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On the Coefficient and Fekete-Szegö Problem of the Pseudo-Starlike and Pseudo-Convex Univalent Function Class

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Abstract: In this paper, we defined a new subclass of starlike and convex univalent functions and examine some geometric properties this function class. For this definition class, we gave some coefficient upper bound estimates and solve Fekete-Sezöge problem.

Keywords: Starlike function, convex function, univalent function, pseudo-starlike function, pseudo-convex function

1. Introduction

We will denote by $H(U)$ the class of analytic functions in the open unit disk $U=\{\zeta\in\mathbb{C}:|z|<1\}$ of the complex plane $\mathbb{C}$. Let A be the class of the functions $f\in H(U)$ given by series expansions

$$f(\zeta)=\zeta+a_2\zeta^2+a_3\zeta^3+a_4\zeta^4+\cdots+a_n\zeta^n+\cdots=\zeta+\sum_{n=2}^{\infty}a_n\zeta^n,\ a_n\in\mathbb{C}\ . \tag{1.1}$$

The subclass of A, which are univalent functions in U is denoted by S in the literature. The class S was introduced by Köebe [1] first time and has become the core ingredient of advanced research in this field. After a short time, in 1916 Bieberbach [2] published a paper in which the coefficient hypothesis was proposed. This hypothesis states that if $f\in S$ and has the series form (1.1), then $|a_n|\le n$ for each $n\ge 2$. There are many articles in the literature regarding to this hypothesis (see [3-14]).

It is well known that the starlike and convex function classes in the open unit disk U are defined analytically as follows

$$S^*=\left\{f\in S:\ \operatorname{Re}\left(\frac{\zeta f'(\zeta)}{f(\zeta)}\right)>0,\ \zeta\in U\right\},$$

$$C=\left\{f\in S:\ \operatorname{Re}\left(\frac{(\zeta f'(\zeta))'}{f'(\zeta)}\right)>0,\ \zeta\in U\right\}$$

and denoted by S^* and C, respectively.

Let's $f,g\in H(U)$, then it is said that f is subordinate to g and denoted by $f\prec g$, if there exists a Schwartz function ω, such that $f(\zeta)=g(\omega(\zeta))$.

In the past few years, numerous subclasses of the class S have been introduced as special choices of the class S^* and C (see for example [3, 8-21]).

2. Materials and Methods

Now, let's we define new subclass of univalent functions in the open unit disk U.

Definition 2.1. For $\beta\in[0,1]$, $\lambda>\frac{1}{2}$ and $\tau\in\mathbb{C}-\{0\}$ the function $f\in S$ is said to be in the class $\chi(\tau,\lambda,\beta;e^{\zeta})$, if the following condition is satisfied

$$(1-\beta)\left\{1+\frac{1}{\tau}\left[\frac{\zeta(f'(\zeta))^{\lambda}}{f(z\zeta)}-1\right]\right\}+\beta\left\{1+\frac{1}{\tau}\left[\frac{\left[\zeta(f'(\zeta))'\right]^{\lambda}}{f'(\zeta)}-1\right]\right\}\prec\exp(\zeta),\ \zeta\in U.$$

In the cases $\beta=0$, $\beta=1$ and $\lambda=1$ from the Definition 2.1, we have the following classes of bi-univalent functions.

Definition 2.2. For $\lambda>\frac{1}{2}$ the function $f\in S$ is said to be in the class $S^*(\tau,\lambda;e^{\zeta})$, if the following conditions are satisfied

$$1+\frac{1}{\tau}\left[\frac{\zeta(f'(\zeta))^{\lambda}}{f(z\zeta)}-1\right]\prec\exp(\zeta),\ \zeta\in U.$$

Definition 1.3. For $\lambda>\frac{1}{2}$ the function $f\in S$ is said to be in the class $C(\tau,\lambda;e^{\zeta})$, if the following conditions are satisfied

$$1+\frac{1}{\tau}\left[\frac{\left[\zeta(f'(\zeta))'\right]^{\lambda}}{f'(\zeta)}-1\right]\prec\exp(\zeta),\ \zeta\in U.$$

Definition 1.4. For $\beta\in[0,1]$ the function $f\in S$ is said to be in the class $\chi(\tau,\beta;e^{\zeta})$, if the following conditions are satisfied

$$(1-\beta)\left\{1+\frac{1}{\tau}\left[\frac{\zeta(f'(\zeta))}{f(z\zeta)}-1\right]\right\}+\beta\left\{1+\frac{1}{\tau}\left[\frac{\zeta(f'(\zeta))'}{f'(\zeta)}-1\right]\right\}\prec\exp(\zeta),\ \zeta\in U.$$

Let P be the class of analytic functions in U satisfied the conditions $p(0)=1$ and $\mathrm{Re}(p(\zeta))>0,\ z\in U.$ It is clear that the functions that satisfy these conditions have the following series expansion

$$p(\zeta)=1+p_1\zeta+p_2\zeta^2+p_3\zeta^3+\cdots=1+\sum_{n=1}^{\infty}p_n\zeta^n,\ \zeta\in U. \tag{2.1}$$

The class P defined above is known as the class Caratheodory functions in the literature [22].
Now, let us give some necessary lemmas for the proof of our main results.

Lemma 2.1 ([23]). Let the function p belong to the class P. Then,

$$|p_n|\le 2 \text{ for each } n\in\mathbb{N},\ |p_n-\nu p_k p_{n-k}|\le 2 \text{ for } n,\ k\in\mathbb{N},\ n>k \text{ and } \nu\in[0,1].$$

The equalities hold for the function

$$p(z)=\frac{1+\zeta}{1-\zeta}.$$

Lemma 2.2 ([23]) Let the an analytic function p be of the form (2.1), then

$$2p_2=p_1^2+\left(4-p_1^2\right)x,$$

$$4p_3=p_1^3+2\left(4-p_1^2\right)p_1x-\left(4-p_1^2\right)p_1x^2+2\left(4-p_1^2\right)\left(1-|x|^2\right)y$$

for some $x,y\in\mathbb{C}$ with $|x|\le 1$ and $|y|\le 1$.

In this paper, we give some coefficient estimates and solve Fekete-Szegö problem for the class $\chi\left(\tau,\lambda,\beta;e^{\zeta}\right)$. Additionally, the results obtained for specific values of the parameters in our study are compared with the results obtained in the literature.

3. Results and Discussion

In this section, we give some coefficient estimates for the functions belonging to the class $\chi\left(\tau,\lambda,\beta;e^{\zeta}\right)$ and solve Fekete-Szegö problem for this class.

Theorem 3.1. Let the function f given by series expansions (1.1) belong to the class $\chi\left(\tau,\lambda,\beta;e^{\zeta}\right)$. Then, we have the following inequalities

$$|a_2|\le\frac{|\tau|}{(2\lambda-1)(1+\beta)} \text{ and}$$

$$|a_3|\le\frac{|\tau|}{(3\lambda-1)(1+2\beta)}\begin{cases}1 & if\ 4a(\tau,\lambda,\beta)\le 1,\\ 4a(\tau,\lambda,\beta) & if\ 4a(\tau,\lambda,\beta)\ge 1,\end{cases} \tag{3.1}$$

Where,

$$a(\tau,\lambda,\beta)=\frac{\left|2\left(2\lambda^2-4\lambda+1\right)(1+3\beta)\tau-(2\lambda-1)^2(1+\beta)^2\right|}{8(2\lambda-1)^2(1+\beta)^2}.$$

Proof. Let $f\in\chi\left(\tau,\lambda,\beta;e^{\zeta}\right)$, then exists Schwartz function $\omega:U\to U$, such that

$$(1-\beta)\left\{1+\frac{1}{\tau}\left[\frac{\zeta\left(f'(\zeta)\right)^{\lambda}}{f(\zeta)}-1\right]\right\}+\beta\left\{1+\frac{1}{\tau}\left[\frac{\left[\left(\zeta f'(\zeta)\right)'\right]^{\lambda}}{f'(\zeta)}-1\right]\right\}=\exp\left(\omega(\zeta)\right),\ \zeta\in U. \tag{3.2}$$

Let's the function $p\in P$ defined as follows:

$$p(\zeta)=\frac{1+\omega(\zeta)}{1-\omega(\zeta)}=1+p_1\zeta+p_2\zeta^2+p_3\zeta^3+\ldots=1+\sum_{n=1}^{\infty}p_n\zeta^n,\ \zeta\in U \tag{3.3}$$

It follows from that

$$\omega(\zeta)=\frac{p(\zeta)-1}{p(\zeta)+1}=\frac{p_1}{2}\zeta+\frac{1}{2}\left(p_2-\frac{p_1^2}{2}\right)\zeta^2+\frac{1}{2}\left(p_3-p_1p_2-\frac{p_1^3}{4}\right)\zeta^3+\ldots,\ \zeta\in U. \tag{3.4}$$

From the (3.2) and (3.4) can written

$$(1-\beta)\left\{1+\frac{1}{\tau}\left[a_2(2\lambda-1)\zeta+\left((3\lambda-1)a_3+\left(2\lambda^2-4\lambda+1\right)a_2^2\right)\zeta^2+\cdots\right]\right\}$$
$$+\beta\left\{1+\frac{1}{\tau}\left[2a_2(2\lambda-1)\zeta+\left(3(3\lambda-1)a_3+4\left(2\lambda^2-4\lambda+1\right)a_2^2\right)\zeta^2+\cdots\right]\right\} \tag{3.5}$$
$$=1+\frac{p_1}{2}\zeta+\frac{1}{2}\left(p_2-\frac{p_1^2}{4}\right)\zeta^2+\cdots,\ \zeta\in U.$$

By equate the coefficients, we obtain the following equalities

$$(2\lambda-1)(1+\beta)a_2=\frac{\tau}{2}p_1, \tag{3.6}$$

$$(3\lambda-1)(1+2\beta)a_3+\left(2\lambda^2-4\lambda+1\right)(1+3\beta)a_2^2=\frac{\tau}{2}\left(p_2-\frac{p_1^2}{4}\right). \tag{3.7}$$

Then, from the equality (3.6), we have

$$a_2=\frac{\tau}{2(2\lambda-1)(1+\beta)}p_1\ . \tag{3.8}$$

Using Lemma 2.1, from the equality (3.8), we have first result of theorem.

Now let's find an upper bound estimate for the coefficient a_3. For the coefficient a_3 from the equalities (3.7) and (3.8) we can write the following expression

$$a_3=\frac{\tau}{(3\lambda-1)(1+2\beta)}\left[\frac{p_2}{2}-\frac{2\left(2\lambda^2-4\lambda+1\right)(1+3\beta)\tau+(2\lambda-1)^2(1+\beta)^2}{8(2\lambda-1)^2(1+\beta)^2}p_1^2\right]. \tag{3.9}$$

Applying Lemma 2.2, equality (3.9) can written as follows

$$a_3=\frac{\tau}{(3\lambda-1)(1+2\beta)}\left[\frac{4-p_1^2}{4}x-\frac{2\left(2\lambda^2-4\lambda+1\right)(1+3\beta)\tau-(2\lambda-1)^2(1+\beta)^2}{8(2\lambda-1)^2(1+\beta)^2}p_1^2\right]$$

for some $x\in\mathbb{C}$ with $|x|\le 1$. Then, applying triangle inequality, we have

$$|a_3|\le\frac{|\tau|}{(3\lambda-1)(1+2\beta)}\left[a(\tau,\lambda,\beta)t^2+\frac{4-t^2}{4}\xi\right]$$

Where, $\xi=|x|$, $t=|p_1|$ and

$$a(\tau,\lambda,\beta)=\frac{\left|2\left(2\lambda^2-4\lambda+1\right)(1+3\beta)\tau-(2\lambda-1)^2(1+\beta)^2\right|}{8(2\lambda-1)^2(1+\beta)^2}.$$

Then, by maximizing the function φ defied as follows, with respect to variables ξ and then t

$$\varphi(\xi,t)=a(\tau,\lambda,\beta)t^2+\frac{4-t^2}{4}\xi,\ \xi\in[0,1],t\in[0,2]$$

it can easily be seen that $\varphi(\xi,t)\le 1$ if $4a(\tau,\lambda,\beta)\le 1$ and $\varphi(\xi,t)\le 4a(\tau,\lambda,\beta)$ if $4a(\tau,\lambda,\beta)\ge 1$.

With this, the proof of second inequality of (3.1) is provided.

Thus, the proof of the theorem is completed.

In the case $\beta=0$, $\beta=1$ and $\lambda=1$ from the Theorem 3.1, we obtain the following results, respectively.

Corollary 3.1. If $f\in S^*\left(\tau,\lambda;e^{\zeta}\right)$, then

$$|a_2| \le \frac{|\tau|}{2\lambda-1} \text{ and } |a_3| \le \frac{|\tau|}{3\lambda-1}\begin{cases} 1 & if \;\; 4a(\tau,\lambda)\le 1, \\ 4a(\tau,\lambda) & if \;\; 4a(\tau,\lambda)\ge 1, \end{cases}$$

Where,

$$a(\tau,\lambda) = \frac{\left|2\left(2\lambda^2-4\lambda+1\right)\tau-\left(2\lambda-1\right)^2\right|}{8\left(2\lambda-1\right)^2}.$$

Corollary 3.2. If $f \in C\left(\tau,\lambda;e^{\zeta}\right)$, then

$$|a_2| \le \frac{|\tau|}{2(2\lambda-1)} \text{ and } |a_3| \le \frac{|\tau|}{3(3\lambda-1)}\begin{cases} 1 & if \;\; 4b(\tau,\lambda)\le 1, \\ 4b(\tau,\lambda) & if \;\; 4b(\tau,\lambda)\ge 1, \end{cases}$$

Where,

$$b(\tau,\lambda) = \frac{\left|2\left(2\lambda^2-4\lambda+1\right)\tau-\left(2\lambda-1\right)^2\right|}{4\left(2\lambda-1\right)^2}.$$

Corollary 3.3. If $f \in \chi\left(\tau,\beta;e^{\zeta}\right)$, then

$$|a_2| \le \frac{|\tau|}{1+\beta} \text{ and } |a_3| \le \frac{|\tau|}{2(1+2\beta)}\begin{cases} 1 & if \;\; 4c(\tau,\beta)\le 1, \\ 4c(\tau,\beta) & if \;\; 4c(\tau,\beta)\ge 1, \end{cases}$$

Where,

$$c(\tau,\beta) = \frac{\left|2\left(1+3\beta\right)\tau+\left(1+\beta\right)^2\right|}{8\left(1+\beta\right)^2}.$$

Now, we give the following theorem on the Fekete-Szegö problem for the class $\chi\left(\tau,\lambda,\beta;e^{\zeta}\right)$.

Theorem 3.2. Let $f \in \chi\left(\tau,\lambda,\beta;e^{\zeta}\right)$ and $\mu \in \mathbb{C}$, then

$$\left|a_3-\mu a_2^2\right| \le \frac{|\tau|}{2(3\lambda-1)(1+2\beta)}\begin{cases} 2 & if \;\; l(\tau,\lambda,\beta,\mu)\le 2, \\ l(\tau,\lambda,\beta,\mu) & if \;\; l(\tau,\lambda,\beta,\mu)\ge 2, \end{cases} \tag{3.10}$$

Where,

$$l(\tau,\lambda,\beta,\mu) = \frac{\left|2\left[\left(2\lambda^2-4\lambda+1\right)\left(1+3\beta\right)+\left(3\lambda-1\right)\left(1+2\beta\right)\mu\right]\tau-\left(2\lambda-1\right)^2\left(1+\beta\right)^2\right|}{\left(2\lambda-1\right)^2\left(1+\beta\right)^2}.$$

Proof. Let $f \in \chi\left(\tau,\lambda,\beta;e^{\zeta}\right)$. Then, applying Lemma 2.2, from the equalities (3.8) and (3.9) the expression $a_3-\mu a_2^2$ can written as follows

$$a_3-\mu a_2^2 = \frac{\tau}{8(3\lambda-1)(1+2\beta)} \times\left\{2\left(4-p_1^2\right)x-\frac{2\left[\left(2\lambda^2-4\lambda+1\right)\left(1+3\beta\right)+\left(3\lambda-1\right)\left(1+2\beta\right)\mu\right]\tau-\left(2\lambda-1\right)^2\left(1+\beta\right)^2}{\left(2\lambda-1\right)^2\left(1+\beta\right)^2}p_1^2\right\} \tag{3.11}$$

for some $x \in \mathbb{C}$ with $|x| \le 1$.

Applying triangle inequality to this equality, we have

$$\left|a_3-\mu a_2^2\right| \le \frac{|\tau|}{8(3\lambda-1)(1+2\beta)} \times$$
$$\times\left\{2\left(4-t^2\right)\xi+\frac{\left|2\left[\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right]\tau-(2\lambda-1)^2(1+\beta)^2\right|}{(2\lambda-1)^2(1+\beta)^2}t^2\right\}, \tag{3.12}$$

Where, $\xi=|x|$ and $t=|p_1|$.

If we maximize the expression on the right side of the inequality (3.12) with respect to the variable ξ, we obtain the following inequality:

$$\left|a_3-\mu a_2^2\right| \le \frac{|\tau|}{8(3\lambda-1)(1+2\beta)}\left\{\left[l(\tau,\lambda,\beta,\mu)-2\right]t^2+8\right\},\ t\in[0,2],$$

Where,

$$l(\tau,\lambda,\beta,\mu)=\frac{\left|2\left[\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right]\tau-(2\lambda-1)^2(1+\beta)^2\right|}{(2\lambda-1)^2(1+\beta)^2}.$$

Maximizing the function $\psi:[0,2]\to\mathbb{R}$ defined as follows

$$\psi(t)=\left[l(\tau,\lambda,\beta,\mu)-2\right]t^2+8,\ t\in[0,2],$$

we can easily see that $\psi(t)\le 8$ if $l(\tau,\lambda,\beta,\mu)\le 2$ and $\psi(t)\le 4l(\tau,\lambda,\beta,\mu)$ if $l(\tau,\lambda,\beta,\mu)\ge 2$.

Thus, the proof of theorem is completed.

In the case $\beta=0$, $\beta=1$ and $\lambda=1$ from the Theorem 3.2, we obtain the following results, respectively.

Corollary 3.4. If $f\in S^*\left(\tau,\lambda;e^\zeta\right)$ and $\mu\in\mathbb{C}$, then

$$\left|a_3-\mu a_2^2\right| \le \frac{|\tau|}{2(3\lambda-1)}\begin{cases}2 & if \quad l_0(\tau,\lambda,\mu)\le 2,\\ l_0(\tau,\lambda,\mu) & if \quad l_0(\tau,\lambda,\mu)\ge 2,\end{cases}$$

Where,

$$l_0(\tau,\lambda,\mu)=\frac{\left|2\left[\left(2\lambda^2-4\lambda+1\right)+(3\lambda-1)\mu\right]\tau-(2\lambda-1)^2\right|}{(2\lambda-1)^2}.$$

Corollary 3.5. If $f\in C\left(\tau,\lambda;e^\zeta\right)$ and $\mu\in\mathbb{C}$, then

$$\left|a_3-\mu a_2^2\right| \le \frac{|\tau|}{6(3\lambda-1)}\begin{cases}2 & if \quad l_1(\tau,\lambda,\mu)\le 2,\\ l_1(\tau,\lambda,\mu) & if \quad l_1(\tau,\lambda,\mu)\ge 2,\end{cases}$$

Where,

$$l_1(\tau,\lambda,\mu)=\frac{\left|\left[4\left(2\lambda^2-4\lambda+1\right)+3(3\lambda-1)\mu\right]\tau-2(2\lambda-1)^2\right|}{2(2\lambda-1)^2}.$$

Corollary 3.6. If $f\in\chi\left(\tau,\beta;e^\zeta\right)$ and $\mu\in\mathbb{C}$, then

$$\left|a_3-\mu a_2^2\right| \le \frac{|\tau|}{4(1+2\beta)}\begin{cases}2 & if \quad l_2(\tau,\beta,\mu)\le 2,\\ l_2(\tau,\beta,\mu) & if \quad l_2(\tau,\beta,\mu)\ge 2,\end{cases}$$

Where,

$$l_2(\tau,\beta,\mu)=\frac{\left|2\left[(1+3\beta)-2(1+2\beta)\mu\right]\tau+(1+\beta)^2\right|}{(1+\beta)^2}.$$

Also, taking $\mu=0$ and $\mu=1$ in the Theorem 3.2, we obtain the following results, respectively.

Corollary 2.7. If $f\in\chi_\Sigma\left(\beta,\lambda;e^z\right)$, then

$$|a_3|\le\begin{cases}\dfrac{1}{(3\lambda-1)(1+2\beta)} & if\ \ 4(3\lambda-1)(1+2\beta)\le(2\lambda-1)^2(1+\beta)^2,\\ \dfrac{4}{(2\lambda-1)^2(1+\beta)^2} & if\ \ 4(3\lambda-1)(1+2\beta)\ge(2\lambda-1)^2(1+\beta)^2.\end{cases}$$

Corollary 2.8. If $f\in\chi_\Sigma\left(\beta,\lambda;e^z\right)$, then

$$\left|a_3-a_2^2\right|\le\frac{1}{(3\lambda-1)(1+2\beta)}.$$

Remark 2.1. We note that Corollary 2.7 confirms the second result of Theorem 2.1.

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