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Review Article

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A Review of Strength Criteria in Rock Mechanics and Their Applicability

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Abstract: As a natural engineering material, rock possesses characteristics such as heterogeneity, anisotropy, discontinuity, and a wide variety of types, with complex mechanical properties that are significantly influenced by hydraulic effects. Rock strength criteria are a crucial aspect of rock mechanics, providing important guidance for rock engineering design and construction. To date, hundreds of models and criteria have been proposed, and there are tens of thousands of research papers on the application of strength criteria, continually modifying and improving existing models. However, no universally applicable strength criterion has been found so far. Almost every strength criterion has its shortcomings and limitations, and existing criteria all have specific applicable ranges and conditions. This paper reviews the shear strength criteria and yield strength criteria, analyzing the adaptability of each strength criterion.

Keywords: Rock mechanics; Shear strength criteria; Yield strength criteria; Applicability.

1. Introduction

Rock mechanics is a newly developed discipline in modern times, and it is a fundamental subject with strong applicability and practicality. Its theoretical foundation is quite broad, involving solid mechanics, fluid mechanics, structural mechanics, elastoplastic theory, engineering geology, and geophysics. Its application range extends to civil engineering, hydraulic and hydropower engineering, mining, railways, highways, geology, seismology, petroleum, and many other fields related to rock mechanics and engineering[1]. It is the collaborative effort of multiple disciplines and the practical activities in various related fields that have continuously promoted the improvement and development of rock mechanics. Rock strength theory is used to determine under what stress and strain conditions rock samples or rock engineering will fail[2]. Of course, rock failure is influenced by various factors such as temperature, strain rate, humidity, strain gradient, etc. Extensive theoretical research and experimental verification have been conducted on strength theories. To date, hundreds of models or criteria have been proposed, but no single model or criterion has been universally accepted. Therefore, it is necessary to conduct in-depth studies on the accuracy of each strength criterion in describing rock strength, explore its applicability, and ensure the objectivity and scientific nature of the application of strength criteria.

2. Current Status of Strength Theory Research

Strength theory studies the yield and failure patterns of materials under complex stress conditions. As a fundamental theory in rock mechanics, it is used for predicting and verifying rock strength, determining whether a rock will fail under a certain stress state. Rock strength theory has long been a hot topic in the field of geotechnical engineering [3]. To date, hundreds of models and criteria have been proposed, and there are tens of thousands of research papers on the application of strength criteria, continually modifying and improving existing models. However, a universally applicable strength criterion has not yet been found. Nearly every strength criterion has its shortcomings and defects, and existing criteria have specific applicable ranges and conditions. Therefore, it is necessary to conduct in-depth research on the accuracy of various strength criteria in describing rock strength, discussing their applicability, and ensuring the objectivity and scientific validity of their use.

As excavation and mining depths continue to increase, rocks at deep (below 2 km depth) or ultra-deep locations exhibit mechanical characteristics that are influenced by geophysical field characteristics, namely changes in the geostress field, geothermal field, and underground water seepage field [4]. Under high temperature and high stress conditions, the strength characteristics of rocks become more complex. Whether classical strength criteria can accurately describe the strength characteristics of deep rocks, and how to objectively describe the strength properties of rocks under high-stress conditions in deep environments, has become a hot topic in engineering in recent years.

The study of rock mass strength theory began as early as the 18th century. Rock mass strength refers to the stress or strain state at which the rock mass fails, or the ultimate capacity of the rock mass to resist failure. Failure is a specific stage in the deformation process of the rock mass. In the context of a practical engineering project, it refers to the loss of the rock mass's ability to carry the expected load [5]. Clearly, the specific phenomena that lead to failure depend on the nature of the load it is bearing. The strength and failure of a rock mass are closely related. Generally, if a specific stress component is increased under certain conditions until failure occurs, the stress value at the point of failure is termed as the material strength under those conditions. The strength under simple stress states can be determined through experiments, while the strength under complex stress states is determined by strength theory. Rock strength criteria, also known as failure criteria, study the relationship between the stress state at the ultimate stress conditions and the rock strength parameters. Typically, this relationship is expressed through the principal stress equation under the limit stress condition:

$$f(\sigma_1, \sigma_2, \sigma_3) = 0$$

Alternatively, it can be expressed through the relationship equation between the shear stress and normal stress on a plane in the state of ultimate equilibrium:

$$t = f(\sigma)$$

During the more than one hundred years of development in rock mass strength theory, many experts and scholars have proposed numerous valuable strength criteria, contributing to the advancement of rock mechanics. Based on their theoretical research and methodologies, rock mass strength theory can be divided into two main categories: "theoretical strength criteria" and "empirical strength criteria."

The former is based on the knowledge system of material mechanics and elastic mechanics and includes four classical strength theories: Maximum Normal Stress Strength Theory, Maximum Normal Strain Theory, Maximum Shear Stress Theory, and Octahedral Shear Stress Theory. Other theories include Mohr-Coulomb Strength Theory, Griffith and Modified Griffith Theory, and the Double Shear Strength Theory. The latter category relies primarily on experimental methods to approximate the description of rock failure mechanisms. Notable examples of empirical strength criteria include the Hoek-Brown empirical strength criterion, among others. There are many rock strength criteria applicable to petroleum engineering, but the suitability of specific criteria for deep rock still requires further discussion. Yu Maohong summarized various strength theories, which can generally be categorized into three main types: Single Shear Strength Theories, such as the classical Mohr-Coulomb failure criterion and the Tresca yield criterion. Double Shear Strength Theories, including the generalized double shear failure criterion and the double shear yield criterion. Triple Shear Strength Theories or Octahedral Shear Stress Strength Theories, such as the Drucker-Prager failure criterion and the Huber-von Mises yield criterion. Among these three categories, the lower bound of all convex theories corresponds to the linear single shear strength theories. These theories are also equivalent to the lower bound of linear double shear strength theories, which represent the upper bound of all convex theories. Between single shear strength theories and double shear strength theories lies the nonlinear octahedral shear stress strength theories. Additionally, there are various other nonlinear criteria, each tailored to specific types of materials and applied within their respective domains. In 2007, Yu Maohong proposed a unified strength theory that includes both linear and nonlinear forms. This unified theory incorporates a broader range of yield and failure criteria, extending its applicability to a wider array of materials. Furthermore, he summarized 12 derivative strength criteria based on this unified framework, further enhancing the versatility and scope of strength theory.

3. The research progress of shear strength criteria.

Coulomb-Navier criterion

Since Mohr proposed the theory of rock strength in 1900, over a century has passed. During this period, numerous scholars worldwide have studied and advanced rock strength theories. This paper focuses on the Coulomb-Navier failure criterion, which is currently the most widely used and simplest criterion in rock mechanics, structural geology, and geomechanics. It is commonly employed to interpret the intersection angle of two sets of shear fractures in rock masses, where the acute angle points toward the direction of maximum compressive stress.

The Coulomb-Navier criterion posits that the failure of rock is a type of shear failure under normal stress. This failure is not only related to the shear stress on the shear plane but also to the normal stress acting on that plane. Rock does not fail along the plane of maximum shear stress; instead, failure occurs along a plane where the combination of shear stress and normal stress is most unfavorable [6]. That is:

$$|\tau| = C + \sigma \tan \varphi$$

This criterion is applicable to materials whose compressive strength is greater than their tensile strength. When considering the pore fluid pressure pp, according to the effective stress law, the pore fluid pressure only reduces the normal stress on any given section but has no effect on the shear stress acting on that section. Therefore, the Coulomb-Navier failure criterion can be expressed as:

$$|\tau_f| = \tau_0 + f(\sigma_n - p)$$

YouHou employed a seismic activity simulation program based on the Coulomb-Navier failure criterion and stress transfer mechanisms, incorporating extensions to improve computational efficiency. After testing the model with two vertical parallel fault models, it was applied to the Taiyuan region. The frequency-magnitude data from the simulation results showed a good fit with the observed catalog, while also tending to follow the characteristic earthquake theory rather than the Gutenberg-Richter law. Additionally, single fault regions with higher slip rates appeared to exhibit lower b-values, indicating a higher risk of strong earthquakes. Data fitting between the simulation results and the corresponding parameters of the Poisson process was also conducted, along with energy conservation tests.

Mohr's failure criterion

Experimental evidence has shown that the relationship between the maximum principal stress and the minimum principal stress is approximately linear under low confining pressure. However, as the confining pressure increases, this relationship becomes distinctly nonlinear [7]. To reflect this characteristic, Mohr's criterion establishes the failure criterion equation based on compression-shear and triaxial failure experiments, namely:

$$|\tau| = f(\sigma)$$

This equation can be specifically simplified into various curve forms, such as inclined straight lines, hyperbolas, parabolas, cycloids, and double-inclined straight lines, depending on the experimental results. Although, from a formal perspective, the difference between the Coulomb criterion and the Mohr criterion lies only in the latter's extension from straight lines to curves, the Mohr criterion expands or extends the envelope into the tensile stress region.

Mohr-Coulomb Strength Criterion

Since the establishment of the Mohr-Coulomb strength criterion in 1900, it has made significant contributions to strength calculation, design, and the development of applied mechanics in engineering structures. Today, it has become one of the most fundamental topics in rock mechanics literature. However, it is still being developed, tested, and discussed by scholars both domestically and internationally, leading to a series of modified criteria [9].

He Zhijun and others, based on the nonlinear Mohr-Coulomb (referred to as nonlinear M-C) failure criterion, combined the limit analysis upper bound method and the Monte Carlo method to perform an upper bound reliability analysis of slopes. The research shows that under the nonlinear M-C failure criterion, the slope reliability decreases as the initial cohesion, internal friction angle, and the variability of nonlinear parameters increase. Slope reliability increases with an increase in initial cohesion and internal friction angle, while it decreases with an increase in the nonlinear parameter.

Fang Liang and others, through comparative analysis, derived failure criteria for masonry under shear-compression combined action, based on the maximum principal stress theory, shear friction theory, Mohr theory, and strain energy theory. Considering the differences between microelements and the macro wall model, they established corresponding shear capacity calculation formulas for masonry based on various failure criteria[10].

Ding Tao and others, in rock slopes, used the Mohr-Coulomb yield criterion for finite element strength reduction and found that soft rock slopes underwent through-failure, while hard rock slopes did not fail even under very high reduction factors. This indicates that the Mohr-Coulomb criterion is not applicable to the strength reduction of hard rock slopes. By applying tensile corrections to the Mohr-Coulomb criterion in the tensile-shear zone, they established a shear and tensile combined failure Mohr-Coulomb criterion. They then re-performed finite element strength reduction on the rock slope model, verifying the reliability of the theoretical results. The modified Mohr-Coulomb criterion demonstrates general applicability for rock slopes subject to both shear and tensile yielding.

Double Shear Strength Criterion

The double shear stress yield criterion was proposed by Chinese scholar Yu Maohong in 1961 as a new yield theory. After more than forty years of development and improvement, a series of double shear stress strength theories have been established. This theory suggests that, in addition to the maximum principal shear stress, the other principal shear stresses also influence the material’s yield. Since there are only two independent quantities among the three principal shear stresses, the theory considers the influence of the two larger principal shear stresses on material yield.

The yield surface in the principal stress space is an equidistant hexagonal pyramid, while in the shear plane, it forms an unequal hexagon with a vertex that is not located on the principal axis but is symmetric with respect to the principal axis. The Double Shear Stress Series Strength Theory, in addition to reflecting the effects of the maximum and second-largest principal shear stresses on yielding, also accounts for the influence of hydrostatic pressure, Lode's angle, normal stress on the shear plane, and varying material tensile and compressive strengths on yielding. It also demonstrates that the double shear yield criterion serves as the upper bound of various existing yield criteria on the shear plane.

4. The Progress in the Research on Yield Strength Criteria.

After the geotechnical material is loaded, as the load increases, it can be considered to go through three stages: elastic, plastic, and failure. The transition from the elastic state to the plastic state is called yielding. At this point, plastic strain begins to develop at a specific point inside the material until it reaches the infinite plastic state (where stress remains constant, but strain increases infinitely), which is termed failure. The conditions that must be met by stress or strain when the material begins to transition into the plastic state are called yield conditions, which define the boundary of the elastic state. Similarly, when a point inside the material transitions from the plastic state to the infinite plastic state, the material enters the failure state, and the stress-strain relationship must satisfy the corresponding failure conditions, which define the boundary of the plastic state[11].

In current geotechnical strength theory, there is a general recognition of the distinction between elasticity and plasticity. As a result, the methods used for calculations differ significantly, and adopting different yield criteria leads to different calculation results. Yielding marks the boundary between elasticity and plasticity, which is why numerous yield criteria have been proposed to meet various needs. In the stress space, the yield function appears in the form of a yield surface. It is a multivariable function of stress, strain, temperature, and time. The initial yield surface is called the initial yield surface, and its equation is as follows:

$$F\left(\sigma_{ij},\ \ \varepsilon_{ij},\ \ t,T\right)=0$$

Under normal circumstances, the yield condition is related to all six components of stress. If the rotation of the principal stress axes is not considered, it becomes related to the three principal stress components or invariants, which then defines the yield function.

$$\begin{cases} F(\sigma_1,\sigma_2,\sigma_3)=0 \\ F(I_1,I_2,I_3)=0 \\ F(I_1,J_2J_3)=0 \\ F(\sigma_m,J_2,\theta_\sigma)=0 \\ F(p,q,\theta_\sigma)=0 \end{cases}$$

Tresca Yield Criterion

The Tresca criterion, proposed by Tresca in 1864, is the earliest yield criterion applied to metallic materials in traditional plasticity theory. This criterion assumes that yielding occurs when the maximum shear stress in the material reaches a certain limit value kk. Its mathematical expression is:

$$F = \tau_{max} = max(\frac{1}{2} \mid \sigma_1 - \sigma_2 \mid, \frac{1}{2} \mid \sigma_2 - \sigma_3 \mid, \frac{1}{2} \mid \sigma_3 - \sigma_1 \mid) = k$$

Mises yield criterion.

The Von Mises criterion was proposed in 1913 based on experimental results. It assumes that yielding begins when the shear stress on the octahedron in the stress space reaches a certain limit. Additionally, this criterion can also be described from the perspective of energy: it indicates that yielding occurs when the sum of the energies on three shear planes of the material reaches a critical value. Its expression is:

$$F = J2 - C = 0$$

Drucker-Prager criterion.

The renowned American scholars Drucker and Prager proposed a strength criterion that considers the effect of hydrostatic pressure in 1952, known as the D-P strength criterion. This criterion approximates the M-C strength criterion by the size of the Mises circle on the π-plane, leading to different D-P series yield criteria. Since the mathematical expression of this criterion includes the intermediate principal stress and hydrostatic pressure, it can reflect the geotechnical friction strength characteristics and has a higher computational efficiency, which has gained popularity among scholars both domestically and internationally [12].

The brittle fracture theory (Griffith criterion).

As early as 1921, Griffith proposed the concept of cracks and their initiation and propagation in materials, explaining the causes of low-stress brittle fracture in many materials, and pointed out an important path for the in-depth study of the fracture failure mechanism of rocks. In 1924, Griffith provided a two-dimensional criterion based on the ideal brittleness assumption [13].

Unlike the classical rock strength theories, the Griffith theory and its modifications and extensions are based on the presence of microcracks within rocks. This opened up a completely new field for the study of rock strength characteristics, namely, the study of fracture strength theory.

Hoek-Brown criterion

The Hoek-Brown strength criterion, developed in the 1970s, was later identified as a criterion for evaluating whether rock masses are damaged, and it has been widely used in underground project design [14]. Currently, the Hoek-Brown strength criterion is not only widely applied in underground projects but is also suitable for the stability analysis of rock slopes. It can be applied to both rocks and rock masses, with parameters obtained through standard laboratory tests, mineral composition, and discontinuity surface descriptions. Compared to the Mohr-Coulomb failure criterion, the Hoek-Brown strength criterion considers the physical and mechanical properties of the rock mass, as well as factors like structural surfaces, providing a more composite constraint. Therefore, this criterion can more accurately describe the failure characteristics of rock masses, making it more aligned with real-world conditions, reducing deviations [15]. At the same time, this criterion exhibits excellent adaptability for rock masses in low-stress regions or those subjected to tensile stresses, making it more suitable for describing the failure characteristics of rock masses in real-world situations. However, the Hoek-Brown criterion is significantly influenced by human subjective factors, which may lead to inaccuracies in determining strength parameters during research, resulting in human-induced deviations.

5. Conclusion

Each strength criterion has certain issues, and current strength standards have specific application ranges and requirements. Therefore, it is necessary to carefully analyze the accuracy with which these strength criteria describe the strength of rock materials, explore their applicability, and ensure the rationality and appropriateness of their use.

The Mohr-Coulomb failure criterion has a relatively simple equation, with clear physical meaning and widespread acceptance. It can be broadly applied in the design of rock mass projects and geological structural studies. It is one of the most classic strength theories in rock mechanics within the geotechnical engineering field. However, the Mohr-Coulomb criterion does not consider the influence of the intermediate principal stress, and it presents singular angles in the plane, which are not smooth functions. When plastic flow occurs, it is difficult to define the direction of incremental changes near sharp corners under yield conditions. While it is relatively safe to use, its calculation results tend to be conservative.

The Drucker-Prager strength criterion is very simple, with very few parameters, and considers the influence of the intermediate principal stress and hydrostatic pressure. It is smooth throughout the π-plane without singular points, making it convenient for defining plastic flow increments. The material's elastic-plastic constitutive relationship can be easily determined, which is why this yield criterion is widely applied. However, the criterion assumes that the intermediate principal stress is equal to the minimum principal stress, which leads to more conservative, potentially unsafe results.

The Hoek-Brown strength criterion reflects the nonlinear failure of rock masses and considers the effects of tensile stress, low stress, and confining pressure on strength. It is suitable for characterizing anisotropic rock masses and is widely used in the design of rock excavation, slopes, and underground chambers. It is the most widely applied rock strength criterion to date and has significant constraint capabilities. However, this strength criterion does not account for the influence of the intermediate principal stress, and its parameters are highly subjective, making it difficult to define them precisely.

The Mohr-Coulomb criterion is a set of linear equations that describes failure in isotropic materials in the principal stress space. It explains well the characteristic that rock materials have much greater compressive strength than tensile strength, and it has become fundamental content in rock mechanics textbooks, widely applied in field engineering practice. However, the MC criterion does not consider the influence of the intermediate principal stress σ_2, and Yu Maohong refers to it as the single-shear strength theory (SSS criterion). It does not account for the effects of structural planes, making it unsuitable for tensile fracture, expansion, creep failure, and splitting failure.

The DP criterion has the advantage of a simple expression and smooth curves, especially the modified DP criterion, which has symmetry in the yield surface in the stress space, making it convenient for programming and implementation in numerical computations. It considers the effect of the intermediate principal stress on material strength. The main limitation of this criterion is that it tends to overestimate rock strength under general stress conditions and produces significant errors in triaxial tension. Additionally, while the DP criterion can describe the uniaxial tensile strength of rock with appropriate parameter selection, it has substantial errors when one or more principal stresses are tensile.

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