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# On the Coefficient and Fekete-Szegö Problem of the Pseudo-Starlike and Pseudo-Convex Bi-Univalent Function Class

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**Abstract:** In this paper, we defined a new subclass of starlike and convex bi-univalent functions and examine some geometric properties this function class. For this definition class, we gave some coefficient upper bound estimates and solve Fekete-Sezöge problem.

**Keywords:** Starlike function, convex function, bi-univalent function, pseudo-starlike function, pseudo-convex function

## 1. Introduction

We will denote by $H(U)$ the class of analytic functions in the open unit disk $U=\{z\in\mathbb{C}:|z|<1\}$ of the complex plane $\mathbb{C}$. Let $A$ be the class of the functions $f\in H(U)$ given by series expansions

$$f(z)=z+a_2z^2+a_3z^3+a_4z^4+\cdots+a_nz^n+\cdots=z+\sum_{n=2}^{\infty}a_nz^n,\ a_n\in\mathbb{C}. \tag{1.1}$$

The subclass of $A$, which are univalent functions in $U$ is denoted by $S$ in the literature. The class $S$ was introduced by Köebe [1] first time and has become the core ingredient of advanced research in this field. After a short time, in 1916 Bieberbach [2] published a paper in which the coefficient hypothesis was proposed. This hypothesis states that if $f\in S$ and has the series form (1.1), then $|a_n|\le n$ for each $n\ge 2.$ There are many articles in the literature regarding to this hypothesis (see [3-14]).

It is known that the function $f$ is called bi-univalent function, if itself and inverse is univalent in $U$ and $f(U)$, respectively. The class of bi-univalent functions in $U$ is denoted by $\Sigma$ in the literature.

For the inverse $g(w)=f^{-1}(w)$ of the function $f\in\Sigma$, can written

$$g(w)=w+A_2w^2+A_3w^3+A_4w^4+\ldots\ =w+\sum_{n=2}^{\infty}A_nz^n,\ w\in f(U)=U_{r_0}, \tag{1.2}$$

Where,

$$A_2=-a_2,\ A_3=2a_2^2-a_3,\ A_4=-a_2^3+5a_2a_3-a_4,\ldots .$$

The bi-starlike and bi-convex function classes in the open unit disk $U$ are defined analytically as follows and denoted by $S_\Sigma^*$ and $C_\Sigma$, respectively

$$S_{\Sigma}^{*}=\left\{ f\in S:\ \operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right)>0,\ z\in U \text{ and } \operatorname{Re}\left(\frac{wg'(w)}{g(w)}\right)>0,\ w\in U_{r_0}\right\}$$

$$C_{\Sigma}\left\{ f\in S:\ \operatorname{Re}\left(\frac{(zf'(z))'}{f'(z)}\right)>0,\ z\in U \text{ and } \operatorname{Re}\left(\frac{(wg'(w))'}{g'(w)}\right)>0,\ w\in U_{r_0}\right\}.$$

Let's $f,g\in H(U)$, then it is said that $f$ is subordinate to $g$ and denoted by $f\prec g$, if there exists a Schwartz function $\omega$, such that $f(z)=g(\omega(z))$.

In the past few years, numerous subclasses of the class $S$ have been introduced as special choices of the class $S_{\Sigma}^{*}$ and $C_{\Sigma}$ (see for example [3, 8-21]).

## 2. Materials and Methods

Now, let's we define new subclass of bi-univalent functions in the open unit disk $U$.

**Definition 2.1.** For $\beta\in[0,1]$ and $\lambda>\frac{1}{2}$ the function $f\in\Sigma$ is said to be in the class $\chi_{\Sigma}(\beta,\lambda;e^{z})$, if the following conditions are satisfied

$$(1-\beta)\frac{z(f'(z))^{\lambda}}{f(z)}+\beta\frac{\left[(zf'(z))'\right]^{\lambda}}{f'(z)}\prec e^{z},\ z\in U \text{ and}$$

$$(1-\beta)\frac{w(g'(w))^{\lambda}}{g(w)}+\beta\frac{\left[(wg'(w))'\right]^{\lambda}}{g'(w)}\prec e^{w},\ w\in U_{r_0}.$$

In the cases $\beta=0$, $\beta=1$ and $\lambda=1$ from the Definition 2.1, we have the following classes of bi-univalent functions.

**Definition 2.2.** For $\lambda>\frac{1}{2}$ the function $f\in S$ is said to be in the class $S_{\Sigma}^{*}(\lambda;e^{z})$, if the following conditions are satisfied

$$\frac{z(f'(z))^{\lambda}}{f(z)}\prec e^{z},\ z\in U \text{ and } \frac{w(g'(w))^{\lambda}}{g(w)}\prec e^{w},\ w\in U_{r_0}.$$

**Definition 2.3.** For $\lambda>\frac{1}{2}$ the function $f\in S$ is said to be in the class $C_{\Sigma}(\lambda;e^{z})$, if the following conditions are satisfied

$$\frac{\left[(zf'(z))'\right]^{\lambda}}{f'(z)}\prec e^{z},\ z\in U \text{ and } \frac{\left[(wg'(w))'\right]^{\lambda}}{g'(w)}\prec e^{w},\ w\in U_{r_0}.$$

**Definition 2.4.** For $\beta\in[0,1]$ the function $f\in S$ is said to be in the class $\chi_{\Sigma}(\beta;e^{z})$, if the following conditions are satisfied

$$(1-\beta)\frac{zf'(z)}{f(z)}+\beta\frac{(zf'(z))'}{f'(z)}\prec e^{z},\ z\in U \text{ and } (1-\beta)\frac{wg'(w)}{g(w)}+\beta\frac{(wg'(w))'}{g'(w)}\prec e^{w},\ w\in U_{r_0}.$$

Let $\mathrm{P}$ be the class of analytic functions in $U$ satisfied the conditions $p(0)=1$ and $\mathrm{Re}(p(z))>0,\ z\in U.$ It is clear that the functions that satisfy the above conditions have the following series expansion

$$p(z)=1+p_1z+p_2z^2+p_3z^3+\cdots=1+\sum_{n=1}^{\infty}p_nz^n,\ z\in U \tag{2.1}$$

The class $\mathrm{P}$ defined above is known as the class Caratheodory functions in the literature [22].

Now, let us give some necessary lemmas for the proof of our main results.

**Lemma 2.1** ([23]). Let the function $p$ belong to the class $\mathrm{P}$. Then,

$$|p_n|\le 2 \text{ for each } n\in\mathbb{N},\ |p_n-\nu p_kp_{n-k}|\le 2 \text{ for } n,\ k\in\mathbb{N},\ n>k \text{ and } \nu\in[0,1].$$

The equalities hold for the function

$$p(z)=\frac{1+z}{1-z}.$$

**Lemma 2.2** ([23]) Let the an analytic function $p$ be of the form (2.1), then

$$2p_2=p_1^2+(4-p_1^2)x,$$

$$4p_3=p_1^3+2(4-p_1^2)p_1x-(4-p_1^2)p_1x^2+2(4-p_1^2)(1-|x|^2)y$$

for some $x,y\in\mathbb{C}$ with $|x|\le 1$ and $|y|\le 1$.

In this paper, we give some coefficient estimates and solve Fekete-Szegö problem for the class $\chi_{\Sigma}(\beta,\lambda;e^z)$. Additionally, the results obtained for specific values of the parameters in our study are compared with the results obtained in the literature.

## 3. Results and Discussion

In this section, we give some coefficient estimates for the functions belonging to the class $\chi_{\Sigma}(\beta,\lambda;e^z)$ and solve Fekete-Szegö problem for this class.

**Theorem 3.1.** Let the function $f$ given by series expansions (1.1) belong to the class $\chi_{\Sigma}(\beta,\lambda;e^z)$. Then, we have the following inequalities

$$|a_2|\le\frac{1}{(2\lambda-1)(1+\beta)} \text{ and}$$

$$|a_3|\le\begin{cases}\dfrac{1}{(3\lambda-1)(1+2\beta)} & if\ \dfrac{3\lambda-1}{(2\lambda-1)^2}\le\dfrac{(1+\beta)^2}{1+2\beta},\\[2ex] \dfrac{1}{(2\lambda-1)^2(1+\beta)^2} & if\ \dfrac{3\lambda-1}{(2\lambda-1)^2}\ge\dfrac{(1+\beta)^2}{1+2\beta}.\end{cases} \tag{3.1}$$

**Proof.** Let $f\in\chi_{\Sigma}(\beta,\lambda;e^z)$, then exists Schwartz functions $\omega:U\to U,\varpi:U_{r_0}\to U_{r_0}$, such that

$$(1-\beta)\frac{z(f'(z))^{\lambda}}{f(z)}+\beta\frac{\left[(zf'(z))'\right]^{\lambda}}{f'(z)}=e^{\omega(z)},z\in U \text{ and}$$

$$(1-\beta)\frac{w(g'(w))^{\lambda}}{g(w)}+\beta\frac{\left[(wg'(w))'\right]^{\lambda}}{g'(w)}=e^{\varpi(w)}, w\in U_{r_0}. \quad (3.2)$$

Let's the functions $p,q\in P$ defined as follows:

$$p(z)=\frac{1+\omega(z)}{1-\omega(z)}=1+p_1z+p_2z^2+p_3z^3+...=1+\sum_{n=1}^{\infty}p_nz^n,\ z\in U,$$

$$q(w)=\frac{1+\varpi(w)}{1-\varpi(w)}=1+q_1w+q_2w^2+q_3w^3+...=1+\sum_{n=1}^{\infty}q_nw^n,\ w\in U_{r_0}. \quad (3.3)$$

It follows from that

$$\omega(z)=\frac{p(z)-1}{p(z)+1}=\frac{p_1}{2}z+\frac{1}{2}\left(p_2-\frac{p_1^2}{2}\right)z^2+\frac{1}{2}\left(p_3-p_1p_2-\frac{p_1^3}{4}\right)z^3...,\ z\in U,$$

$$\varpi(w)=\frac{q(w)-1}{q(w)+1}=\frac{q_1}{2}w+\frac{1}{2}\left(q_2-\frac{q_1^2}{2}\right)w^2+\frac{1}{2}\left(q_3-q_1q_2-\frac{q_1^3}{4}\right)w^3...,w\in U_{r_0}. \quad (3.4)$$

From the (3.2) and (3.4) can written

$$\begin{aligned}
&(1-\beta)\left\{1+a_2(2\lambda-1)z+\left((3\lambda-1)a_3+(2\lambda^2-4\lambda+1)a_2^2\right)z^2+\cdots\right\}\\
&+\beta\left\{1+2a_2(2\lambda-1)z+\left(3(3\lambda-1)a_3+4(2\lambda^2-4\lambda+1)a_2^2\right)z^2+\cdots\right\}\\
&=1+\frac{p_1}{2}z+\frac{1}{2}\left(p_2-\frac{p_1^2}{4}\right)z^2+\cdots,\ z\in U,\\
&(1-\beta)\left\{1+A_2(2\lambda-1)w+\left((3\lambda-1)A_3+(2\lambda^2-4\lambda+1)A_2^2\right)w^2+\cdots\right\}\\
&+\beta\left\{1+2A_2(2\lambda-1)w+\left(3(3\lambda-1)A_3+4(2\lambda^2-4\lambda+1)A_2^2\right)w^2+\cdots\right\} \quad (3.5)\\
&=1+\frac{q_1}{2}w+\frac{1}{2}\left(q_2-\frac{q_1^2}{4}\right)w^2+\cdots,\ w\in f(U).
\end{aligned}$$

By equating the same coefficients, we obtain the following equalities

$$a_2=\frac{p_1}{2(2\lambda-1)(1+\beta)},$$

$$(3\lambda-1)(1+2\beta)a_3+(2\lambda^2-4\lambda+1)(1+3\beta)a_2^2=\frac{1}{2}\left(p_2-\frac{p_1^2}{4}\right), \quad (3.6)$$

$$a_2=-\frac{q_1}{2(2\lambda-1)(1+\beta)},$$

$$(3\lambda-1)(1+2\beta)A_3+(2\lambda^2-4\lambda+1)(1+3\beta)A_2^2=\frac{1}{2}\left(q_2-\frac{q_1^2}{4}\right). \quad (3.7)$$

Then,

$$\frac{p_1}{2(2\lambda-1)(1+\beta)}=a_2=-\frac{q_1}{2(2\lambda-1)(1+\beta)}\text{; that is, } p_1=-q_1. \quad (3.8)$$

Using Lemma 2.1 to the equality (3.8), we have first result of theorem.

Considering $A_2=-a_2$, $A_3=2a_2^2-a_3$, from the equalities (3.6) and (3.7) we have

$$a_3=\frac{1}{4(2\lambda-1)^2(1+\beta)^2}p_1^2+\frac{p_2-q_2}{4(3\lambda-1)(1+2\beta)}. \tag{3.9}$$

Applying Lemma 2.2, we can write

$$a_3=\frac{1}{4(2\lambda-1)^2(1+\beta)^2}p_1^2+\frac{4-p_1^2}{8(3\lambda-1)(1+2\beta)}(x-y) \tag{3.10}$$

for some $x,y\in\mathbb{C}$ with $|x|\le 1$ and $|y|\le 1$.

Then, applying triangle inequality to the last equality, we have

$$|a_3|\le\frac{t^2}{4(2\lambda-1)^2(1+\beta)^2}+\frac{4-t^2}{8(3\lambda-1)(1+2\beta)}(\zeta+\varsigma),\ \zeta,\varsigma\in[0,1], \tag{3.11}$$

where $\zeta=|x|$, $\varsigma=|y|$ and $t=|p_1|$. The inequality (3.11) can written

$$|a_3|\le\frac{a(\lambda,\beta)}{4}t^2+\frac{1}{(3\lambda-1)(1+2\beta)},\ t\in[0,2],$$

Where,

$$a(\lambda,\beta)=\frac{1}{(2\lambda-1)^2(1+\beta)^2}-\frac{1}{(3\lambda-1)(1+2\beta)}.$$

Then, maximizing the function

$$\varphi(t)=\frac{a(\lambda,\beta)}{4}t^2+\frac{1}{(3\lambda-1)(1+2\beta)},\ t\in[0,2]$$

it can easily be seen that $\varphi(t)\le\frac{1}{(3\lambda-1)(1+2\beta)}$ if $a(\lambda,\beta)\le 0$ and $\varphi(t)\le\frac{1}{(2\lambda-1)^2(1+\beta)^2}$ if $a(\lambda,\beta)\ge 0.$

With this, the proof of second inequality of (3.1) is provided.

Thus, the proof of theorem is completed.

In the case $\beta=0$, $\beta=1$ and $\lambda=1$ from the Theorem 3.1, we obtain the following results, respectively.

**Corollary 3.1.** If $f\in S_\Sigma^*(\lambda;e^z)$, then

$$|a_2|\le\frac{1}{2\lambda-1}\ \text{and}\ |a_3|\le\begin{cases}\dfrac{1}{(2\lambda-1)^2} & if \quad \lambda\in\left[\dfrac{7-\sqrt{17}}{8},\dfrac{7+\sqrt{17}}{8}\right],\\[2ex] \dfrac{1}{3\lambda-1} & if\ \lambda\in\left(\dfrac{1}{2},\dfrac{7-\sqrt{17}}{8}\right] or\, \lambda\ge\dfrac{7+\sqrt{17}}{8}.\end{cases}$$

**Corollary 3.2.** If $f\in C_\Sigma(\lambda;e^z)$, then

$$|a_2| \le \frac{1}{2(2\lambda-1)} \text{ and } |a_3| \le \begin{cases} \dfrac{1}{4(2\lambda-1)^2} & if \quad \lambda \in \left[\dfrac{25-\sqrt{177}}{32}, \dfrac{25+\sqrt{177}}{32}\right], \\ \dfrac{1}{3(3\lambda-1)} & if\, \lambda \in \left(\dfrac{1}{2}, \dfrac{25-\sqrt{177}}{32}\right] or\, \lambda \ge \dfrac{25+\sqrt{177}}{32}. \end{cases}$$

**Corollary 3.3.** If $f \in \chi_\Sigma\left(\beta; e^z\right)$, then

$$|a_2| \le \frac{1}{1+\beta} \text{ and } |a_3| \le \frac{1}{(1+\beta)^2}.$$

Now, we give the following theorem on the Fekete-Szegö problem for the class $\chi_\Sigma\left(\beta, \lambda; e^z\right)$.

**Theorem 3.2.** Let $f \in \chi_\Sigma\left(\beta, \lambda; e^z\right)$ and $\mu \in \mathbb{C}$, then

$$|a_3 - \mu a_2^2| \le \begin{cases} \dfrac{1}{(3\lambda-1)(1+2\beta)} & if \;\; |1-\mu| \le \dfrac{(2\lambda-1)^2(1+\beta)^2}{(3\lambda-1)(1+2\beta)}, \\ \dfrac{|1-\mu|}{(2\lambda-1)^2(1+\beta)^2} & if \;\; |1-\mu| \ge \dfrac{(2\lambda-1)^2(1+\beta)^2}{(3\lambda-1)(1+2\beta)}. \end{cases} \tag{3.12}$$

**Proof.** Let $f \in \chi_\Sigma\left(\beta, \lambda; e^z\right)$, then from the equalities (3.8) and (3.9), we can write

$$a_3 - \mu a_2^2 = \frac{(1-\mu)\, p_1^2}{4(2\lambda-1)^2(1+\beta)^2} + \frac{p_2 - q_2}{4(3\lambda-1)(1+2\beta)}. \tag{3.13}$$

Then, applying Lemma 2.2 the expression $a_3 - \mu a_2^2$ can written as follows

$$a_3 - \mu a_2^2 = \frac{(1-\mu)\, p_1^2}{4(2\lambda-1)^2(1+\beta)^2} + \frac{4-p_1^2}{8(3\lambda-1)(1+2\beta)}(x-y) \tag{3.14}$$

for some $x, y \in \mathbb{C}$ with $|x| \le 1$ and $|y| \le 1$.

Applying triangle inequality to this equality, we have

$$|a_3 - \mu a_2^2| \le \frac{|1-\mu| t^2}{4(2\lambda-1)^2(1+\beta)^2} + \frac{(4-t^2)(\xi+\eta)}{8(3\lambda-1)(1+2\beta)}, \; \xi, \eta \in [0,1], \tag{3.15}$$

where $\xi = |x|$, $\eta = |y|$ and $t = |p_1|$.

Maximizing the right-hand side of the inequality (3.15) with respect to the variables $\xi$ and $\eta$, we can write:

$$|a_3 - \mu a_2^2| \le \frac{b(\lambda, \beta; \mu)}{4} t^2 + c(\lambda, \beta), \; t \in [0,2],$$

Where,

$$b(\lambda, \beta; \mu) = \frac{|1-\mu|}{(2\lambda-1)^2(1+\beta)^2} - \frac{1}{(3\lambda-1)(1+2\beta)} \text{ and } c(\lambda, \beta) = \frac{1}{(3\lambda-1)(1+2\beta)}.$$

Maximizing the function $\psi : [0,2] \to \mathbb{R}$, defined as follows

$$\psi(t) = \frac{b(\lambda, \beta; \mu)}{4} t^2 + c(\lambda, \beta), \; t \in [0,2],$$

we can easily see that

$$\psi(t) \le \frac{1}{(3\lambda-1)(1+2\beta)}$$

if $b(\lambda,\beta;\mu) \le 0$; that is if

$$(3\lambda-1)(1+2\beta)|1-\mu| \le (2\lambda-1)^2(1+\beta)^2$$

and

$$\psi(t) \le \frac{|1-\mu|}{(2\lambda-1)^2(1+\beta)^2}$$

if

$$(3\lambda-1)(1+2\beta)|1-\mu| \ge (2\lambda-1)^2(1+\beta)^2.$$

Thus, the proof of theorem is completed.

In the case $\beta=0$, $\beta=1$ and $\lambda=1$ from the Theorem 3.2, we obtain the following results, respectively.

**Corollary 3.4.** If $f \in S_\Sigma^*(\lambda;e^z)$ and $\mu \in \mathbb{C}$, then

$$|a_3-\mu a^2| \le \begin{cases} \dfrac{1}{3\lambda-1} & if \ |1-\mu| \le \dfrac{(2\lambda-1)^2}{4(3\lambda-1)}, \\ \dfrac{4|1-\mu|}{(2\lambda-1)^2} & if \ |1-\mu| \ge \dfrac{(2\lambda-1)^2}{4(3\lambda-1)}. \end{cases}$$

**Corollary 3.5.** If $f \in C_\Sigma(\lambda;e^z)$ and $\mu \in \mathbb{C}$, then

$$|a_3-\mu a_2^2| \le \begin{cases} \dfrac{1}{3(3\lambda-1)} & if \ |1-\mu| \le \dfrac{(2\lambda-1)^2}{3(3\lambda-1)}, \\ \dfrac{4|1-\mu|}{4(2\lambda-1)^2} & if \ |1-\mu| \ge \dfrac{(2\lambda-1)^2}{3(3\lambda-1)}. \end{cases}$$

**Corollary 3.6.** If $f \in \chi_\Sigma(\beta;e^z)$ and $\mu \in \mathbb{C}$, then

$$|a_3-\mu a_2^2| \le \begin{cases} \dfrac{1}{2(1+2\beta)} & if \ |1-\mu| \le \dfrac{(1+\beta)^2}{8(1+2\beta)}, \\ \dfrac{4|1-\mu|}{(1+\beta)^2} & if \ |1-\mu| \ge \dfrac{(1+\beta)^2}{8(1+2\beta)}. \end{cases}$$

Also, taking $\mu=0$ and $\mu=1$ in the Theorem 3.2, we obtain the following results, respectively.

**Corollary 3.7.** If $f \in \chi_\Sigma(\beta,\lambda;e^z)$, then

$$|a_3| \le \begin{cases} \dfrac{1}{(3\lambda-1)(1+2\beta)} & if \ \dfrac{3\lambda-1}{(2\lambda-1)^2} \le \dfrac{(1+\beta)^2}{1+2\beta}, \\ \dfrac{1}{(2\lambda-1)^2(1+\beta)^2} & if \ \dfrac{3\lambda-1}{(2\lambda-1)^2} \ge \dfrac{(1+\beta)^2}{1+2\beta}. \end{cases}$$

**Corollary 3.8.** If $f \in \chi_{\Sigma}\left(\beta, \lambda; e^{z}\right)$, then

$$\left|a_3 - a_2^2\right| \le \frac{1}{(3\lambda - 1)(1 + 2\beta)}.$$

**Remark 3.1.** We note that Corollary 3.7 confirms the second result of Theorem 3.1.

**References**

[1]. Köebe, P. (1909). Über die Uniformisierrung der algebraishen Kurven, durch automorpher Funktionen mit imaginarer Substitutions gruppe. Nachr. Akad. Wiss. Göttingen Math. -Phys. 68-76.

[2]. Bieberbach, L. (1916). Über die Koeffizienten derjenigen Potenzreihen welche eine schlichte Abbidung des Einbeiskreises vermitteln. Sitzungsberichte Preuss. Akad. Der Wiss. 138, 940-955.

[3]. Sokol, J. (2011). A certain class of starlike functions. Comput. Math. Appl. 62, 611-619.

[4]. Janowski, W. (1970). Extremal problems for a family of functions with positive real part and for some related families. Ann. Pol. Math. 23, 159-177.

[5]. Arif, M., Ahmad, K., Liu, J.-L., Sokol, J. (2019). A new class of analytic functions associated with Salagean operator. J. Funct. Spaces https://doi.org/10.1155/2019/6157394, 1-8.

[6]. Brannan, D. A., Kirwan, W. E. (1969). On some classes of bounded univalent functions. J. Lond. Math. Soc. 2, 431-443.

[7]. Sharma, K., Jain, N. K., Ravichandran, V. (2016) Starlike function associated with a cardioid. Afr. Math. 27, 923-939.

[8]. Kumar, S. S., Arora, K. (2020). Starlike functions associated with a petal shaped domain. ArXiv 2020, arXiv: 2010.10072.

[9]. Mendiratta, R., Nagpal, S., Ravichandran, V. (2015). On a subclass of strongly starlike functions associated with exponential function. Bull. Malays. Math. Soc. 38, 365-386.

[10]. Bano, K., Raza, M. (2020). Starlike functions associated with cosine function. Bull. Iran. Math. Soc. 47, 1513-1532.

[11]. Alotaibi, A., Arif, M., Alghamdi, M. A., Hussain, S. (2020). Starlikness associated with cosine hyperbolic function. Mathematics 8, 1118.

[12]. Ullah, K., Zainab, S., Arif, M., Darus, M., Shutayi, M. (2021). Radius Problems for Starlike Functions Associated with the Tan Hyperbolic Function. J. Funct. Spaces, Article ID 9967640.

[13]. Cho, N. E., Kumar, V., Kumar, S. S., (2019). Ravichandran, V. Radius problems for starlike functions associated with the sine function. Bull. Iran. Math. Soc. 45, 213-232.

[14]. Mustafa, N., Nezir, V., Kankılıç. A. (2023). Coefficient estimates for certain subclass of analytic and univalent functions associted with sine hyperbolic function. 13th International Istanbul Scientific Research Congress on Life, Engineering and Applied Sciences on April 29-30, 234-241, Istanbul, Turkey.

[15]. Mustafa, N., Nezir, V., Kankılıç. A. (2023). The Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine hyperbolic function. 13th International Istanbul Scientific Research Congress on Life, Engineering and Applied Sciences on April 29-30, 242-249, Istanbul, Turkey.

[16]. Mustafa, N., Nezir, V. (2023). Coefficient estimates and Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine hyperbolic function. 13th International Istanbul Scientific Research Congress on Life, Engineering and Applied Sciences on May 1-2, 475-481, Istanbul, Turkey.

[17]. Mustafa, N., Demir, H. A. (2023). Coefficient estimates for certain subclass of analytic and univalent functions with associated with sine and cosine functions. 4th International Black Sea Congress on Modern Scientific Research on June 6-8, 2555-2563, Rize Turkey.

[18]. Mustafa, N., Demir, H. A. (2023). Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine and cosine functions. 4th International Black Sea Congress on Modern Scientific Research on June 6-8, 2564-2572, Rize Turkey.

[19]. Mustafa, N., Nezir, V., Kankılıç. A. (2023). Coefficient estimates for certain subclass of analytic and univalent functions associated with sine hyperbolic function with complex order. Journal of Scientific and Engineering Research, 10(6), 18-25.

[20]. Mustafa, N., Nezir, V., Kankılıç. A. (2023). The Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine hyperbolic function with complex order. Eastern Anatolian Journal of Science, 9(1), 1-6.

[21]. Mustafa, N., Demir, H. A. (2023). Coefficient estimates for certain subclass of analytic and univalent functions with associated with sine and cosine functions with complex order. Journal of Scientific and Engineering Research, 10(6), 131-140.

[22]. Miller, S. S. (1975). Differential inequalities and Caratheodory functions. Bull. Am. Math. Soc. 81, 79-81.

[23]. Duren, P. L. (1983). Univalent Functions. In Grundlehren der Mathematischen Wissenschaften, New York, Berlin, Heidelberg and Tokyo, Springer-Verlag, Volume 259.