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# Predefined-Time Tracking Consensus Control for Integrator-Type Multi-Agent Systems

**Jianfeng Xu, Junqi Yang***

School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo, China, 454003
*Email: yjq@hpu.edu.cn

**Abstract:** This paper addresses the issue of single-integrator multi-agent systems tracking consensus by proposing a predefined-time control protocol. The proposed control protocol ensures that the states of all agents in the system can track the target state within the predefined-time. Moreover, this predefined-time is independent of the system's initial conditions and can be arbitrarily specified by the user in advance. Finally, simulations show that the actual convergence time is not conservative compared to the predefined-time.

**Keywords:** Multi-agent systems, Predefined-time stability, Distributed control, Predefined-time tracking control

## 1. Introduction

Multi-agent systems have garnered significant attention in recent years due to their wide-ranging applications in various fields, including intelligent robotics [1-2], unmanned aerial vehicles (UAVs) [3-4] and spacecraft formations [5-6]. These systems are composed of multiple interacting agents that work together to achieve shared objectives, utilizing their collective abilities to perform tasks that are difficult or unattainable for a single agent. The decentralized characteristic of multi-agent systems improves scalability, robustness, and flexibility, rendering them highly suitable for complex and dynamic environments.

The convergence rate is a critical performance metric in consensus control. Papers [7-8] improved convergence speed by optimizing connection weights or interaction topologies. However, these approaches only achieve asymptotic convergence, which may not suffice for time-critical applications. In contrast, a finite-time formation control framework was proposed in [9] to ensure that all agents in a multi-agent system reach consensus within a finite time. Significant research has been conducted on finite-time consensus control, as evidenced by [9-13]. Nevertheless, finite-time consensus control is constrained by its reliance on initial conditions, where larger initial values may hinder convergence. To address this limitation, Paper [14] introduced a fixed-time control algorithm that guarantees convergence within a fixed time, independent of initial conditions. As a result, fixed-time consensus control has garnered substantial attention [15-19].

However, fixed-time stability fails to establish a clear relationship between the upper bound of convergence time and system parameters, and it does not support the arbitrary specification of convergence time in advance. To overcome these limitations, a control method known as predefined-time stability was proposed in [20], which has attracted considerable attention. The upper bound of the settling time for predefined-time stability is equal to a predefined-time parameter and can be specified by the designer, allowing the convergence characteristics to be adjusted arbitrarily. Given the advantages of predefined-time stability, it has been widely applied in the consensus control of multi-agent systems. Paper [21] designed a linear feedback control protocol with time-varying gains for single-integrator multi-agent systems. However, this protocol fails to maintain control over the system after the predefined convergence time, which limits its practical applicability. Paper [22] introduced a free-will arbitrary-time consensus protocol, effectively resolving the limitations identified in [21]. Paper [23] proposed a smooth distributed control architecture to address the tracking problem of single-integrator multi-agent systems, achieving predefined-time tracking consensus through a time transformation method. However, similar to the issue in [21], this control architecture cannot continue to control the system after the predefined-time. This motivates us to investigate a predefined-time tracking consensus problem of multi-agent systems.

The main contributions are summarized as follows:

1. This paper proposes a tracking consensus protocol for single-integrator multi-agent systems, ensuring that the error between the states of the agents and the tracking target converges to zero within a predefined-time. Moreover, this predefined-time is independent of the system's initial states and can be specified in advance by the designer.
2. Unlike the work in [23], the proposed predefined-time tracking consensus protocol in this paper guarantees that the system states continue to track the target states even after the predefined-time has elapsed.

The remainder of this paper is organized as follows. Section 2 provides some definitions, lemmas, and the problem formulation. Section 3 will propose predefined-time controller. Numerical simulation results are presented in Section 4. Finally, Section 5 gives a summarization of this paper.

**Notation:** For any $\alpha \ge 0$, we define $\lceil x \rfloor^{\alpha} = |x|^{\alpha} sign(x)$ for $x \in \mathrm{R}$ and

$$\lceil x \rfloor^{\alpha} = \left[ |x_1|^{\alpha} sign(x_1), |x_2|^{\alpha} sign(x_2), \cdots, |x_N|^{\alpha} sign(x_N) \right]^T$$

for $x = [x_1, x_2, \cdots, x_N]^T \in \mathrm{R}^N$. The *p*-norm of a vector $x \in \mathrm{R}^N$ is given as $\|x\|_p$, where $p \ge 1$. $\mathbf{1}_N$ are commonly used to denote *N*-dimensional column vectors where all elements are 1.

## 2. Preliminaries and Problem formulation

### Preliminaries

We firstly recall some knowledge about graph theory. A graph $\mathrm{G} = (\mathrm{V}, \mathrm{E}, \mathrm{A})$ is used to model the communication topology of a multi-agent system with $N$ followers, where $\mathrm{V} = \{v_1, v_2, \ldots, v_N\}$ and $\mathrm{E} \subseteq \mathrm{V} \times \mathrm{V}$ denote the node set and edge set, respectively. $\mathrm{A} = [a_{ij}]$ is the adjacency matrix with weights $a_{ij} \ge 0$, where $i, j = 1, 2, \cdots, N$. Node $v_i$ represents the $i$ th agent, and an edge $(v_j, v_i)$ indicates information flow from the $j$ th agent to the $i$ th agent. The adjacency element $a_{ij}$ is positive if and only if $(v_j, v_i) \in \mathrm{E}$. The neighbor set of agent $i$ is defined as $\mathrm{N}_i = \{ j | v_j \in \mathrm{V}, (v_j, v_i) \in \mathrm{E} \}$. The Laplacian matrix $\mathrm{L}$ of $\mathrm{G} = (\mathrm{V}, \mathrm{E}, \mathrm{A})$ is given by $\mathrm{L} = \mathrm{D} - \mathrm{A}$, where $\mathrm{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$ is the degree matrix with diagonal elements $d_i = \sum_{j=1}^{N} a_{ij}$. In this paper, the interaction topology is assumed to be undirected and free of self-loops, that is $a_{ij} = a_{ji}$ and $a_{ii} = 0$ for all $i, j \in \mathrm{N}$. As a result, the Laplacian matrix $\mathrm{L}$ is symmetric.

Consider a class of nonlinear system

$$\dot{x} = f(x(t)), \quad x(t_0) = x_0, \tag{1}$$

where $x \in \mathrm{R}^n$ and $x_0 \in \mathrm{R}^n$ represent system state and initial state, respectively. The nonlinear function $f : \mathrm{R}^n \to \mathrm{R}^n$ satisfies $f(t, 0) = 0$ for $t > 0$. Suppose the origin $x = 0$ is the equilibrium point.

**Definition 1[14]:** If the system (1) is globally asymptotically stable and any of its solutions reach equilibrium point at some finite time, i.e., $x(t, x_0) = 0$, $\forall t \ge T(x_0)$, then the equilibrium point $x = 0$ of system (1) is said to be globally finite-time stable, where $T$: $\mathrm{R}^n \to \mathrm{R}_{\ge 0}$ is the so-called settling time function.

**Definition 2[14]:** The equilibrium point $x=0$ of system (1) is said to be globally fixed-time stable if it is globally finite-time stable and the settling time function $T(x_0)$ is bounded, i.e., there exists constant $T_{max}>0$ such that $\forall x_0 \in \mathrm{R}^n$, $T(x_0)\le T_{max}$ holds.

**Definition 3[20]:** For a fixed-time stable system (1), if there exists an arbitrarily selected parameter $T_c>0$ that is independent of any system parameters and initial conditions, such that $T_{max}\le T_c$ holds, then the origin of the nonlinear system (1) is said to be predefined-time stable.

**Lemma 1[24]:** For $\forall \xi \in \mathrm{R}^n$, the inequalities $\|\xi\|_2^{2+\gamma} \le n^{\gamma/2}\|\xi\|_{2+\gamma}^{2+\gamma}$ and $\|\xi\|_{p_1} \le \|\xi\|_{p_2}$ hold for any positive constant $\gamma>0$ and $p_1 \ge p_2 \ge 1$.

**Lemma 2:** If there exists a Lyapunov function $V$ such that

$$\dot{V} \le -\frac{\pi}{rT_c}\left(V^{1+\frac{r}{2}}+V^{1-\frac{r}{2}}\right), \tag{2}$$

where $0<r<1$, $T_c$ is positive constants, then $V$ is predefined-time stable with the predefined-time $T_c$.

The result of Lemma 2 is a simplified version of Lemma 2 in [25]. Please see Appendix for its proof.

**Problem formulation**

In this paper, we consider a multi-agent system consisting of $N$ agents. The dynamics of the system is described as

$$\dot{x}_i(t)=u_i(t), \quad x_i(0)=x_{i,0}, \quad i=1,\cdots,N \tag{3}$$

where $x_i(t)\in \mathrm{R}$ and $x_{i,0}\in \mathrm{R}$ are the state and initial state of agent $i$, respectively, and $u_i(t)\in \mathrm{R}$ is the control input of agent $i$. Consider a tracking target with the following dynamics

$$p(t)=\int_0^t v(\tau)d\tau + p(0) \tag{4}$$

where $v(t)\in \mathrm{R}$ denotes the velocity of the target, $p(t)\in \mathrm{R}$ represents the position of the target, and $p(0)$ signifies the initial position of the target.

**Assumption 1:** Graph G is an undirected connected graph, and at least one agent can obtain the state information of the tracking target.

**Assumption 2:** The velocity of the target is unknown to any agent, but its upper bound $v(t)\le v_{\max}$ is known in advance.

To represent the information transmission between the tracking target and followers, let $\mathrm{K}=diag\{k_1,k_2,\cdots,k_N\}$ and $k_i>0$ indicate that the $i$ th follower can receive the tracking target's state information; otherwise, $k_i=0$. Accordingly, $\mathrm{M}=\mathrm{L}+\mathrm{K}$ denotes the interaction matrix of the tracking system.

**Lemma 3 [26]:** If Assumption 1 holds, then $\mathrm{M}\in \mathrm{R}^{N\times N}$ is a positive definite matrix.

The control objective of this paper is to design a predefined-time consensus protocol for the multi-agent system (3), ensuring that the system states can track the target state within any predefined-time $T$, i.e.,

$$\begin{cases} \lim\limits_{t\to T} x_i(t)=p(t), & i=1,2,\cdots,N \\ x_i(t)\equiv p(t), & \forall t>T, \end{cases} \tag{5}$$

where $T\in(0,\infty)$ is the user-predefined convergence time, which does not depend on system parameters or initial conditions. Different from paper [23], the control protocol proposed in this paper ensures that the agent error remains zero relative to the tracking target for $\forall t>T$.

## 3. Predefined-Time Tracking Consensus Control Protocol

To achieve predefined-time consensus tracking, a predefined-time consensus tracking protocol is proposed for the multi-agent system (3) as follow

$$
\begin{aligned}
u_i(t) = &-\alpha_1 \left\lceil \sum_{j=1}^{N} \left[ a_{ij}\left(x_i(t)-x_j(t)\right)+k_i\left(x_i(t)-p(t)\right)\right] \right\rceil^{1-r} \\
&-\alpha_2 \left\lceil \sum_{j=1}^{N} \left[ a_{ij}\left(x_i(t)-x_j(t)\right)+k_i\left(x_i(t)-p(t)\right)\right] \right\rceil^{1+r} \\
&-\alpha_3 \operatorname{sign}\left( \sum_{j=1}^{N} \left[ a_{ij}\left(x_i(t)-x_j(t)\right)+k_i\left(x_i(t)-p(t)\right)\right] \right),
\end{aligned} \tag{6}
$$

where $0<r<1$. The gains of the controller $\alpha_1$, $\alpha_2$, and $\alpha_3$, are selected as follow

$$\alpha_1 = (\pi/rT)(2\underline{\lambda})^{\frac{r}{2}-1}, \tag{7}$$

$$\alpha_2 = (\pi/rT)(2\underline{\lambda})^{-\frac{r}{2}-1}(N)^{\frac{r}{2}}, \tag{8}$$

$$\alpha_3 = v_{\max}, \tag{9}$$

where $\underline{\lambda}$ denotes the minimum eigenvalue of the matrix $\mathrm{M}$.

**Theorem 1:** If Assumptions 1 and 2 hold, then under the control protocol (6) with gains (7)-(9), the multi-agent system (3) can achieve predefined-time tracking consensus.

**Proof:** Define an error variable as

$$\tilde{x}_i(t) = x_i(t) - p(t). \tag{10}$$

Differentiating (10) yields

$$\dot{\tilde{x}}_i(t) = \dot{x}_i(t) - \dot{p}(t) = u_i(t) - v(t). \tag{11}$$

Substituting the control input (6) into (11) results in

$$
\begin{aligned}
\dot{\tilde{x}}_i(t) = &-\alpha_1 \left\lceil \sum_{j=1}^{N} \left[ a_{ij}\left(x_i(t)-x_j(t)\right)+k_i\left(x_i(t)-p(t)\right)\right] \right\rceil^{1-r} \\
&-\alpha_2 \left\lceil \sum_{j=1}^{N} \left[ a_{ij}\left(x_i(t)-x_j(t)\right)+k_i\left(x_i(t)-p(t)\right)\right] \right\rceil^{1+r} \\
&-\alpha_3 \operatorname{sign}\left( \sum_{j=1}^{N} \left[ a_{ij}\left(x_i(t)-x_j(t)\right)+k_i\left(x_i(t)-p(t)\right)\right] \right) - v(t).
\end{aligned}
$$

Defining $r_{\tilde{x}_i} = \sum_{j=1}^{N}\left[ a_{ij}\left(\tilde{x}_i(t)-\tilde{x}_j(t)\right)+k_i\tilde{x}_i(t)\right]$, a compact form can be obtained as

$$
\begin{aligned}
\dot{\tilde{x}} &= \begin{bmatrix} u_1 \\ u_1 \\ \vdots \\ u_N \end{bmatrix} - \mathbf{1}_N v(t) = \begin{bmatrix} -\alpha_1 \lceil r_{\tilde{x}_1} \rceil^{1-r} - \alpha_2 \lceil r_{\tilde{x}_1} \rceil^{1+r} - \alpha_3 \operatorname{sign}(r_{\tilde{x}_1}) \\ -\alpha_1 \lceil r_{\tilde{x}_2} \rceil^{1-r} - \alpha_2 \lceil r_{\tilde{x}_2} \rceil^{1+r} - \alpha_3 \operatorname{sign}(r_{\tilde{x}_2}) \\ \vdots \\ -\alpha_1 \lceil r_{\tilde{x}_N} \rceil^{1-r} - \alpha_2 \lceil r_{\tilde{x}_N} \rceil^{1+r} - \alpha_3 \operatorname{sign}(r_{\tilde{x}N}) \end{bmatrix} - \mathbf{1}_N v(t) \\
&= -\alpha_1 \begin{bmatrix} r_{\tilde{x}_1} \\ r_{\tilde{x}_2} \\ \vdots \\ r_{\tilde{x}_N} \end{bmatrix}^{1-r} - \alpha_2 \begin{bmatrix} r_{\tilde{x}_1} \\ r_{\tilde{x}_2} \\ \vdots \\ r_{\tilde{x}_N} \end{bmatrix}^{1+r} - \alpha_3 \operatorname{sign} \begin{bmatrix} r_{\tilde{x}_1} \\ r_{\tilde{x}_2} \\ \vdots \\ r_{\tilde{x}_N} \end{bmatrix} - \mathbf{1}_N v(t) \\
&= -\alpha_1 \lceil \mathrm{M}\,\tilde{x} \rceil^{1-r} - \alpha_2 \lceil \mathrm{M}\,\tilde{x} \rceil^{1+r} - \alpha_3 \operatorname{sign}(\mathrm{M}\,\tilde{x}) - \mathbf{1}_N v(t).
\end{aligned}
$$

where $\tilde{x} = [\tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_N] \in \mathrm{R}^N$, $u = [u_1, u_2, \cdots, u_N] \in \mathrm{R}^N$. Define a Lyapunov function as

$$V = \frac{1}{2}\tilde{x}^T \mathrm{M}\,\tilde{x}. \tag{12}$$

Differentiating (12) yields

$$\begin{aligned}\dot{V} &= \tilde{x}^T \mathrm{M}\left(-\alpha_1 \lceil \mathrm{M}\,\tilde{x}\rfloor^{1-r} - \alpha_2 \lceil \mathrm{M}\,\tilde{x}\rfloor^{1+r} - \alpha_3 \,\mathrm{sign}(\mathrm{M}\,\tilde{x}) - \mathbf{1}_N v\right) \\ &= -\alpha_1 \tilde{x}^T \mathrm{M} \lceil \mathrm{M}\,\tilde{x}\rfloor^{1-r} - \alpha_2 \tilde{x}^T \mathrm{M} \lceil \mathrm{M}\,\tilde{x}\rfloor^{1+r} - \alpha_3 \tilde{x}^T \mathrm{M}\,\mathrm{sign}(\mathrm{M}\,\tilde{x}) - \tilde{x}^T \mathrm{M}\,(\mathbf{1}_N v).\end{aligned} \tag{13}$$

Let $(\mathrm{M}\,\tilde{x})_k$ be the $k$ th $(k = 1, 2, \cdots N)$ entry of vector $\mathrm{M}\,\tilde{x}$, then one has

$\left|(\mathrm{M}\,\tilde{x})_k\right|^{1-r} (\mathrm{M}\,\tilde{x})_k \, sign\left((\mathrm{M}\,\tilde{x})_k\right) = \left|(\mathrm{M}\,\tilde{x})_k\right|^{1-r} \left|(\mathrm{M}\,\tilde{x})_k\right| = \left|(\mathrm{M}\,\tilde{x})_k\right|^{2-r}$.

This implies that

$$\tilde{x}^T \mathrm{M}\,\mathrm{sign}(\mathrm{M}\,\tilde{x}) = \sum_{k=1}^{N} \left|(\mathrm{M}\,\tilde{x})_k\right| = \|\mathrm{M}\,\tilde{x}\|_1, \tag{14}$$

and

$$\tilde{x}^T \mathrm{M} \lceil \mathrm{M}\,\tilde{x}\rfloor^{1-r} = (\mathrm{M}\,\tilde{x})^T \lceil \mathrm{M}\,\tilde{x}\rfloor^{1-r} = \sum_{k=1}^{N} \left|(\mathrm{M}\,\tilde{x})_k\right|^{2-r} = \|\mathrm{M}\,\tilde{x}\|_{2-r}^{2-r}. \tag{15}$$

By using a similar approach, it can be derived that

$$\tilde{x}^T \mathrm{M} \lceil \mathrm{M}\,\tilde{x}\rfloor^{1+r} = (\mathrm{M}\,\tilde{x})^T \lceil \mathrm{M}\,\tilde{x}\rfloor^{1+r} = \sum_{k=1}^{N} \left|(\mathrm{M}\,\tilde{x})_k\right|^{2+r} = \|\mathrm{M}\,\tilde{x}\|_{2+r}^{2+r}. \tag{16}$$

From Assumption 2, it is known that $|v| \le v_{\max}$, and thus

$$-\tilde{x}^T \mathrm{M}\,(\mathbf{1}_N v) \le v_{\max} \|\mathrm{M}\,\tilde{x}\|_1, \tag{17}$$

Combining (14)-(17) and (13), we obtain

$$\dot{V} \le -\alpha_1 \|\mathrm{M}\,\tilde{x}\|_{2-r}^{2-r} - \alpha_2 \|\mathrm{M}\,\tilde{x}\|_{2+r}^{2+r} - \alpha_3 \|\mathrm{M}\,\tilde{x}\|_1 + v_{\max} \|\mathrm{M}\,\tilde{x}\|_1. \tag{18}$$

Since $\alpha_3 = v_{\max}$, (18) becomes

$$\dot{V} \le -\alpha_1 \|\mathrm{M}\,\tilde{x}\|_{2-r}^{2-r} - \alpha_2 \|\mathrm{M}\,\tilde{x}\|_{2+r}^{2+r} \tag{19}$$

From Lemma 1 and (19), we obtain

$$\dot{V} \le -\alpha_1 \|\mathrm{M}\,\tilde{x}\|_2^{2-r} - \alpha_2 N^{-r/2} \|\mathrm{M}\,\tilde{x}\|_2^{2+r}. \tag{20}$$

Furthermore, based on $\|\mathrm{M}\,\tilde{x}\|_2 \ge \underline{\lambda}^{\frac{1}{2}} \left(\tilde{x}^T \mathrm{M}\,\tilde{x}\right)^{\frac{1}{2}} \ge (2\underline{\lambda})^{\frac{1}{2}} (V)^{\frac{1}{2}}$, it can be concluded that

$$\|\mathrm{M}\,\tilde{x}\|_2^{2-r} \ge (2\underline{\lambda})^{1-\frac{r}{2}} \left(V^{1-\frac{r}{2}}\right) \tag{21}$$

and

$$\|\mathrm{M}\,\tilde{x}\|_2^{2+r} \ge (2\underline{\lambda})^{1+\frac{r}{2}} V^{1+\frac{r}{2}}. \tag{22}$$

Substituting (21) and (22) into (20), we can derive

$$\dot{V} \le -\alpha_1 (2\underline{\lambda})^{1-\frac{r}{2}} V^{1-\frac{r}{2}} - \alpha_2 N^{-r/2} (2\underline{\lambda})^{1+\frac{r}{2}} V^{1+\frac{r}{2}}. \tag{23}$$

By substituting $\alpha_1$ and $\alpha_2$ from (7) and (8) into (23), we obtain

$$\dot{V} \le -\frac{\pi}{rT}\left(V^{1+\frac{r}{2}} + V^{1-\frac{r}{2}}\right). \tag{24}$$

From (24) and Lemma 2, it is evident that as $t \to T$, the Lyapunov function $V \to 0$, and when $t \ge T$, the Lyapunov function $V = 0$. Therefore, the state error $\tilde{x}_i(t)$ will converge to 0 within the predefined-time $T$, which implies that (5) is satisfied. Thus, it can be concluded that under the control protocol (6), the multi-agent system (3) achieves predefined-time tracking consensus. This completes the proof.

The control protocol designed in [23] enables the multi-agent system states to track the dynamic target within a prescribed time $T$. However, due to a flaw in its controller, the tracking performance is no longer guaranteed after $T$. In contrast, the proposed controller (6) in this paper not only ensures that the system states track the target within the predefined-time $T$ but also maintains the tracking error at zero for all time $t > T$.

**4. Simulation**

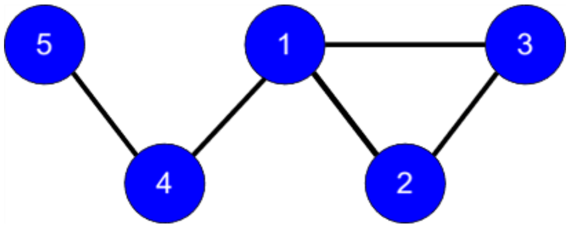


*Figure 1: Multi-agent system communication topology*

To validate the effectiveness of the proposed control protocol (6) for the multi-agent system (3), a multi-agent system consisting of 5 agents is considered, as illustrated in Figure 1. For simplicity and without loss of generality, it is assumed that the connection weights between the agents are all equal to 1. Building upon the preliminary introduction, the Laplacian matrix can be derived from the communication topology depicted in Figure 1 as follows

$$\mathrm{L} = \mathrm{D} - \mathrm{A} = \begin{bmatrix} 3 & -1 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 & 0 \\ -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

In this system, there are two agents capable of perceiving the position of the time-varying target, namely $k_i = 1(i = 1,3)$, while $k_i = 0(i = 2,4,5)$. Consequently, the interaction matrix is given by

$$\mathrm{M} = \mathrm{L} + \mathrm{K} = \begin{bmatrix} 4 & -1 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 3 & 0 & 0 \\ -1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

The minimum eigenvalue of the interaction matrix is calculated as $\underline{\lambda} = 0.2281$. The target position is set as $p(t) = 4 + 8sin(t)$, and thus $v_{\max}$ is computed as 12. The parameters in the control protocol (6) are selected as $r = 0.5$. The initial state values of the system are set to $x_0 = [-5,15,-7,11,-10]^T$. With these initial values, predefined-times $T = 1s$ and $T = 0.1s$ are chosen to verify that the convergence time can be arbitrarily selected. The simulation results are shown in Figure 2-3.

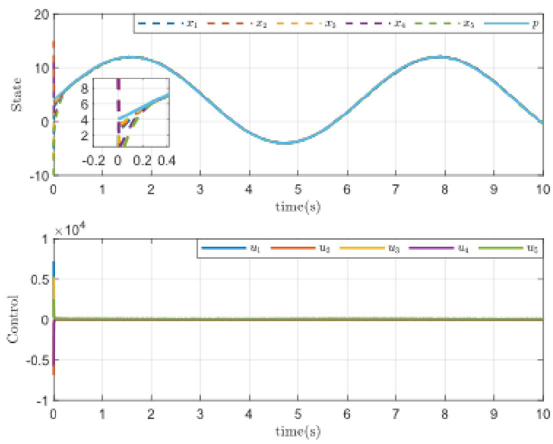


*Figure 2: Agent and target state & control at*

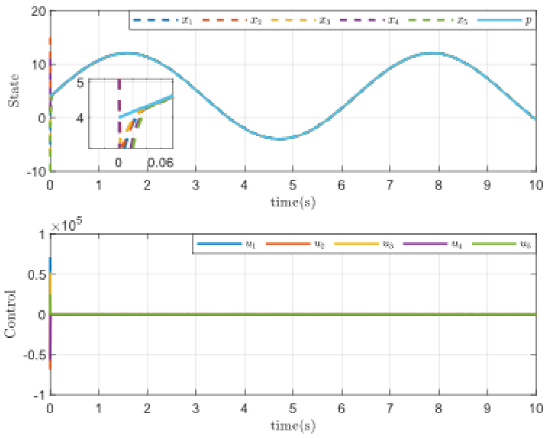


*Figure 3: Agent and target state & control at*

From Figure 2 and 3, it can be observed that regardless of whether the predefined-time is chosen as or, the agents are able to track the dynamic target within the predefined-time, and the actual convergence time is not conservative compared to the predefined-time.

## 5. Conclusion

This paper investigates the predefined-time consensus tracking problem for a class of single-integrator multi-agent systems. A predefined-time consensus tracking protocol is designed for the multi-agent system, ensuring that all agents can track the dynamic target within a prespecified time. Under the proposed protocol, the tracking convergence time is independent of the system's initial states and parameters, being solely determined by the designed controller gains.

**Appendix: the proof of Lemma 2**

Let $T_*$ be the time that $V$ first goes into the region $V=0$ (note that $V\left(T_*\right)=0$ ). Integrating (2) from 0 to $T_*$, we can obtain

$$\int_0^{T_*}\frac{\dot{V}dt}{V^{1+\frac{r}{2}}+V^{1-\frac{r}{2}}}\le-\int_0^{T_*}\frac{\pi}{rT_c}dt\Rightarrow T_*\le-\frac{rT_c}{\pi}\int_0^{T_*}\frac{\dot{V}dt}{V^{1+\frac{r}{2}}+V^{1-\frac{r}{2}}}.$$

Let $\vartheta=V$ with $\vartheta_0=V\left(0\right)$, one has

$$T_*\le-\frac{rT_c}{\pi}\int_{\vartheta_0}^{0}\frac{d\vartheta}{\vartheta^{1+\frac{r}{2}}+\vartheta^{1-\frac{r}{2}}}=-\frac{2T_c}{\pi}\int_{\vartheta_0}^{0}\frac{d\vartheta^{\frac{r}{2}}}{1+\vartheta^{r}}=-\frac{2T_c}{\pi}\left[\arctan\left(0\right)-\arctan\left(\vartheta_0^{\frac{r}{2}}\right)\right]\le T_c.$$

This completes the proof.

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