,q) representation:

$$h_t = \omega + \left(\sum_{i=1}^{p} \alpha_i L^i\right)\varepsilon_t^2 + \left(\sum_{j=1}^{q} \beta_j L^j\right)h_t \quad (13)$$

2.3.3. The Exponential GARCH (EGARCH) Model

The EGARCH model was proposed by [7] to allow for asymmetric effects between positive and negative asset returns. It can be expressed as:

$$\ln(h_t) = \omega + \sum_{i=1}^{p} \alpha_i \left\{\left|\frac{\varepsilon_{t-i}}{\sigma_{t-i}}\right|\right\} + \sum_{j=1}^{q} \beta_j \ln(h_{t-j}) + \sum_{k=1}^{r} \gamma_k \left[\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right] \quad (14)$$

Here γ represents the asymmetric coefficient in the model. Also the difference between α_i and γ_k can be expressed as impact of shocks on conditional volatility. Here β coefficient represents the measure of persistence. The conditional variance equation for EGARCH (1,1) model specification is given as:

$$\ln(h_t) = \omega + \alpha_1 \left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \beta_1 \ln(h_{t-1}) + \gamma \left[\frac{\varepsilon_{t-k}}{\sigma_{t-k}}\right] \quad (15)$$

2.3.4. Threshold GARCH (TGARCH) Model

Threshold GARCH (TGARCH) was introduced independently by [8 & 10]. The generalized specification of TGARCH for the conditional variance is given by:

$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} + \sum_{k=1}^{v} \gamma_k \varepsilon_{t-k}^2 \, \mathbb{I}_{t-k}^- \quad (16)$$

where $\mathbb{I}_t^- = 1$ if $\varepsilon_t < 0$ and 0 otherwise.

In this model, good news, $\varepsilon_{t-i} > 0$, and bad news, $\varepsilon_{t-i} < 0$, have differential effects on the conditional variance; good news has impact on α_i, while bad news has an impact of $\alpha_i + \gamma_i$. If $\gamma_i > 0$, bad news increases volatility, and we say that there is a leverage effect for the $i-th$ order. If $\gamma \neq 0$, the news impact is asymmetric.

The conditional variance equation for the TGARCH (1,1) model specification is given by:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \gamma \varepsilon_{t-1}^2 \mathbb{I}_{t-1}^- \quad (17)$$

2.4. Estimation and Distributional Assumptions of GARCH Models

We obtain the estimates of GARCH process by maximizing the log likelihood function:

$$Log(L\theta_t) = -\,{}^1\!/_2 \sum_{t-1}^{T}\left(\ln 2\pi + lnh_t + \frac{\varepsilon_t^2}{h_t}\right) \quad (18)$$

(i) Normal distribution is given by:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, -\infty < z < \infty \quad (19)$$

and the normal distribution to the log likelihood for observation t is:

$$l_t = \frac{-\frac{1}{2}\log(2\pi) - \frac{1}{2}\log h_t - \frac{1}{2}\left(y_t - X_t'\theta\right)^2}{h_t} \quad (20)$$

(ii) The student's $t-$ distribution (STD) is given by:

$$f(z) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(1 + \frac{z^2}{v}\right)^{-\left(\frac{v+1}{2}\right)}, -\infty < z < \infty \quad (21)$$

and the student's $t-$ distribution to the log-likelihood contributions is of the form:

$$l_t = \frac{1}{2}\log\left[\frac{\pi(v-2)\Gamma\left({}^{v}\!/_2\right)^2}{\Gamma\left({}^{(v+1)}\!/_2\right)}\right] - \frac{1}{2}\log h_t - \frac{(v+1)}{2}\log\left[1 + \frac{\left(y_t - X_t'\theta\right)^2}{h_t(v-2)}\right] \quad (22)$$

where the degree of freedom $v > 2$ controls the tail behaviour. The $t-$distribution approaches the normal distribution as $v \to \infty$.

(iii) The Generalized Error Distribution (GED) is given as:

$$f(z, \mu, \sigma, v) = \frac{\sigma^{-1} v e^{\left(-\frac{1}{2}\left|\frac{\left(\frac{z-\mu}{\sigma}\right)}{\lambda}\right|^{v}\right)}}{\lambda 2^{(1+(1/v))}\Gamma\left(\frac{1}{v}\right)}, 1 < z < \infty \quad (23)$$

$v > 0$ is the degrees of freedom or tail -thickness parameter and $\lambda = \sqrt{2^{(-2/v)}\Gamma\left(\frac{1}{v}\right)/\Gamma\left(\frac{3}{v}\right)}$

and the GED distribution to the log-likelihood contributions is given by:

$$l_t = -\frac{1}{2}\log\left[\frac{\Gamma\left({}^{1}\!/_v\right)^3}{\Gamma\left({}^{3}\!/_v\right)\left({}^{v}\!/_2\right)^2}\right] - \frac{1}{2}\log h_t - \left[\frac{\Gamma\left({}^{3}\!/_v\right)\left(y_t - X_t'\theta\right)^2}{h_t \Gamma\left({}^{1}\!/_v\right)}\right]^{\frac{v}{2}} \quad (24)$$

The GED is a normal distribution if $v = 2$, and fat-tailed if $v < 2$.

3. Results and Discussion

3.1. Descriptive Statistics of Exchange Rate Returns

We employ the descriptive statistics in order to explore the distributional characteristics of the exchange rate log return series. The results are summarized in Table 1.

Table 1: Descriptive Statistics of Exchange Rate Log Returns

Statistic	EUR	GBP	USD	WAUA
Mean	0.031684	0.011265	0.018026	0.015015
Maximum	852.1012	921.1516	230.8997	851.5490
Minimum	-851.5205	-921.6347	-231.0241	-852.2692
Std. Dev.	19.74169	29.01413	5.683067	32.94952
Skewness	0.040929	0.005849	-0.017126	-0.030060
Kurtosis	1847.135	938.3340	1463.109	586.4039
Jarque-Bera	5.32E+08	1.37E+08	3.34E+08	53252167
P-value	0.000000	0.000000	0.000000	0.000000
N	3755	3755	3755	3755

From the summary statistics results presented in Table 1, it is observed that the means of the four exchange rate log returns are all very close to zero with high standard deviations indicating high level of dispersion from the average log returns in the exchange market over the period under review. The wide gap between the maximum log returns and minimum log returns supports the high level of variability of exchange rates fluctuations over the period under review. The USD and WAUA log returns exhibit negative skewness while the EUR and GBP log returns exhibit positive skewness which are all very close to zero. The positive or negative skewness indicate asymmetry in the log return series. All the four exchange rate log returns have high kurtosis values. The high values of kurtosis coefficients which deviate from normal suggest that big shocks of either signs are more likely to be present in the series and that the log returns series are clearly leptokurtic. The Jarque -Bera test which is performed at 1 percent significance level rejects the null hypothesis of zero skewness and kurtosis value of 3 in all cases. This confirms departure from normality.

3.2. Phillips – Perron Unit Root Test Results

To investigate the unit root and stationarity properties of the log return series, we employ Phillips – Perron parametric unit root test. The result is presented in Table 2.

Table 2: Phillips & Perron Parametric Unit Root Test Results for the Log Returns

	Option	PP Test Statistic	P-value	Critical values	
				1%	5%
EUR	Intercept only	-61.2835	0.0001	-3.4319	-2.8621
	Intercept & trend	-885.769	0.0001	-3.9605	-3.4110
GBP	Intercept only	-894.4631	0.0001	-3.4319	-2.8621
	Intercept & trend	-890.8340	0.0001	-3.9605	-3.4110
USD	Intercept only	-292.1670	0.0001	-3.4319	-2.8621
	Intercept & trend	-310.9658	0.0001	-3.9605	-3.4110
WAUA	Intercept only	-1288.929	0.0000	-3.4319	-2.8621
	Intercept & trend	-1446.621	0.0000	-3.9605	-3.4110

The Phillips – Perron unit root test result presented in Table 2 reveals that all the four exchange rate log returns series are stationary. This is indicated by their PP test statistics being smaller than their corresponding critical values at 1% and 5% significance levels with p-values of the test statistics less than $\alpha = 0.05$. The stationarity of the returns series means that no unit root is presence in the log returns.

3.3. Engle's LM Heteroskedasticity Test for ARCH Effects

To test for heteroskedasticity, the presence of ARCH effects in the residuals of the log return series of the foreign exchange rates, we obtain an optimal autoregressive moving average ARMA (1,1) model for the conditional mean in the return series as initial regressions in all cases, we then test the null hypothesis of no ARCH effects in the residual series up to lag 31 which corresponds to one trading month using Engle's LM ARCH test. The result of the ARCH-LM test is presented in Table 3.

Table 3: Heteroskedasticity Test for ARCH Effects

Log Returns	Lag	F-statistic	P-value	nR^2	P-value
EUR	1	314.0075	0.0000	289.9061	0.0000
	31	14.8498	0.0000	412.8433	0.0000
GBP	1	312.3404	0.0000	288.4852	0.0000
	31	14.7024	0.0000	409.1959	0.0000
USD	1	341.8363	0.0000	313.4529	0.0000
	31	14.9752	0.0000	415.9418	0.0000
WAUA	1	312.3473	0.0000	288.4911	0.0000
	31	14.7892	0.0000	411.2964	0.0000

From the ARCH – LM test result presented in Table 3, both the F – statistics and nR^2 statistics in the entire four log returns series reject the null hypothesis of homoskedasticity. When the null hypothesis is rejected, it indicates the presence of ARCH effects in the residuals of returns. This means that the daily exchange rate log returns are non-constant and can only be modeled using Autoregressive Conditional Heteroskedastic Models.

3.4. Historical Volatility Results

The results of volatility as measured by daily variance, daily standard deviation and annualized volatility for the four exchange rate log returns series are presented in Table 4.

Table 4: Volatility Rankings as Measured by Standard Deviation and Annualized Volatility

Rank	FXR	Daily Variance	Daily Standard Deviation	$\sigma_{annualized}$
1	WAUA	1085.6709	32.94952	523.0574
2	GBP	841.8197	29.01413	460.5850
3	EUR	389.7343	19.74169	313.3896
4	USD	32.2973	5.683067	90.2159

The result of Table 4 shows the historical rankings of volatility for the four foreign exchange rate log returns in Nigeria as measured by daily variance, daily standard deviation and annualized volatility methods. The result shows that WAUA with daily standard deviation of 32.94952% and annualized volatility of 523.0574 ranked first as having the highest volatility among the four foreign exchange rates in Nigeria under review. Next to WAUA is GBP with daily standard of 29.01413% and annualized volatility of 460.5850 and is ranked second. The third volatile foreign exchange rate return series is the EUR with a daily standard deviation of 19.74169% and annualized volatility of 313.3896. The USD has the least daily variance measure, standard deviation and annualized volatility among the four exchange rate log returns under review. Historical volatility is used as a criterion to study the risk associated with a financial asset. It is widely accepted as a practical measure of risk. Stocks with a high historical volatility are usually associated with a higher level of risk tolerance. From the result of historical volatility presented in Table 4, the USD is the only foreign exchange rate log return associated with less level of risk. The historical volatility is reported graphically in Figure 1 using the daily standard deviation measure.

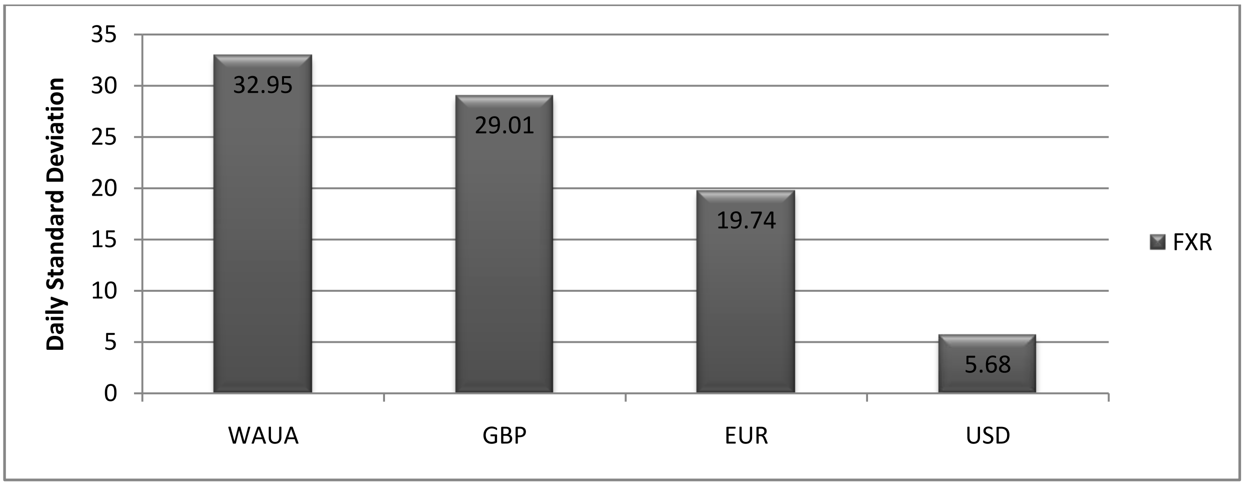

Figure 1: Bar Graph of Historical Volatilities of Foreign Exchange Rate Returns in Nigeria

3.5. Symmetric and Asymmetric GARCH Models Order Selection

To select a model that best fit a given log return series, we employ log likelihoods and information criteria such as Akaike information criterion (AIC) due to [35], Schwarz information criterion (SIC) due to [36] and Hannan Quinn criterion (HQC) due to [37] with different error distributions. The best fitted model is given by the highest log likelihoods and smallest information criteria. Only the selected results are presented in Table 5.

Table 5: Symmetric and Asymmetric GARCH Models Order Selection

Return	Model	Distribution	LogL	AIC	SIC	HQC
EUR	GARCH (1,1)	ND	-15545	8.2817	8.2884	8.2841
	EGARCH(1,1)	STD	-3887	2.0737	2.0837	2.0772
	TGARCH(1,1)	ND	-16136	8.5972	8.6055	8.6002
GBP	GARCH (1,1)	STD	-15233	8.1160	8.1243	8.1190
	EGARCH(1,1)	STD	-3708	1.9781	1.9880	1.9816
	TGARCH(1,1)	ND	-17576	9.3640	9.3720	9.3670
USD	GARCH (1,1)	STD	-6502	3.4657	3.4740	3.4687
	EGARCH(1,1)	STD	12474	-6.6405	-6.6306	-6.6370
	TGARCH(1,1)	STD	-5583	2.9771	2.9870	2.9806
WAUA	GARCH (1,1)	STD	-15828	8.4329	8.4412	8.4358
	EGARCH(1,1)	GED	1558	-0.8265	-0.8165	-0.8229
	TGARCH(1,1)	STD	-15772	8.4039	8.4139	8.4075

The symmetric and asymmetric GARCH models presented in Table 5 depicts the selected models with innovation densities for each exchange rate log return series.

3.5.1. Symmetric GARCH Models Results

To investigate the symmetric properties of the log return series we employ the basic GARCH (1,1) model with the selected error distributions. The results are presented in Table 6.

Table 6: Parameter Estimates of Basic GARCH (1,1) Models

	EUR	GBP	USD	WAUA
Distribution	ND	STD	STD	STD
μ	0.4897*	-1.2849	0.0272*	-0.0731
ω	0.2461*	0.6039*	0.5291*	0.8147*
α_1	0.1447*	0.1426*	0.1319*	0.1761*
β_1	0.5186*	0.5015*	0.7518*	0.3637*
λ	0.6633	0.6441	0.8837	0.5398
Shape v	---	10.1924*	11.4378*	9.8719*
ARCH Test	0.037734	0.083444	0.000868	0.147326
P-value	0.8460	0.7727	0.9765	0.7011

Note: $\lambda = \alpha_1 + \beta_1$ measures shock persistence in volatility; v = shape parameter. * denotes the significant of the parameter at 1% marginal significance levels.

The parameter estimates of the symmetric GARCH (1,1) models shows that the coefficients of the ARCH terms (α_1) in all the four models are positive and statistically significant at 1% levels showing that news about previous volatilities have explanatory powers on current volatilities. The coefficients of GARCH terms (β_1) are also positive and statistically significant at 1% levels, showing that past volatilities of stock market returns are significant and influence current volatilities. The sums of ARCH and GARCH coefficients are $\alpha_1 + \beta_1 = 0.6633 < 1$ for EURO, $\alpha_1 + \beta_1 = 0.6441 < 1$ for GBP, $\alpha_1 + \beta_1 = 0.8837 < 1$ for US Dollar and $\alpha_1 + \beta_1 = 0.5398 < 1$ for West African Unit of Account (WAUA) which are measures of volatility persistence. Since the sum of ARCH and GARCH coefficients are less than one in all the four GARCH (1,1) models, it means that the models are stable and the conditional variances are stationary. This also implies that the volatilities are significantly quite persistence in Nigerian foreign exchange market. The results of the GARCH (1,1) models thus indicates that memories of shocks are remembered in Nigerian foreign Exchange Market.

3.5.2. Test for Asymmetry

To test for asymmetric effects on the conditional volatility in the four foreign exchange rates log return series, we employ the sample correlation between squared returns and lagged returns $\mathrm{Corr}(r_t^2, r_{t-1})$. According to [38], when the correlation coefficient is negative, then there is evidence for potential asymmetry and leverage effects in the log returns. The sample correlation results for the four time series are presented in Table 7.

Table 7: Asymmetric Test on Conditional Volatility for the four Exchange Rate Log Returns

Log Return Series	$Corr(r_t^2, r_{t-1})$
EUR	-0.04783
GBP	-0.06592
USD	-0.08539
WAUA	-0.03618

From the sample correlation results reported in Table 7, we observe that the correlations between squared returns (r_t^2) and lagged returns (r_{t-1}) have small negative values in all cases, indicating weak evidence for asymmetry. Asymmetric GARCH models can therefore perform better in explaining conditional volatility for the four foreign exchange rate log return series.

3.5.3. Asymmetric GARCH Models Results

To further investigate some stylized facts of financial returns such as asymmetry and leverage effects, we employ the popular Exponential GARCH (EGARCH) and the Threshold GARCH (TGARCH) models. The parameter estimates of the EGARCH (1,1) models for the four foreign exchange rate log returns are presented in Table 8 while the parameter estimates of the TGARCH (1,1) models are reported in Table 9.

Table 8: Estimates of EGARCH (1,1) with Varying Innovation Densities

	EUR*	GBP*	USD*	WAUA*
Distribution	STD	STD	STD	GED
μ	-0.0000	0.0063	0.0011	0.1103
ω	-0.2959	-0.2218	2.1002	0.1727
α_1	0.0005	0.1366	0.1256	0.4099
β_1	0.7888	0.4932	0.7272	0.1584
γ	-0.0006	-0.1208	-0.0147	-2.2616
λ	0.7893	0.6298	0.8528	0.5683
v	0.3100	2.5481	1.4422	11.8126
ARCH Test	0.001179	0.001068	0.006419	0.000118
P-value	0.8926	0.9739	0.6112	0.9913

Note: $\lambda = \alpha_1 + \beta_1$; $v =$ shape parameter. *means all parameters in the variance equation are significance at 1% level.

Table 9: Estimates of TGARCH (1,1) with Varying Innovation Densities

	EUR*	GBP*	USD*	WAUA*
Distribution	ND	GED	STD	STD
μ	0.6048	0.9404	0.0085	0.0441
ω	0.2521	0.8356	3.2542	0.7629
α_1	0.1503	0.1391	0.8831	0.1715
β_1	0.5978	0.5909	0.0004	0.4798
γ	-0.1928	-0.2232	-0.0711	-0.0517
λ	0.7481	0.7300	0.8835	0.6513
v	----	1.9914	10.2743	11.5439
ARCH Test	0.179418	1.188874	0.000547	0.116391
P-value	0.6719	0.2756	0.9813	0.7330

Note: $\lambda = \alpha_1 + \beta_1$; $v =$ shape parameter. *means all parameters in the variance equation are significance at 1% level.

The asymmetric effect parameters γ captured by EGARCH (1,1) and TGARCH (1,1) models are both negative and significant as expected thereby providing supportive evidence for the existence of asymmetric and leverage effects in the returns during the study period. This implies that previous period's positive and negative shocks have different effects on the conditional variance. This also means that there is a tendency for changes in stock prices to be negatively correlated with changes in volatility. This result is consistent with the findings of [39 & 40] that also found asymmetry and leverage effects in Nigerian Stock Market.

The results of the ARCH LM tests reported in the lower panels of Tables 6, 8 and 9 of the estimated GARCH (1,1), EGARCH (1,1) and TGARCH (1,1) models showed that the GARCH family models have captured all the ARCH effects in the residuals of the exchange rate log return series. This is indicated by the p-values of the ARCH LM test statistics which are highly statistically insignificant. This shows that our estimated GARCH-type models are good, adequate, valid and accurate in describing the volatility situation in Nigerian Foreign Exchange Market.

3.6. Mean Reversion and Half-Life of Volatility

To test whether the log returns volatilities of the EURO, GBP, USD and WAUA foreign exchange rates finally revert to their long-run averages, we apply the volatility mean reversion rates of the estimated models given by the sum of ARCH and GARCH coefficients $(\alpha_1 + \beta_1)$. We also compute the volatility half-life for the four exchange rate return series and the results are reported in Table 10.

Table 10: Mean Reversion and Volatility Half-Life of Log Return Series

Log Returns	GARCH (1,1)		EGARCH (1,1)		TGARCH (1,1)	
	λ	L_{half}	λ	L_{half}	λ	L_{half}
EUR	0.6633	2	0.7893	3	0.7481	2
GBP	0.6441	2	0.6298	2	0.7300	2
USD	0.8837	6	0.8528	5	0.8835	6
WAUA	0.5398	1	0.5683	1	0.6513	2

Note: $\lambda = \alpha_1 + \beta_1$ measures mean reversion; L_{half} = Volatility half-life measured in days.

Results of the stationary GARCH – type models indicates that volatility mean reversion rate $(\lambda = \alpha_1 + \beta_1)$ which is less than one for all the four foreign exchange rates log returns series is satisfied. The result of half life of volatility shock estimated by $L_{half} = \ln\left[\frac{0.5}{\alpha_1 + \beta_1}\right]$ to measure the average number of time periods it takes the volatility to revert to its long run average is presented. When the value of $\alpha_1 + \beta_1$ is close to 1, the half life of a volatility shock is longer. If $(\alpha_1 + \beta_1) > 1$, the GARCH model is said to be non-stationary and the volatility eventually explodes to infinity. Results of estimated GARCH (1,1) model shows volatility half lives of 2 days for EURO and GBP, 6 days for USD and 1 day for WAUA. EGARCH (1,1) model indicates volatility half-lives of 3, 2, 5 and 1 days for EURO, GBP, USD and WAUA respectively while TGARCH (1,1) model shows volatility half-lives of 2 days for EURO, GBP and WAUA and 6 days for USD. Thus, the log return series of the foreign exchange rates under review are stationary and mean reverting. As policy implication for investors, stationary and mean reverting returns are good opportunities for long term investment.

To investigate whether differences exist between the best fitting and the best forecast performance volatility models, we use Akaike information criterion (AIC) and Root Mean Square Error (RMSE). The best fitting model and the best forecast performance model are given by the smallest values of AIC and RMSE. Results are presented in Table 11.

Table 11: Model Order Selection for Best Fitting and Best Forecast Performance

Log Returns	Density	GARCH (1,1)		EGARCH (1,1)		TGARCH (1,1)	
		AIC	RMSE	AIC	RMSE	AIC	RMSE
EUR	ND	8.2817	19.7444	4.5998	19.7292	**8.5972**	**19.7464**
	STD	7.5275	20.6620	**2.0737**	**19.7291**	8.9152	19.7482
	GED	**6.9584**	**19.7391**	2.3221	19.7391	8.9041	19.7474
GBP	ND	9.0669	29.0194	8.7933	29.0630	**9.3640**	**29.0252**
	STD	8.1160	29.0392	**1.9781**	**29.0103**	9.8462	29.0258
	GED	**2.2143**	**29.0103**	2.2898	29.0104	9.6643	29.0255
USD	ND	5.9920	5.6866	5.1895	5.6823	5.9094	5.6841
	STD	5.4657	5.6823	**-6.6405**	**5.6823**	**2.9771**	**5.6823**
	GED	**4.1753**	**5.6823**	3.8267	5.6823	4.3854	5.6823
WAUA	ND	9.5592	32.9453	9.1580	32.9464	9.5699	32.9499
	STD	8.4329	32.9453	0.3860	32.9452	8.4039	32.9452
	GED	**-0.2220**	**32.9452**	**-0.8265**	**32.9452**	**-0.2305**	**32.9452**

The results comparison of the best fitting GARCH model and the best volatility forecast performance model presented in Table 11 shows that symmetric GARCH (1,1) models with GED distributions produces the best volatility forecast performance for all the four exchange rate return series, asymmetric EGARCH (1,1) models with STD distributions produces the best volatility forecast performance for EURO, GBP and USD exchange rate return series, EGARCH (1,1) model with GED distribution produces the best volatility forecast performance for the WAUA exchange rate return series whereas asymmetric TGARCH (1,1) models with ND, STD and GED distributions produces the best volatility forecast performance for EURO, GBP; USD and WAUA exchange rate return series respectively. These results clearly indicate that the best fitted models are not necessarily the best volatility forecast performance models. This result collaborate the findings of [41 & 42].

4. Conclusion

This study has attempted to model the exchange rate volatility of Naira against Euro, Great British Pounds, US Dollar and West African Unit of Account (WAUA) in Nigeria using symmetric and asymmetric Autoregressive Conditional Heteroskedasticity (GARCH) models in the presence of Gaussian and non-Gaussian errors. The study utilizes daily quotations of these exchange rates from 12/11/2001 to 07/04/2017 making a total of 3755 observations each. Symmetric GARCH as well as asymmetric EGARCH and TGARCH specifications were used to model the exchange rates log return series. Results shows that basic GARCH (1,1) model with Gaussian error was the best fitted symmetric model for Naira/EUR exchange rate while basic GARCH (1,1) with student-t innovation was the best fitted symmetric model for Naira/GBP, Naira/USD and Naira/WAUA exchange rates. The asymmetric EGARCH (1,1) with student-t distribution was found to fit Naira/EUR, Naira/GBP and Naira/USD while EGARCH (1,1) with GED was found to fit Naira/WAUA exchange rate return series. The asymmetric TGARCH (1,1) with normal and GED distribution fitted Naira/EUR and Naira/GBP exchange rate respectively while TGARCH (1,1) model with student-t innovation fitted the Naira/USD and Naira/WAUA exchange rate returns. All the estimated models were found to be stable, stationary and mean reverting indicating that exchange rate volatility is predictable in Nigerian foreign exchange market. The asymmetric EGARCH (1,1) and TGARCH (1,1) models show supportive evidence for the existence of asymmetry and leverage effects suggesting that negative shocks produces more volatility in Nigerian foreign exchange market than positive shocks of the same magnitude. Results also show that the best fitted models were not necessarily the best models selected for forecasting exchange rate volatility in Nigeria.

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