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Novel Concepts and Modeling of Devices Based on Nonlinear Phase Contrast Technique

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Abstract: Nonlinear Phase Contrast Technique has recently been in focus among the researchers working in nonlinear optics. This paper presents the detailed scientific analysis of the related phenomena and important breakthroughs in the subject, especially nonlinear phase contrast technique, autoresonance in nonlinear systems, nonlinear optics in high-Q optical microspheres, photonic nanowires, and quantum cascade laser systems The technical analysis of these parameters, and modeling for of the devices based on them have been briefly presented. The paper should be useful for the researchers and designers in this evolving field.

Keywords: Nonlinear Phase Contrast, Autoresonance in nonlinear systems, Nonlinear optics in photonic nanowires, Nonlinear optics in high-Q optical microspheres, Four wave mixing conversion efficiency, Self-calibrating common-path interferometry, Spiral phase contrast imaging in nonlinear optics

1. Nonlinear Optics

The field of nonlinear optics is really very interesting and has drawn the attention of various researchers for many applications. The nonlinear optics is different from the linear optics, in the sense that whereas the linear optics is the ‘optics of weak light’, in which the light is deflected or delayed but its frequency is unchanged; the nonlinear optics is the ‘optics of intense light’, in which the effects induced by the light itself, as it propagates through the medium, are observed and studied. In case of the nonlinear region, the interaction of light in matter is studied, and it is observed that it is possible to control the refractive index by the light itself, and also to manipulate one beam with the other, which in fact leads to many technical innovations. However, this nonlinearity is observed only at very high light intensities i.e. values of the electric field must be comparable to the interatomic electric fields, typically ~ 108 V/m, and these are provided by pulsed lasers. Recently, Thomson et al (1) have presented detailed description of many topics in nonlinear optics. Payam et al (2) have made a very interesting study of the Type-0 second order nonlinear interaction in monolithic waveguides of isotropic semiconductors, Recently, a lot of interest has been shown in the studies (3-11) connected with the different aspects of nonlinear phase contrast., as the technique has a great potential for very useful applications including all-optical information processing systems, and the imaging neuronal activity.

Porras-Aguilar et al (3) have investigated Self-calibrating common-path interferometry. They have presented a quantitative phase measuring technique that estimates the object phase from a series of phase shifted interferograms that are obtained in a common-path configuration with unknown phase shifts. They have derived random phase shifting algorithm for common-path interferometers, which is based on the Generalized Phase Contrast theory [pl. Opt. 40(2), 268 (2001)10.1063/1.1404846], which accounts for the particular image formation and includes effects that are not present in two-beam interferometry. They have shown experimentally that this technique can be used within common-path configurations employing nonlinear liquid crystal materials as self-induced phase filters for quantitative phase imaging without the need of phase shift calibrations, and have also emphasized on the advantages of such liquid crystal elements compared to spatial light modulator based solutions e.g. self-alignment, and the generation of diminutive dimensions of the phase filter size, giving unique performance advantages. Their Schematic of the nonlinear phase contrast microscope with a polarization-controlled phase shift, used by them has been given below:

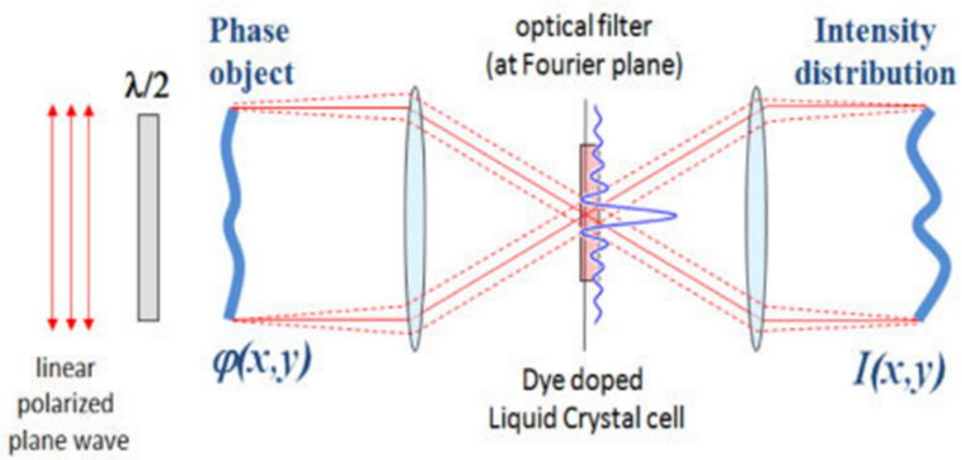

Figure 1: Schematic of the nonlinear phase contrast microscope with a polarization controlled phase shift. Figure courtesy Schematic of the nonlinear phase contrast microscope with a polarization controlled phase shift. Figure courtesy Porras-Aguilar Rosario, Falaggis Konstantinos, Ramirez-San-Juan Julio Cesar, Ramos-Garcia Ruben, Self-calibrating common-path interferometry, Optics Express, February 2015, 23(3):3327, DOI:10.1364/OE.23.003327A.

Molecular nonlinear optics relate optical properties of bulk matter to their microscopic molecular properties. Just as the polarizability can be described as a Taylor series expansion, one can expand the induced dipole moment in powers of the electric field:

$$\mu = (\mu_0 + \alpha E + 1/2\, \beta\, E.E + \dots) \qquad (1)$$

where μ is the polarizability, α is the first hyperpolarizability, β is the second hyperpolarizability, and so on.

2. Important Experimental Breakthroughs and Results in Nonlinear Phase Contrast

Some important experimental breakthroughs and results in Nonlinear Phase Contrast have been discussed below:

Energy vs. time variation in Autoresonant Pendulum

Autoresonance is a very interesting phenomenon of nonlinear physics, in which a perturbed nonlinear system is made to resonate and stay phase-locked with the perturbing oscillations/waves continuously, even if the system's parameters are changed. This continuous phase-locking, also termed as adiabatic synchronization, implies that the controlled excursion in system's solutions space takes place, resulting in the frequent emergence of coherent structures. Friedland (4) has investigated in detail the Autoresonance in nonlinear systems, by considering the case of the Autoresonant Pendulum, and studying the energy vs. time variation. His results are very interesting, which have been reproduced below:

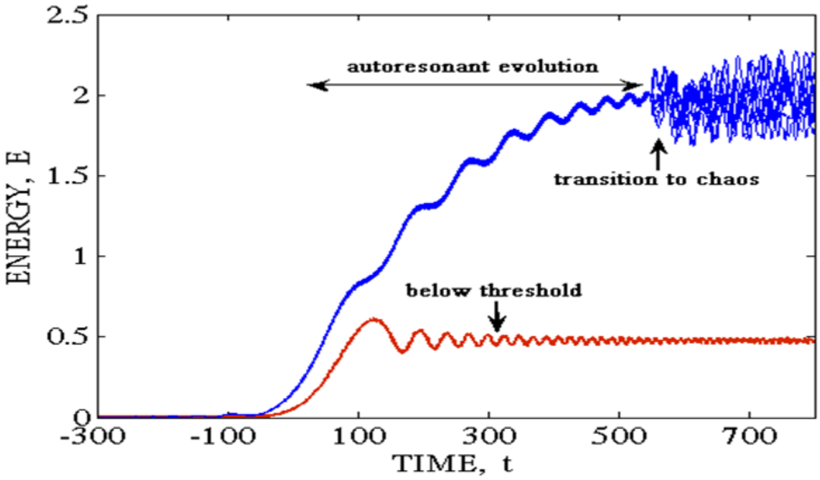

Figure 2: Energy E (A U) vs time (s) variation for autoresonant pendulum; Figure courtesy Friedland Lazar, Scholarpedia, 4(1) (2009) 5473.

Energy of the driven pendulum vs. time for 10 different initial driving phases has been illustrated by applying the autoresonant approach to stochastic instability. The bottom curve corresponds to the case of below threshold (Sub-threshold evolution); and the top curve represents autoresonant evolution up to a certain value of time, after which it shows the transition to chaos.

The analysis of the autoresonance is done by following the simple approach (4), in which the driven pendulum is described by the equation:

$$d2u/dt2+ \sin u = \varepsilon\cos\varphi(t) \qquad (2)$$

where the driving perturbation has constant amplitude ε, and a slowly varying frequency: $\omega(t)=d\varphi/dt$.

The figure illustrates the evolution of energy $E = [1/2.\ \{(du/dt)-\cos u+1\}^2]$ of the pendulum (starting in equilibrium $u = 0$ at $t =-300$), as the driving frequency(ω) varies in time given by : $\omega(t) = 1 - \alpha t$ (α being the frequency chirp rate), and passes the linear resonance at $t = 0$. These results are for the parameters α=0.001 and ε=0.03, which are considered to be the most practical values. It has to be noted that the threshold for autoresonance is a weakly nonlinear phenomenon, which for a driven, weakly nonlinear pendulum may be represented mathematically as:

$$(d2u/dt2+u-\beta u^3) = \varepsilon\cos\varphi \qquad (3)$$

where $\varphi = t-(1/2/\alpha t^2$ (chirped frequency drive), and ε, β, and α, are respectively the driving amplitude, nonlinearity parameter of the pendulum, and the driving frequency chirp rate. The simple solution to this problem is in the form a single parameter nonlinear Schrodinger-type equation:

$$id\Psi/d\tau + (|\Psi|^2 -\tau)\Psi=\mu \qquad (4)$$

which describes the capture into autoresonance, i.e. transition to phase locked solution $\Phi\rightarrow 0$, $A\rightarrow \sqrt{\tau}$, as τ passes from $-\infty$ to $+\infty$ through the linear resonance at τ=0, τ being the rescaled time $\tau =(\alpha.1/2.t)$. Thus, it is clear that the threshold for autoresonace has to be calculated by making a proper choice of driving amplitude, nonlinearity parameter of the pendulum, and the driving frequency chirp rate.

Optical parametric oscillations and Comb generation dynamics

High-Q optical microcavities can be used for the resonance enhancement of the nonlinear effect in certain materials. A lot of interest has been shown in conducting the studies on this interesting topic. A very important development has taken place - the wavelength conversion at ultra-low pump powers in micrometer-size photonic devices has been demonstrated (5). This capability is useful in the sense that it enables the integration of nonlinear optical components on-chip for all optical functions like wavelength conversion, signal regeneration, and switching. In addition, the Conversion efficiencies of the order of 0 dB are achievable by operating a reverse biased PIN device with waveguides exhibiting low anomalous group-velocity dispersion (GVD), which has led to the possibility of the integrated silicon optical parametric oscillators and optical frequency combs. The achievement of this efficiency in a silicon CMOS-compatible photonic device in a practical, continuous-wave, low-power regime has led to the efforts in developing an all-optical on-chip communication and signal-processing scheme, by overcoming the bandwidth limitations of CMOS electronics. By combining this enhancement and the high transverse confinement achieved in nanowaveguides, four wave mixing has been observed (5) with extremely low peak powers (<below 1-mW) in the dispersion engineered silicon microrings.

Silicon nitride is another CMOS-compatible material, which is considered very promising for nonlinear optics due to its (i) large nonlinear refractive index, (ii) low loss, and (iii) characteristic of showing the absence of the two photon absorption at the communication wavelengths. Recently, experiments have been conducted, and the parametric gain has been demonstrated in nitride microrings, which has resulted in achieving optical parametric oscillation (6) and frequency comb generation with spectral ranges spanning an octave (7). The on-chip, ultrahigh-bandwidth optical networks can now be fabricated by using Silicon photonics, which are very useful for the microelectronics, and also critical for its future. The field has witnessed so much progress that many optical components required for implementing a wavelength division multiplexing network have already been demonstrated in silicon. Still, for a long time after these developments, a fully integrated multiple-wavelength source capable of driving such a network had not been realized. Levy et al (6) have demonstrated the first monolithically integrated CMOS-compatible source in the form of an optical parametric oscillator fabricated by a silicon nitride ring resonator on silicon, which can generate more than 100 new wavelengths with operating powers < 50 mW, and hence is considered to be extremely useful for a high-bandwidth optical network on a microelectronic chip.

Okawachi et al (7) have demonstrated a frequency comb spanning an octave by the parametric process of cascaded four-wave mixing in a monolithic, high-Q silicon nitride microring resonator, the comb being generated from a single frequency pump laser at 1562nm, spanning 128THz and having a spacing of 226 GHz, which is tunable slightly with the pump power. The RF amplitude noise characteristics of the parametric comb have also been investigated and it has been observed that the comb can be made to operate in a low noise state with a 30dB reduction in noise, by tuning the pump frequency into the cavity resonance.

The results of Levy et al (6) and Okawachi et al (7) as available in the literature have been reproduced below:

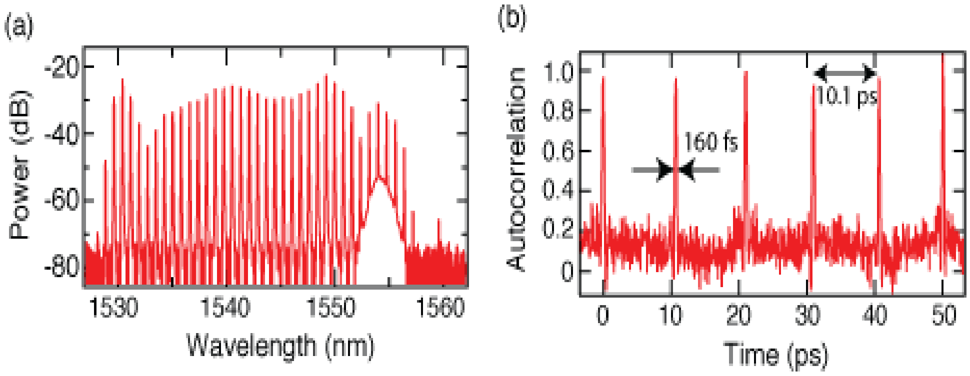

Figure 3: Variation of power (dB) with wavelength (a); and Variation of Autocorrelation with time (pS); Figure courtesy Levy et al, Nature Photonics 4, 37 - 40 (2010), and Okawachi et al, Opt. Lett. 36 (2011) 3398-3400.

The optical parametric oscillations and the various peaks in the autocorrelation vs. time curve (width ~ 160fs at half height and spacing ~ 10.1ps) can be clearly seen.

Actually, this has been the first demonstration of a CMOS-compatible monolithically integrated source, and this technology has useful applications in the high bandwidth integrated optical networks. Subsequently, this comb generation has also been achieved (8) at different wavelengths, and with different comb spacings . Saha et al (8) have reported the demonstration of broadband frequency comb generation from a single-frequency pump laser at 1-µm using parametric oscillation in a high-Q silicon-nitride ring resonator. Also, the resonator dispersion has been engineered to have a broad anomalous group velocity dispersion region near the pump wavelength for the efficient parametric four-wave mixing, with the comb spanning 55 THz, and a 230-GHz free spectral range. On the basis of the results demonstrated, it has been emphasized that dispersion engineering in chip-based devices for producing combs with a wide range of pump wavelengths has many advantages in the possible applications. Their results have been reproduced below:

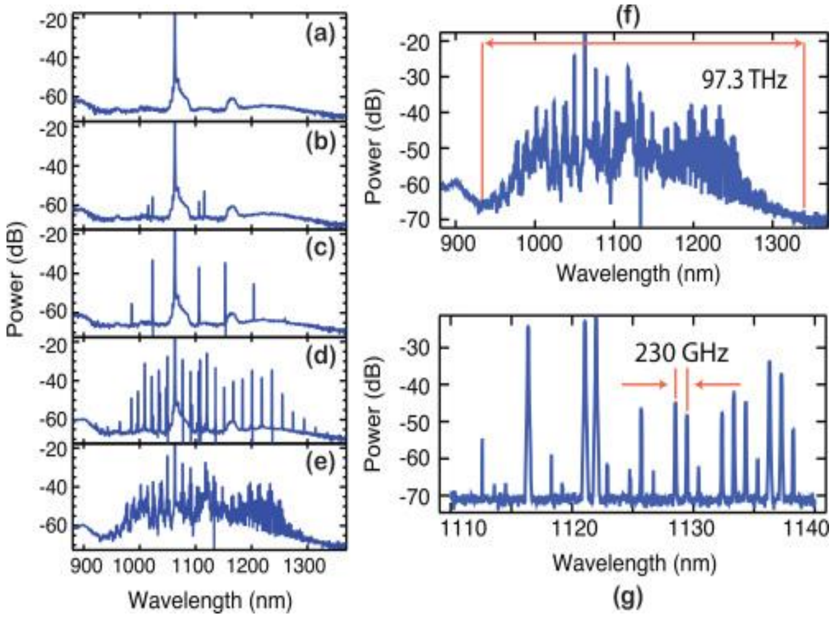

Figure 4: Comb generation dynamics (left – a to e); comb spanning 97.3 THz with a spacing of 230 GHz (right - f); and A zoomed in spectrum at the longer wavelength (right- g). Figure courtesy Saha et al , Opt. Express, 20 (2012) 26935-26941.

This figure shows the Comb generation dynamics from top to bottom (left – a to e); with a pump laser at 1064 nm. It has been observed that as the power oscillating inside the micro ring increases, and the threshold is reached, the cavity modes having maximum phase-matching experience gain and hence start oscillating. On tuning the pump deeper into resonance, and also further increasing the power in the side modes, cascaded four-wave mixing takes place, resulting in the multiple cascaded oscillations, and also the development of a wide bandwidth comb The curve (right f) shows the Frequency comb spectrum generated with 2 W of pump power, the comb spanning 97.3 THz with a spacing of 230 GHz; and finally the curve (right g) displays a zoomed in spectrum at the longer wavelength end, where the comb teeth do not appear at every free spectra range (FSR), because of the fact that the phase-matching is not proper, and also there is power buildup.

By fabricating high- Q silicon-nitride spiral resonators, Johnson et al (9) have demonstrated the frequency combs spanning over 200 nm with FSRs of 80, 40, and 20 GHz by using cascaded four-wave mixing. Besides, they have characterized the RF beat note for the 20 GHz FSR comb, and have verified that the measured linewidth of 3.6 MHz is consistent with thermal fluctuations in the resonator due to the amplitude noise of the pump source. It has been concluded that the combs represent an important advance towards developing a CMOS-based system capable of linking the optical and the electronic regimes.

Also, a transition of these generated frequency combs to a mode-locked state has been observed (10), which leads to the formation of the coherent pulses in the time domain. Saha et al (10) have investigated simultaneously the temporal and optical and radio-frequency spectral properties of parametric frequency combs generated in silicon-nitride microresonators, and have observed that the system undergoes a transition to a mode-locked state. They have also demonstrated the generation of sub-200-fs pulses at a repetition rate of 99 GHz, and have stated on the basis of their calculations that the pulse generation in this system is consistent with the soliton modelocking. It has been emphasized that such parametric devices have the potential of producing ultra-short laser pulses from the visible to mid-infrared regime at repetition rates ranging from GHz to THz.

Lamontet al (11) have performed the first theoretical modeling of the full spectral–temporal dynamics of octave-spanning parametric microresonator comb generation through use of the Lugiato–Lefever model extended to include higher-order dispersion and self-steepening, and have shown that the three distinct stages are required to achieve single-pulse modelocking, besides discussing the dispersion characteristics needed for the ultrabroadband, stabilized comb generation. It has been concluded that their simulations agree well with the already reported experimental demonstrations, and also are able to predict many observed features, including multipulse generation, dispersive wave generation, modelocking, and comb stabilization.

Badrodien et al (12) have been able to achieve Improved image contrast in nonlinear light-sheet fluorescence microscopy using i2PIE Pulse compression Nonlinear microscopy, which has become an invaluable tool for biological imaging, offering high-resolution visualization of biological specimens. They have presented the application of a spectral phase measurement technique, i^2PIE, to compress broad-bandwidth supercontinuum pulses for two-photon excitation fluorescence light-sheet fluorescence microscopy. Their results have shown a significant improvement in the two-photon excitation response achieved. They have emphazed that the implementation of i2PIE allowed for enhanced image contrasts when compared to conventional compression techniques, with i^2PIE producing an image contrast improvement over conventional methods by about 50%>

Sharply autofocused ring-Airy beams transforming into non-linear intense light bullets

It is very difficult to control the propagation of intense optical wavepackets in transparent media , since during propagation, low- high-order non-linear effects, including the Kerr effect, and multi photon absorption and ionization, result in an uncontrolled complex reshaping of the optical wave packet, involving pulse splitting, refocusing cycles in space and also considerable focus variations. It has been established that new type of light beam called a ring-Airy beam is able to self-focus into intense light bullets which propagate over extended distances. These well-defined, high-intensity optical wavepackets have a great potential for the applications in several areas, like laser micromachining and harmonic generation. Panagiotopoulos et al (13) have studied the sharply autofocused ring-Airy beams transforming into non-linear intense light bullets, and their results have been shown below:

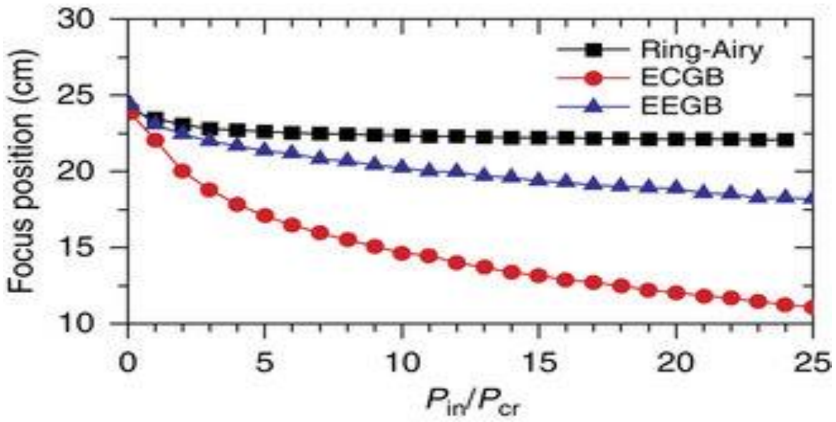

Figure 5: Focus shift in the non-linear regime. Figure courtesy Panagiotopoulos et al, Nature Commun. 4, Article No. 2622, doi:10.1038/ncomms3622, 17 Oct. 2013.

The Focus position as a function of the input power for the autofocused ring-Airy beam (top curve) is quite flat, as compared to the curves for the two types of Gaussian beams- EEGB (middle curve), and ECGB (bottom curve). This clearly shows the effectiveness of the autofocusing technique for the successful control of the propagation of intense optical wave packets in transparent media. As is clear, there is negligible focus shift in the region of the values of the ratio Pin/Pcr (Pin being the input power, and Pcr the critical power for self-focusing) from 0 to 5; and practically no shift at all from 5 to 25. It is observed that the power of this self-trapped solution is exactly equal to one critical power for self-focusing, Pcr; and a beam with larger power is found to collapse at a finite distance on-axis. Also, the experimental results have revealed that the self-focusing of optical beams in Kerr media lead to a catastrophic collapse for Pin > Pcr, and diffraction for Pin < Pcr. Nonlinear losses result in arresting the collapse, and generating the ring structures, which are azimuthally unstable. In case of the peak power exceeding the critical power for self-focusing, the formation of one or several narrow structures is observed, with a hot core (diameter less than 100 microns), which are able to propagate over extended distances without diffraction This regime is called femtosecond filamentation and is the subject of an intense research and development. Femtosecond filamentation is useful because of the spatial and temporal localization of the pulse energy associated with high intensities over long propagation distances

However, the beams carrying powers more than Pcr have a tendency to break transversally into multiple filaments. These filaments in transparent media have great advantages, which make them useful for various applications in many fields such as nanotechnology and remote sensing of the atmosphere.

It is now understood that the Imaging for weak-phase objects is a challenging issue in the linear imaging process. With this view, Wang et al (14) have demonstrated a high-contrast phase imaging method based on a nonlinear holographic hot image model. As discussed by them that due to the nonlinear Kerr effect, the holographic hot image can transform a weak phase into strong amplitude as a signal amplifier, and the phase information is iteratively obtained from the light field distribution of the holographic hot image. By their study, they have observed that the strong signal-to-noise ratio helps improve the imaging contrast. Using a tunable photorefractive crystal,They have numerically and experimentally demonstrate the advantage of this method for imaging weak-phase objects.These observations should prove to be very useful for further research applications in this field.

Qiu et al (15) have studied Spiral phase contrast imaging in nonlinear optics for seeing phase objects using invisible illumination. Though Spiral phase contrast (SPC) imaging offers a vital, convenient tool for edge detection in image processing, and also despite significant experimental and theoretical progress in this area, SPC imaging with invisible light is still lacking. With this objective in view, in contrast to the general SPC scheme, Qiu et al (15) have constructed a nonlinear spatial filter by equivalently imprinting the vortex phase plate onto the potassium titanyl phosphate (ktp) crystal using second harmonic generation (SHG). In this technique, the phase or intensity objects are displayed by a spatial light modulator (SLM), and illuminated with 1064 nm infrared light. Therefore, the combination of the nonlinear filter with SHG in the Fourier domain enables concise, and highly efficient SPC imaging, leading to a visible edge enhancement with invisible illumination. They have also demonstrated the capacity of their scheme to detect edges and contours in real time by programming a running dog cartoon with SLM, Their results have the potential of finding direct applications in infrared monitoring. Their employed system in this technique is as shown below:

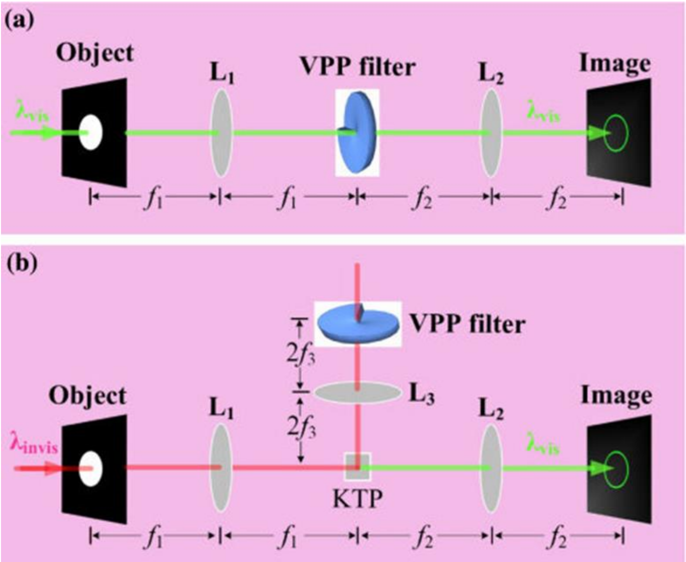

Figure 6: Schematic illustration of spiral phase contrast imaging. (a) The generic case in linear optics. (b) The case in nonlinear optics, Figure courtesy Qiu Xiaodong, Li Fangshu, Zhang Wuhong, Zhu Zhihan, and Chen Lixiang, Spiral phase contrast imaging in nonlinear optics: seeing phase objects using invisible illumination, Optica, Vol. 5, Issue 2, pp. 208-212, (2018) , https://doi.org/10.1364/OPTICA.5.000208.

Following the approach given by Khonina et al (16) and Abramowitz and Stegun (17) , the input object, can be mathematically described by $E\text{in}(r,\phi)$, which is illuminated by a plane wave, generally, with a visible wavelength of λvis. The first lens L1 carries out the Fourier transform, so that the spatial Fourier spectrum of the object can be written as: $\tilde{E}\text{in}(\rho,\varphi)=\mathscr{F}[E\text{in}(r,\phi)]$, in its back-focal plane. The VPP filter has to be located in the Fourier plane, and its transmission function can be written as: $F(\rho,\varphi)=\text{circ}(\rho/R)\exp(i\varphi)$, with $\text{circ}(\rho/R)$ being the aperture of radius R. Further, the total field in the Fourier plane can be written as:

$$\tilde{E}f(\rho,\varphi)=\tilde{E}\text{in}(\rho,\varphi)F(\rho,\varphi). \qquad (5)$$

The second lens L2 reverses the Fourier transform and forms the filtered image in its focal plane, which gives

$$E\text{out}(r,\phi)=E\text{in}(r,\phi)*\mathscr{F}[F(\rho,\varphi)] \qquad (6)$$

where * denotes the convolution. Obviously, the Fourier spectrum of the vortex filter plays the role of the point spread function (PSF) of the whole imaging system, which can be specified as (16)

$$\mathscr{F}[\text{circ}(\rho/R)\exp(i\varphi)]=\pi R2r[J0(kRrf)H1(kRrf)-J1(kRrf)H0(kRrf)]\exp(i\phi) \qquad (7)$$

where k=(2π/λ) is the wave vector, f is the focal length of lens,

Jm is mth-order Bessel function of the first kind, and Hm is mth-order Struve function (17). So, the designer has to take into consideration e.g. f1, f2, 2f1, and 2f2, as shown in the Figure above.

Effect of Nonlinear optical effect on high-Q optical microspheres and photonic nanowires

Significantly lower thresholds are required for the initiating the nonlinear optical phenomena in case of High-Q optical microcavities. Though a lot of work has been done on studying the quality factors (QFs) in the high-Q structures, and QF exceeding 10 billion has been demonstrated, not much work was initially done on the dispersive properties of these structures. Demonstrations had previously been limited to phenomena like Raman oscillation and optical bistability, both of which do not require phase-matching. Agha et al (18) have investigated theoretically the mode volume and quality factor (Q) of the fundamental whispering gallery mode of a silica microsphere and have studied the optimum size of silica microspheres for both linear and nonlinear optical interactions. In addition, they have stated that they can exhibit anomalous dispersion, which allows for phase and energy matching. In the nonlinear case, an optimal wavelength exists that corresponds to the low-loss range for silica, and for the linear optical interactions, the optimal size is found to be relatively insensitive to the choice of wavelength. Agha et al (19) have demonstrated theoretically and experimentally that under certain conditions high-Q microspheres can show the anomalous dispersion, which permits phase and energy matching for nonlinear optical interactions and have used such dispersion control to produce either Raman or four-wave mixing parametric oscillations by changing the size of the microsphere. In addition, Agha et al (20) have analyzed the process of cascaded four-wave mixing in a high-Q microcavity and have shown that under conditions of suitable cavity-mode dispersion, broadband frequency combs can be generated. They have also experimentally demonstrated the broadband, cascaded four-wave mixing parametric oscillation in the anomalous group-velocity dispersion regime of a high-Q silica microsphere with an overall bandwidth greater than 200 nm.

Silicon photonics has displayed a lot of promise, by showing a lot of technological progress in the form of seamless integration of photonic elements with electronics. It is really important to note that the large index contrast of silicon-on-insulator nanowaveguides combined with the large Kerr nonlinearity of silicon, has provided effective nonlinearities > three orders of magnitude larger than those achievable in case of the silica glass fibers. Foster et al (21) have reviewed recent research on nonlinear optical interactions in waveguides with sub-micron transverse dimensions, called photonic nanowires. These nanowaveguides, fabricated from glasses or semiconductors, provide the maximal confinement of light for index guiding structures, leading to large enhancement of nonlinear interactions and group-velocity dispersion engineering. Because of these two properties, photonic nanowires are ideally suited for many nonlinear optical applications like the generation of single-cycle pulses and optical processing with sub-mW powers. Foster et al (21) have investigated the nonlinear optical devices by using this very power efficient platform like- the devices based on self-phase modulation, stimulated Raman scattering and especially four-wave mixing (FWM).

Interestingly, the technique of the FWM in silicon waveguides is based on the ultrafast third-order Kerr nonlinearity, which has proved to be very efficient and broadband in silicon, when the conditions of phase-matching are satisfied by properly choosing the group-velocity dispersion (GVD). A very interesting result has been achieved by the Dispersion engineering of silicon photonic nanowires - a zero-GVD point or an anomalous-GVD region has been located in the C telecommunications band. Already, in case of the waveguides with near-zero-GVD, conversion of continuous-wave light has been achieved with efficiencies of -10 dB over bandwidths > 150 nm. Moreover, in case of the silicon nanowaveguides , pump power of only ~ 100 mW is required for the wavelength conversion. Foster et al (22) have demonstrated highly broad-band frequency conversion via four-wave mixing in silicon nanowaveguides. Through appropriate engineering of the waveguide dimensions, conversion bandwidths greater than 150 nm are achieved and peak conversion efficiencies of -9.6 dB are demonstrated. Furthermore, utilizing fourth-order dispersion, wavelength conversion across four telecommunication bands from 1477 nm (S-band) to 1672 nm (U-band) is demonstrated with an efficiency of -12 dB. The effectiveness of the Dispersion engineering technique for wavelength conversion is so strong that it has allowed us to achieve over two-thirds of an octave. Turner et al (23) have demonstrated ultrabroad-bandwidth low-power frequency conversion of continuous-wave light in a dispersion engineered silicon nanowaveguide via four-wave mixing and have produced continuously tunable four-wave mixing wavelength conversion over two-thirds of an octave from 1241-nm to 2078-nm wavelength light, with a pump wavelength in the telecommunications C-band.

Development of an optical amplifier on silicon is required for the success of silicon-on-insulator (SOI) photonic integrated circuits. Foster et al (24) have demonstrated (i) net on/off gain over a wavelength range of 28 nm through the optical process of phase-matched four-wave mixing in suitably designed SOI channel waveguides; and (ii) wavelength conversion in the range 1,511–1,591 nm with peak conversion efficiencies of +5.2 dB, which represents more than 20 times improvement on previous four-wave-mixing efficiencies in SOI waveguides. It has been emphasized that these advances are useful for the implementation of dense wavelength division multiplexing in an all-silicon photonic integrated circuit. Foster et al (24) have observed parametric amplification with peak powers ~ 1 W in waveguides exhibiting anomalous-GVD. These results and observations have proved critical and useful for the research and development of all-optical information processing systems.

An optical microscope has enabled image-based findings and diagnosis on microscopic targets, which is indispensable in many scientific, industrial and medical settings. It has been observed that a standard benchtop microscope platform, equipped with bright-field and phase-contrast modes, is of importance and convenience for various users because the wide-field and label-free properties allow for morphological imaging without the need for specific sample preparation. But the problem in this case is that, these microscopes never have capability of acquiring molecular contrast in a label-free manner. To solve this problem, Toda et al (25) have developed a simple add-on optical unit, comprising of an amplitude-modulated mid-infrared semiconductor laser, that is attached to a standard microscope platform to deliver the additional molecular contrast of the specimen on top of its conventional microscopic image, based on the principle of photothermal effect. By attaching this unit, termed molecular-contrast unit, to a standard phase-contrast microscope, they have demonstrated high-speed label-free molecular-contrast phase-contrast imaging of silica-polystyrene microbeads mixture and molecular-vibrational spectroscopic imaging of HeLa (HeLa is an immortalized cell line used in scientific research) cells. They have claimed that their simple molecular-contrast unit can empower existing standard microscopes and deliver convenient accessibility to the molecular world. This should be really helpful for carrying out further research in this area.

Designing of the nonlinear optical devices based on fully resonant interaction of light with electrons confined in quantum wells

A novel approach in designing of the nonlinear optical devices based on fully resonant interaction of light with electrons confined in quantum wells, and monolithic integration of nonlinear elements and pump sources, aimed at combining the tunability and flexibility of nonlinear optical sources with advantages of the semiconductor injection lasers like compactness, high efficiency, injection pumping scheme, low cost, and possibility of monolithic integration with electronic chips has recently been evolved. In fact, two groups, one of Federico Capasso (Harvard) and the other of Claire Gmachl (Princeton), have been working in collaboration (26) on this topic, with a great amount of success. This research work (26) requires a standard setup for nonlinear optical generation including a passive nonlinear crystal, for the optical pump beams from one or more high-power external lasers for generating a nonlinear signal at the (i) sum-frequency, (ii) difference-frequency, or (iii) second and higher harmonics of the fundamental pump frequencies. One of the requirements of the nonlinear crystal for avoiding absorption is that it should be transparent to all the beams involved in the nonlinear mixing process, which implies that all the frequencies should not be near any resonances in the nonlinear medium. The other major requirement is that of the injection-pumped quantum-cascade lasers, in which the laser field works as an optical pump for the resonant, nonlinear optical interaction in the same active region, which implies the integration of the optical pump source and the nonlinear optical medium Since, such a system experiences net gain and no absorption, therefore, all the fields participating in the nonlinear interaction are amplified and resonate with the corresponding inter- subband transitions, leading to the tremendous enhancement of the second-order nonlinear susceptibility (~ up to hundreds nm/V), which is the enhancement by many orders of magnitude.

Mathematical modeling of four wave mixing

The modeling of the observed interaction is done by following the well-known approach (27), based on assuming negligible depletion of the pump or signal due to the generation of the idler, and thus the Four wave mixing (FWM) conversion efficiency (□) using an optical resonator can be written as;

$$\eta = Ii(out)/Is(in) = [\gamma\ Pp\ Leff]^2.(Fp.FsFi)^2 \qquad (8)$$

$$L_{eff}^2 = L^2.\exp(-\alpha L).\ [1-\exp(-\alpha L + j\Delta kL)/\alpha L - j\Delta kL]^2 \qquad (9)$$

And

$$Fp,s,i = [\sigma/[1-\tau \exp(-\alpha L2 + jkp,s,iL)]]^2 \qquad (10)$$

where Ii(out) is the output intensity of the idler, Is(in) is the input intensity of the signal, γ is the effective nonlinearity given by γ =(n2.ω0)/(cAeff), n2 is the nonlinear index coefficient, ω0 is the center frequency, c is the speed of light in vacuum, Aeff is the effective mode area, Pp is the input pump power, Leff is the effective length, Fp, Fs, and Fi are the resonant intensity enhancement factors for the pump, signal and idler, respectively, L is the circumference of the ring, σ and τ are the coupling and transmission coefficients, respectively, with |σ|2+|τ|2=1 and kp,s,i are the wavenumbers of the pump, signal, and idler fields with the phase mismatch given by:

$$\Delta k = (2k_p - k_s - k_i\ ) \qquad (11)$$

These parameters are all useful for achieving the desired conversion efficiency, and also for the optimization. A large number of parameters, and their interdependence for achieving the desired η, present a challenging problem to the designer, who has to choose the best possible combination from the various possible solutions.

One method of generating narrowband entangled photons is by exploiting the FWM process in an atomic ensemble, in which the frequency bandwidth of the pairs is determined by the dispersion of the vapour and is quite narrow for matching the atomic absorption profile. Based on FWM, signal and idler photons can be generated by two pump laser beams P1 and P2 by satisfying the following two conditions:

Phase-matching: $K_{P1} + K_{P2} = K_S + K_I$ **(12)**

and

Energy conservation: $\omega_{P1} + \omega_{P2} = \omega_S + \omega_I$ **(13)**

It has to be noted that the feedback from the experimental results obtained are sometimes slightly different from those expected on the theoretical designing, and hence corrections have to be applied so that the optimum desired results are obtained.

3. Qualitative Review of Some Important Novel Investigations on Nonlinear Phase Contrast and Concluding Remarks

Recently, many very novel investigations have been made, and some of the important ones are reviewed and discussed here. Goy and Psaltis (28) have demonstrated a simple method to detect small phase steps using propagation in Kerr media and have performed experiments to show that the nonlinear diffraction in acetone produces high contrast signals for a certain type of phase mask. Samineni et al (29) have demonstrated nonlinear phase contrast imaging in highly scattering media using rapid femtosecond pulse shaping of mode-locked laser pulses and have also discussed the potential applications of this technique for intrinsic functional neuronal imaging.

Wilson et al (30) have discussed that nonlinear phase contrast is achieved by a simple modification to a nonlinear pump-probe microscope, and this technique measures cross-phase modulation by detecting a pump-induced spectral shift in the probe pulse. Importantly, images with nonlinear phase contrast are acquired both in transparent and absorptive media. They have emphasized that in paraffin-embedded biopsy sections, cross-phase modulation complements the chemically specific pump-probe images with structural context. These observations have great potential for the various applications in research areas. Their optical set-up for XPMSS imaging involves adding a shortpass filter to an existing pump-probe microscope, has been shown below:

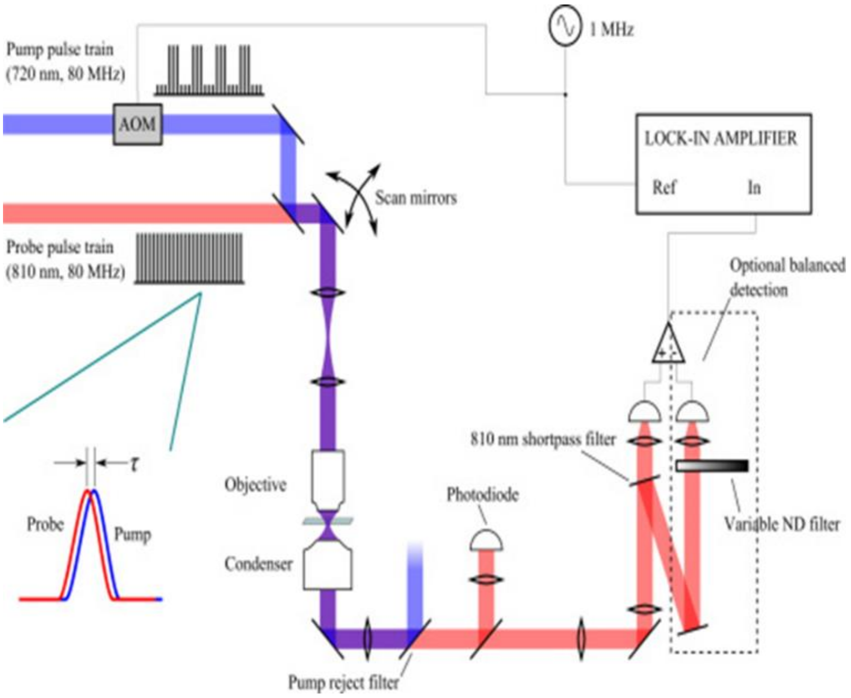

Figure 7: The optical set-up for XPMSS imaging involves adding a shortpass filter to an existing pump-probe microscope. Optionally a balanced photodiode can be used to isolate nonlinear phase modulation from nonlinear Figure courtesy Wilson Jesse W., Samineni Prathyush, Warren S. Warren, and Fischer Martin C., Cross-phase modulation spectral shifting: nonlinear phase contrast in a pump-probe microscope, Optics Express, Vol. 3, Issue 5, pp. 854-862, (2012).

The setup is quite complex, and it's geometry is designed by an expert to get optimum efficiency. Another difficulty in this experiment is the alignment of various components to get the desired results.

The X-ray phase contrast imaging technique is based on measuring the Fresnel diffraction intensity patterns associated with the phase shift induced by the object. The simultaneous recovery of the phase and of the absorption is a difficult non linear inverse problem. Sixou et al (31) have investigated the resolution of this problem with non linear Tikhonov regularization and with the Iterative Gauss Newton method, by evaluating the algorithm using the simulated noisy data. Silva et al (32) have shown that the statistics of the idler wave generated through FWM follows a thermal distribution when the fiber input power is maintained in a low level; and have explained that this arises from the thermal nature of the photons generated through spontaneous FWM and through Raman scattering inside the fiber. Nonlinear imaging is based on taking advantage of the localized nature of the interaction to achieve high spatial resolution, optical sectioning, and deeper penetration in tissue. However, it is quite challenging to measure the nonlinear contrast because it is overwhelmed by the large background of detected illumination light, the measurement of the nonlinear refractive index being especially difficult Fischer et al (33) have developed a technique to suppress the background in these types of measurements by using femtosecond pulse shaping to encode nonlinear interactions in background-free regions of the frequency spectrum, and have been able to measure the self-phase modulation (SPM) in highly scattering environments, like biological tissue, with very modest power levels. In addition, they have demonstrated strong intrinsic SPM signatures of glutamate-induced neuronal activity in hippocampal brain slices. It has been emphasized that this technique should be very useful for imaging neuronal activity.

Bublis et al (34) proposed the use of photothermal Zernike phase-contrast filter for measuring the absorption of the medium and have provided the results of numerical simulation. They have shown that the efficiency is comparable to the imaging scheme that uses linear filters, with a significant simplification of the process of its adjustment. This result has great potential for applications in various disciplines. Moosmann et al (35) have investigated the case of the coherent X-ray imaging, based on phase contrast through free-space Fresnel propagation, and have discussed two noniterative, nonlinear approaches to the phase-retrieval problem from a single-distance intensity map of a pure-phase object. They have proposed (i) a perturbative set-up where nonlinear corrections to the linearized transport-of-intensity situation are expanded in powers of the object-detector distance z and are evaluated in terms of the linear estimate; and (ii), a nonperturbative projection algorithm, which is based on the linear and local contrast transfer function (CTF), working with an effective phase in Fourier space, obeying a modified CTF relation between intensity contrast at $z > 0$ and phase contrast at $z = 0$. In addition, by identifying the positions of the zeros of the Fourier transformed intensity contrast as order parameters for the dynamic breaking of scaling symmetry, they have investigated the phase structure of the forward propagation problem in case of being interpreted as a statistical system. Lai et al (36) have used (i) high speed X-ray phase-contrast imaging, (ii) weakly nonlinear analysis, and (iii) boundary integral simulations; and thus, have characterized the final stage of underwater bubble break-up. Their X-ray imaging study has illustrated that (i) an initial azimuthal perturbation to the shape of the bubble neck gives rise to oscillations which increasingly distort the cross-section shape; and (ii) these oscillations terminate in a pinch-off where the bubble surface develops concave regions. They have also presented a weakly nonlinear analysis which shows that this coalescence-like mode of pinch-off occurs, when the initial shape oscillation interferes constructively with the higher harmonics generated by it. In addition, they have demonstrated that when the oscillations interfere destructively, a qualitatively different mode of pinch-off results, in which case, the cross-section profile of the bubble neck develops sharply-curved regions.

Differential phase-contrast imaging with X-ray tubes based on Talbot Interferometry, is affected by conventional X-ray imaging setups. The problem encountered is that the parameters optimized for conventional setups are generally not optimal for differential phase-contrast imaging. Though, an objective function can be formed by combining quantities like visibility, contrast to noise ratio, and dose, they are generally not known analytically, and also are expected to be non-linear, leading to some problems in case of the differential phase-contrast imaging. Ritter et al (37) have discussed that the optimization of differential phase-contrast is still possible because the quantities necessary to form an objective function, can be obtained by a simulation, and also the setup parameters can be varied within the simulation more purposefully than possible in an experimental setup. They have used a Monte-Carlo simulation framework and a wave field simulation framework to take particles as well as wave contributions into account. Shui et al (38) have studied third-order nonlinearities of various samples by using the nonlinear-imaging technique with a phase object (NIT-PO). Their technique is based on (i) developing an approximate method to calculate the nonlinear refractive coefficient analytically for the pure nonlinear refractive materials; and (ii) decomposing the object field passing through the phase object into two top-hat beams of different phases and beam radii, and thus acquiring the approximate phase contrast, from which the nonlinear refractive coefficient can be extracted, the approximation being valid when the on-axis nonlinear phase shift by the sample is less than π. The existence of stable autoresonance (AR) with continuously growing energy is directly connected with the inherent property of nonlinear systems to remain in resonance when the driving frequency varies in time, though the physical mechanism underlying the transformation of bounded oscillations into AR is still somewhat unclear. Kovaleva and Manevitch (39) have discussed that the emergence of AR from stable bounded oscillations is basically analogous to the transition from quasilinear to nonlinear oscillations in the time-invariant oscillator driven by an external harmonic excitation with constant frequency, and AR can occur as a result of the loss of stability of the so-called limiting phase trajectory. They have obtained the parametric threshold, which determines the transition from bounded oscillations to AR in the time-dependent system, and also have confirmed the accuracy of the obtained approximations by numerical simulations. Results of various studies on nonlinear phase contrast techniques can also be usefully employed in Quantum Optoelectronics (40).

Bian et al (41) have proposed a concept of nonlinear phase shifter to provide a new idea for the hardware architecture of sensing reconfigurable intelligent surfaces (RISs). As suggested by them, the equivalent circuit of the proposed phase shifter is first given to provide theoretical guidance for the design, after which the nonlinearity analysis of the nonlinear phase shifter is carried out. They have fabricated two simplified sensing RIS elements operating at 4 GHz and validated the design by measurements. They have chosen the measured scanning range from 16° to −18° with the input power increasing from 5 to 23 dBm. Their results of the simulated and measured S parameters of the two nonlinear phase shifters at 5 dBm input power have been reproduced below:

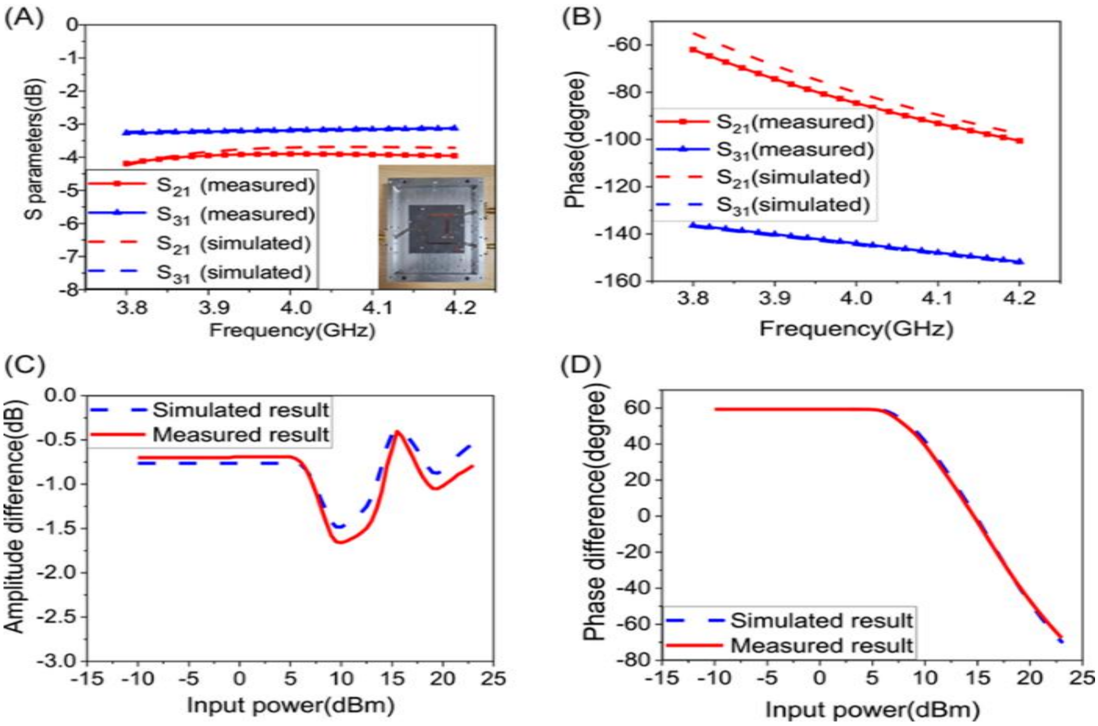

Figure 8: The simulated and measured S parameters of the two nonlinear phase shifters at 5 dBm input power (A) S parameters and (B) phase. The simulated and measured amplitude and phase differences of the two nonlinear phase shifters at 4 GHz (C) amplitude (D) phase.Figure courtesy Bian Chenge , Zhou Dongfang, Zhang Dewei , Liu Qing , Yi Zhang and Lv Dalong, The concept of nonlinear phase shifter and its application to element of sensing RISs. Microwave and Optical Technology Letters 66(1):n/a-n/a. October 2023, DOI:10.1002/mop.33924.

It has been claimed that the nonlinear phase shifter shows great application potential in sensing RISs. The measured phase shift of the transmitted beam by the sample ϕ, is related to the index of refraction δ, and sample thickness, t, by:

$$\phi = \delta . kt. \qquad (14)$$

where $k = 2\pi/\lambda$.

So, the designer has to consider all these parameters to get the opt the optimized results. In view of these recent investigations, it can be safely concluded that the topic of nonlinear phase contrast is drawing the attention of various workers, especially for the new important applications; and hence the evolution of the subject is on a firm footing.

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