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Prediction of the Crack Initiation Plan in Material Fatigue

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Abstract: The multiaxial fatigue criteria make it possible to determine the lifespan of a material subjected to multiaxial stresses. Among these we distinguish the critical plane type criteria whose formalism makes it possible to identify the crack initiation plane. Robert's criterion is used for this study. It is applied to multiaxial stress loadings either at constant amplitude, where we are interested in the distribution of its damage indicator by physical plane, or at variable amplitude, where we use a method based on a concept of calculation and accumulation of damage plan by plan. Whether it is one or the other of the two loadings, the maximum damage plan constitutes the critical plan, that is to say the initiation plan. An application is given for each load. In the context of multiaxial stresses of variable amplitude, a validation of the estimation of the orientations of the initiation planes is carried out based on experimental results on cruciform specimens. The estimated orientations are close to those observed experimentally.

Keywords: Multiaxial fatigue, Fatigue criterion, Critical plane, Variable amplitude, Constant amplitude.

1. Introduction

The current trend is towards lighter structures and mechanical components, particularly in the transport industry which must face two constraints. One is economic (cost of raw materials) and the other is environmental (reduction of energy consumed and therefore air pollution). Both trends have the same objective: to reduce the quantity of raw material used for the manufacture of a component with constant specifications, that is to say for a given mechanical loading. This trend leads to an increase in cases of fatigue failure when the stresses vary. Indeed, the reduction in the quantity of raw material has the immediate consequence of an increase in the level of mechanical stresses within a structure, thus making it more subject when they are variable to the appearance of the fatigue phenomenon. A part stressed cyclically and presenting a fatigue failure has generally undergone three successive stages of degradation: first the initiation of a crack, then the slow propagation of this until the sudden rupture of the part. In many cases the initiation of the crack is sufficient to account for the lifespan of a member, particularly when it concerns a metallic material such as carbon steel, for which the initiation phase represents most of the life of the component. The appearance of a macrocrack, thus defining initiation, is most frequently located on the surface of the parts and is directly linked to the nature of the material and the loading. The phenomenon of crack initiation under cyclic loading follows a metallurgical process that is now well identified [1]. Thus for an isotropic steel where the stacking faults of the metal and the inclusions are uniformly distributed, the cyclic loading imposes deformations on the material activating sliding planes. Persistent sliding bands appear, characteristic of damage to the material. Their orientation depends on successive states of stress and the sensitivity of the material to them.

For many years, many multiaxial fatigue criteria making it possible to evaluate the fatigue life of materials subjected to periodic multiaxial stresses have been developed. Originally, these made it possible to predict the initiation of a crack at the endurance threshold, a threshold below which it is commonly accepted that macrocracks no longer appear. Now extended to the field of limited endurance, they make it possible to situate any cycle of multiaxial stresses in relation to the fatigue limit of the material corresponding to a finite lifespan. In other words, they make it possible to estimate the lifespan of a material in terms of the number of multiaxial stress cycles to which it can be subjected [2]. The formalism of these fatigue criteria is available in different types of modeling. We can cite empirical type criteria, which describe the experimental behavior of a set of tests of a given type, such as those of Gough and Pollard in 1935 [3], Nishihara-Kawamoto in 1941 [4] or even Lee in 1989 [5]. Other criteria based on the concept of the critical plane come from metallurgical findings stipulating that fatigue damage results in the appearance of persistent sliding bands or microdeformations linked to the action of normal and tangential stresses acting on a particular physical plane. In these models, the fatigue behavior of the material is controlled by the plane most stressed in the sense of the criterion and called critical plane. Among these criteria, we can cite those of Findley in 1957 [6], Dang Van in 1973 [7] or Robert in 1992 [8]. A third type of multiaxial criteria concerns those of a global approach. The latter use either characteristic quantities reflecting the overall action of stresses (invariants of the stress tensor or its deviator), or an average of the damage indicators associated with all the physical planes passing through a material point. The Sines criteria in 1955 [9], Fogue in 1987 [10] or Papadopoulos in 1993 [11] are part of these global approach criteria. The use of a multiaxial critical plane type criterion makes it possible to account for the sensitivity of the material to normal and tangential stresses acting on any physical plane and playing a predominant role in fatigue because the different failure modes depend on them. By scanning a large number of physical planes, the criterion makes it possible to evaluate the damage relative to each plane cycle after cycle thanks to a damage indicator Eh per plane. The most damaged plane constitutes the critical plane, that is to say the plane most stressed with regard to the fatigue resistance capacity of the material. The interest of critical plane type fatigue criteria used in limited endurance lies in the dual prediction of the lifespan of the material and the orientation of the crack initiation plane.

2. Estimation of Material Damage in Multiaxial Fatigue

The fatigue damage suffered by a material subjected to a stress cycle is generally evaluated using a damage law. The common point of the laws generally used is to establish the damage induced by the cycle from the lifespan of the material, obtained in the case where it would be subjected to periodic stress including only the cycle in question. This lifespan can be established using a fatigue criterion used in limited endurance, which can also determine any equivalent cycle from the lifespan point of view. It is in fact for this purpose that the extension of multiaxial fatigue criteria to limited lifespans was proposed and developed.

Three damage laws are mainly used for fatigue life estimations nowadays. These are Miner's law [13], the nonlinear law of Lemaitre and Chaboche [14,15] and the one proposed from Chaboche's law. These three laws were originally proposed only for uniaxial stress cycles. The principle of equivalent stress obtained by means of a multiaxial fatigue criterion makes it possible to extend their field of application to multiaxial stresses.

Miner law

The lifespan of the material is defined by the number of cycles at the initiation of a crack (or rupture) NR. Thus, the application of n cycles (n < NR) results in partial deterioration or damage of the material. Knowledge of this damage is important because it makes it possible to evaluate the residual lifespan and to decide whether or not to replace the component to avoid fatigue damage. The simplest rule for evaluating the degradation of the fatigue resistance capabilities of the material consists of considering a linear evolution of the damage. It stipulates that the damage suffered by the material at each cycle is a function of the level of effort representative of this cycle. For n cycles applied, the quantity is called damage:

$$D = \sum_{i=1}^{k} d_i = \sum_{i} \frac{n_i}{N_i} \qquad (1)$$

Where: n_i is the number of applied cycles identical to that considered,

N_i is the number of cycles, identical to that considered, supported by the material at the initiation of a crack.
According to this concept, the initiation of a crack appears in theory when the damage D is equal to unity.
Simple and very widely used, this method of predicting lifespan according to Miner presents proven and recognized defects in the presence of "low-high" and "high-low" type loadings, that is to say loadings at increasing or decreasing loading amplitude respectively. Miner's law ignores the interactions, from a fatigue point of view, between the different successive loading levels. What is more, the forecast error generated by Miner's law is approximately of the same order but with opposite trends for these two types of "extreme" loading, with monotonous variation in amplitude. When these two trends are encountered during the same loading history, the successive forecast errors partially compensate for each other. This explains the success still encountered today by this law of fatigue damage of variable amplitude.

Chaboche law

The fatigue life of a material plane is derived by the method described above for all the cycles this material plane experiences. A damage law is used to assess the damage induced by each cycle and a cumulation rule allows one to obtain the amount of damage corresponding to the whole sequence.
Two damage rules may be used for this step of the method. The first one is the well-known linear Miner's rule [10], the second one is the non-linear low proposed by Lemaitre and Chaboche [11, 12]. This law gives the increase δD of damage due to δN identical uniaxial stress cycles defined by their amplitude σ_a and their mean value σ_m as:

$$\delta D = \left[1-(1-D)^{\beta+1}\right]^{\alpha}\left[\frac{\sigma_{ai}}{M_0\left(1-b\sigma_{mi}\right)(1-D)}\right]^{\beta}\delta N \quad (2)$$

Where: $\alpha = 1-a\left\langle\frac{\sigma_{ai}-\sigma_A\left(\sigma_{mi}\right)}{R_m-\sigma_{ai}-\sigma_{mi}}\right\rangle$ with $\sigma_A\left(\sigma_{mi}\right)=\sigma_D\left(1-b\sigma_{mi}\right)$.

$\sigma_A\left(\sigma_m\right)$ is the amplitude of the average stress endurance cycle. The MacCauley function (< >) gives rise to two scenarios:

- case of a large cycle: $\sigma_{ai} > \sigma_A\left(\sigma_{mi}\right)$ et $\alpha = 1-a\left(\frac{\sigma_{ai}-\sigma_A\left(\sigma_{mi}\right)}{R_m-\sigma_{ai}-\sigma_{mi}}\right)$,
- case of a small cycle: $\sigma_{ai} \le \sigma_A\left(\sigma_{mi}\right)$ et $\alpha = 1$.

β, a and M_0 are coefficients specific to the material and introduced by law. Rm is the maximum tensile strength of the material. is the endurance limit of the material (in symmetrical alternating traction). b is a material coefficient representing the influence of the average stress on the fatigue limit.
This non-linear damage allows small amplitude cycles to contribute to the material damage and takes into account the occurrence order of the cycles.

Law proposed by Tikri

The proposed nonlinear law, put in its differential form, expresses the increase in damage introduced by identical cycles according to:

$$\delta D = \left[1 - (1 - D)^{\beta+1}\right]^{\alpha(F_{max},F_D,F_u)} \cdot \left[\frac{F_u - F_D}{F_{max} - F_C} \cdot \left(\frac{F_{max}}{\sqrt{M_0\left(1-\frac{0.55 \cdot F_{max}}{A \cdot F_u}\right)} \cdot (1 - D)}\right)^{\beta}\right]\delta N \quad (3)$$

Where:
- δD is the increase in damage D of the material, due to δN identical cycle(s),
- F_u is the maximum tensile breaking strength of the specimen,
- F_C is the conventional endurance limit at $2{,}10^6$ cycles,
- F_D is the endurance limit determined by smoothing the experimental points of the F-N curve using the ESOPE software,

- the coefficient α, function of the maximum effort F_{max}, the conventional endurance limit F_C, the endurance limit F_D and the maximum tensile resistance Fu, reflects the non-linearity of the accumulation of damage; it is defined by:

$$\alpha = 1 - a \langle \frac{F_{max} - F_C}{F_u - F_D} \rangle \tag{4}$$

a, A, β et M_o sont des coefficients propres au matériau.

3. Lifespan Forecast using Damage-by-Plan Approach

The fatigue phenomenon is the consequence of a variable, cyclic or random loading suffered by the material, and more precisely of the variable stresses that it induces at the material level. It leads to the degradation of the material, which is called damage and which generally and ultimately leads to a crack, initiated at the physical point considered on the component.

Each material plane or facet passing through this point, due to its orientation, is not subjected during loading to the same tangential and normal constraints. Consequently, each facet will suffer damage that is specific to it and which is directly linked to the constraints applied to it.

Formalism of a critical plan type criterion

The critical plane type criteria generally define a damage indicator Eh per plane which brings together several terms representative of the multiaxial stress cycle [σ(t)] acting on this material plane. A linear combination of these elements (normal and tangential constraints) is produced for each plane, which is defined by the orientation of its normal h in relation to the global reference frame linked to the material (figure 1).

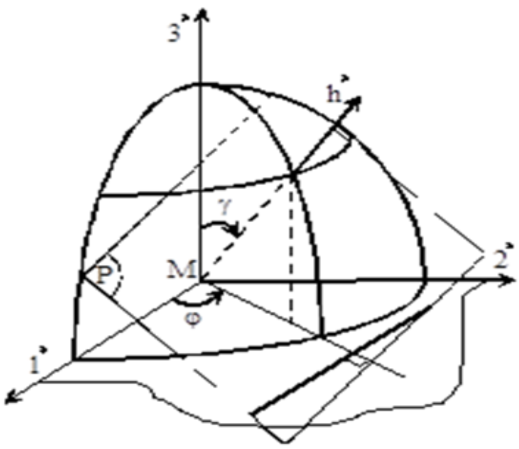

Figure 1 *– Orientation of a physical plane P passing through a material point M of a structure in a global reference frame.*

The plane damage indicator E_h is therefore expressed in the form of a function depending on the multiaxial stress cycle [σ (t)] and the fatigue limits at N cycles σ_{-1} (N), σ_0(N) and σ_{-1}(N) of the material:

$$E_h = E_h\left([\sigma(t)], \sigma_{-1}(N), \sigma_0(N), \tau_{-1}(N)\right) \tag{5}$$

The critical normal plane h_c corresponds to the plane presenting the highest damage indicator E_{hc}, which constitutes the fatigue function E of the criterion:

$$E = E_{hc} = \underset{h}{Max}(E_h) \tag{6}$$

The fatigue function E equals unity when the N-cycle fatigue limit of the material is reached for the multiaxial stress cycle [σ (t)] considered; it thereby indicates the lifespan of the material for this multiaxial cycle because it represents an implicit equation in N whose solution is the lifespan of the material.

$$E = 1 \tag{7}$$

A critical plane type multiaxial fatigue criterion therefore makes it possible, from the knowledge of several fatigue curves of the material, to determine both the lifespan and the orientation of the crack initiation plane of a material point subjected to a multiaxial stress cycle. The application will subsequently be made using Robert's criterion. It is transposable to other criteria based on the concept of the critical plane. The damage indicator of a fatigue criterion concretely translates this idea, thus making it possible to evaluate the severity of the constraints, from the point of view of fatigue, acting on each plane.

It must also be recognized that this evaluation is only realistic under the condition of anisotropy of the material. Figure 2 recalls the distribution of the damage indicator of the LMSo critical plane type criterion for two particular cases of cyclic stresses, one with principal directions of fixed stresses, the other with principal directions of moving stresses. These distributions are given over the interval of variation $[0,\pi[\times[0,\pi[$ of the two angles defining the normal h to the plane considered.

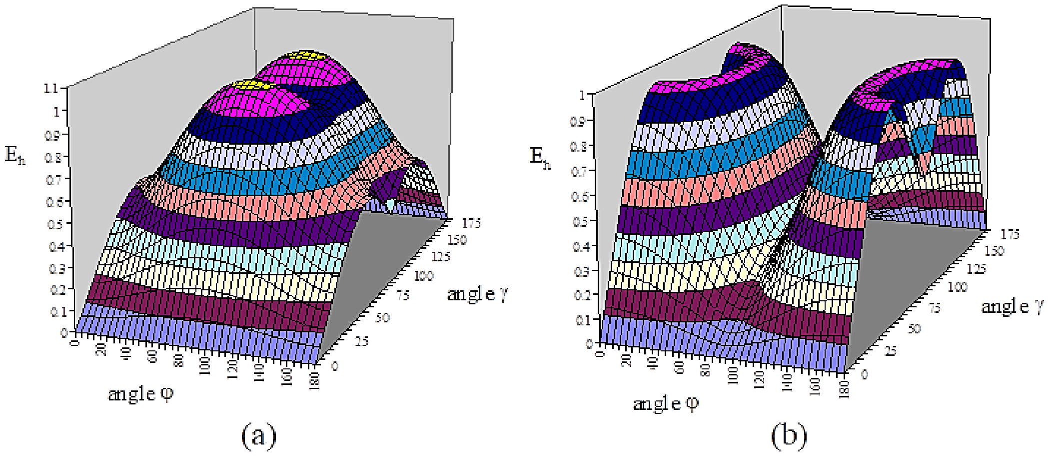

Figure 2 – *Distribution of the damage indicator: (a) for a constant principal stress directions cycle, (b) for a rotating principal stress directions cycle.*

The fatigue function of a multiaxial criterion establishes the lifespan or, which amounts to the same thing, the damage of the cycle as a whole. It uses either the critical plane or all of the planes, depending on the type of criterion, to estimate the overall damage of the material. If this concept is valid for a multiaxial stress cycle of periodic loading, it is less so in random multiaxial fatigue where cycles of different nature can follow one another. In this context, as we can see in Figure 2, the most requested plans are not always the same from one cycle to the next. The damage caused by a cycle is not global from the point of view of the material but distributed in a non-homogeneous manner over all possible planes. From there comes the idea of taking this effective distribution into account by accumulating the respective contributions of the damage from each cycle plan by plan. Unless all cycles of a sequence are identical (in which case it is a constant amplitude loading sequence), the prediction of the damage obtained on the critical level with regard to the entire sequence is less conservative than that carried out with damage established globally by the fatigue criterion (classic or initial method). The figure 2 shows flowchart of the damage-based lifetime prediction by plan. For each extracted cycle, the lifespan is determined on the facet considered using the damage indicator of the critical plane criterion. It should be noted here that only a critical plan type criterion extended to limited endurance can be used. Until now, the criterion makes it possible to calculate the lifespan of the material from the damage indicator obtained on the critical normal plane and the lifespan is calculated by solving the implicit equation. This principle in fact can be extended to any material plane. The lifespan N calculated on a plane is that which corresponds to the initiation of a crack on this plane when the stresses applied to it during a cycle are repeated N times.

4. Application to Periodic Multiaxial Loading

Numerous tests implementing periodic multiaxial loading have been carried out, both to better understand the real fatigue behavior of materials and to validate the multiaxial criteria. These tests generally correspond to combined stresses of traction-torsion, flexion-torsion or even biaxial traction in phase or out of phase. They can be classified into two groups:

- tests with main directions of fixed stresses,
- tests with principal directions of moving stresses

The evolution of the constraints $\sigma_{ij}(t)$ generated experimentally can generally be expressed in the form of sinusoidal functions of the type:

$$\sigma_{ij}(t) = \sigma_{ijm} + \sigma_{ija} \sin(\omega t + \phi_{ij}) \quad (8)$$

Where σ_{ijm}, σ_{ija} et σ_{ij} represent respectively the average value, the amplitude and the proper phase shift of the stress component $\sigma_{ij}(t)$.

The tests are carried out and given generally in series, corresponding to the same lifespan. Among these tests are those used as wedging tests for fatigue criteria, i.e. those of symmetrical alternating traction-compression (σ_{-1}), repeated traction (σ_0) and symmetrical alternating torsion (τ_{-1}). The other tests then serve to validate the multiaxial fatigue criteria because they describe the real fatigue behavior of the material and make it possible to assess the accuracy of the criterion by the difference between its prediction and the experiment.

5. Application to the Froustey and Lasserre Tests

Consider the results of the Froustey and Lasserre flexion-torsion tests presented in Table 1 below:

Table 1 – Presentation of the tests by Froustey and Lasserre [11].

Tests	σ_{11m}	σ_{11a}	σ_{22m}	σ_{22a}	σ_{12m}	σ_{12a}	ϕ_{11}	ϕ_{12}	ϕ_{22}
1		485				280			
2	300	630							
3	450	550							
4	510	525							
5	600	535							
6		480				277		90°	
7	300					395			
8	300	211				365			
9	300	222				385		90°	
10	300	480				277			
11	300	480				277		45°	
12	300	470				271		60°	
13	300	473				273		90°	
14	300	590				148			
15	300	565				141		45°	
16	300	540				135		90°	
17	300	455			200	263			
18	300	465			200	269		90°	
19	450					395			
20	450	415				240			
21	450	405				234		90°	
22	600					350			
23	600	370				214			
24	600	390				225		90°	

These are tests carried out on a 30NCD16 steel with mechanical resistance Rm=1160MPa. Crack initiation was observed for 100 000 cycles. The fatigue limits for this life and material are: 695MPa, 1040MPa 415MPa.

Figure 3 gives for each test the distribution of damage indicators by plane Eh as a function of the orientation of the normal to the plane h, itself defined using two two angles φ and γ (figure 2).

Two trends stand out across all distributions. The distributions show either a finite number or an infinite number of planes where the damage indicator Eh is maximum. When the number of plans is finite, initiation takes place according to the criterion on the one most unfavorably oriented, i.e. on the one which suffers maximum damage. If an infinite number of plans are maximally equi-damaged, determining the starting plan is delicate and even unpredictable. In this case and for a perfect material, i.e. without defects, initiation can take place following the average direction of all the critical planes.

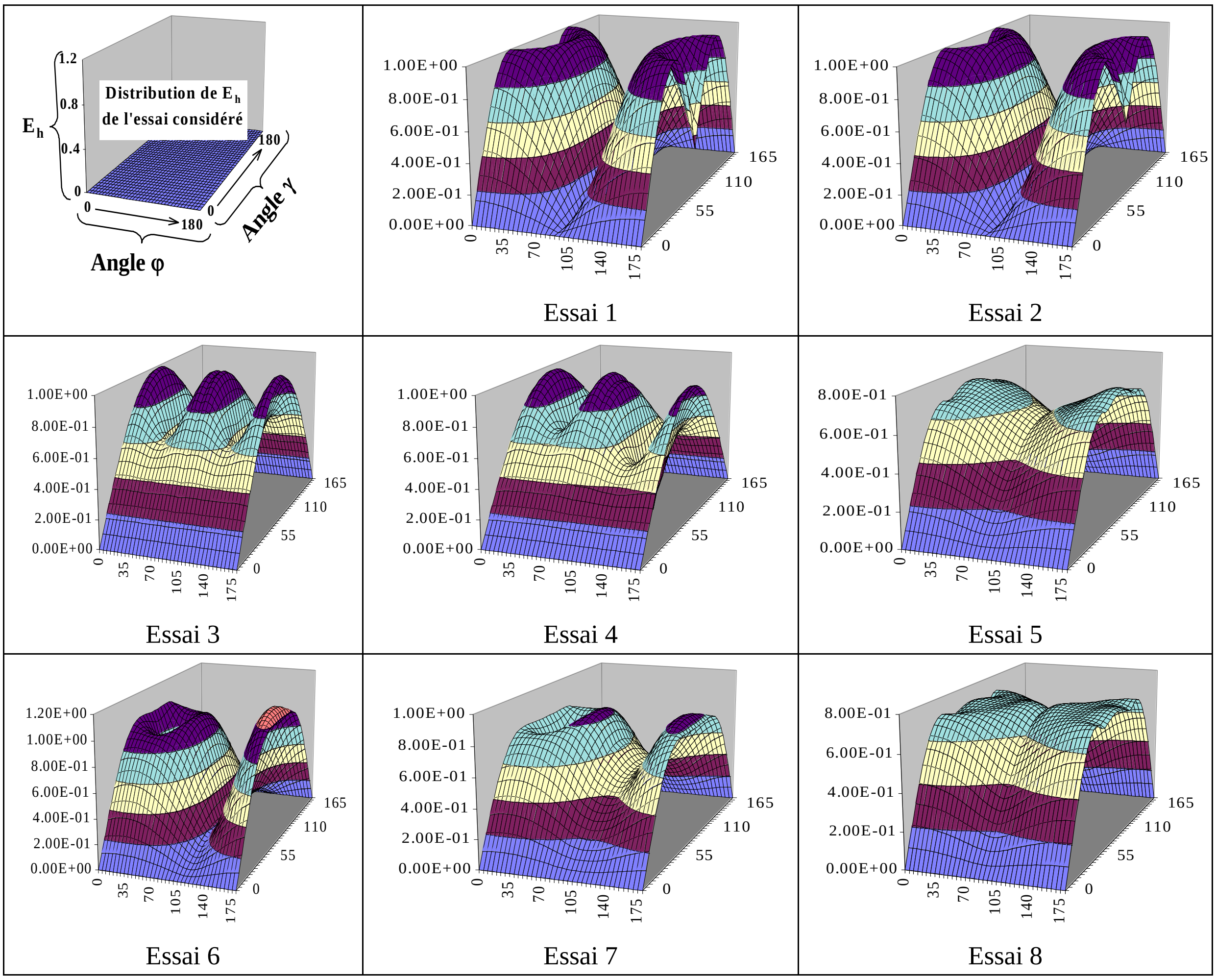

***Figure 3 –** Distributions of damage indicators Eh per plane as a function of the two angles □ and □ for all of the Simbürger tests.*

6. Validation of the Method on Cruciform Test Pieces

Tests on cruciform specimens were carried out by Bedkowski and Macha [14-15] at the Technical University of Opole (Poland). The grade tested (10 HNAP) is a low carbon steel whose chemical composition is given in Table 2.

Table 2 – Chemical composition of 10 HNAP steel.

Elements	C	Mn	Si	P	S	Cr	Cu	Ni
Percentages [%]	0.115	0.71	0.41	0.082	0.028	0.81	0.30	0.50

Ten random biaxial sequences were generated and applied to the cruciform specimens. The orientation, by the angle α of the trace of the cracking plane was noted for each specimen relative to the mark (O, x, y) on the free surface (figure 4) and in its thinned central part.

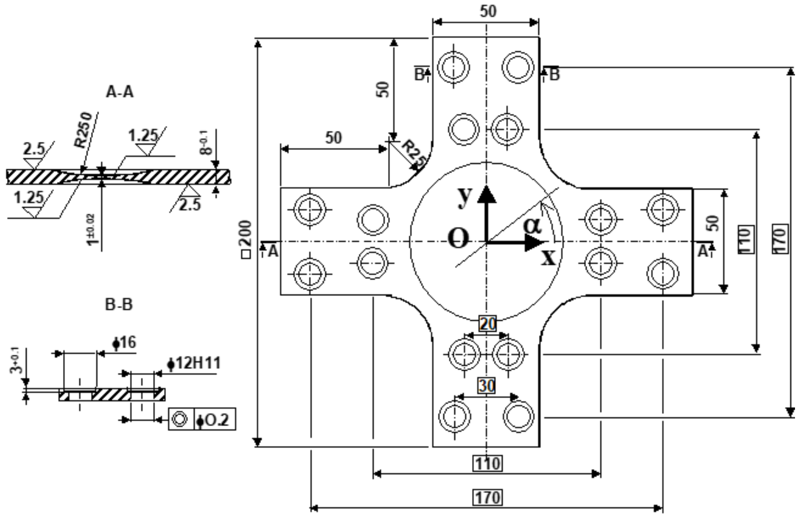

***Figure 4 –** Orientation (par l'angle* α*) de la trace de la fissure par rapport au repère (O, x, y) lié à l'éprouvette.*

7. Lifetime prediction

The estimation of lifespan in random multiaxial fatigue as described previously is the subject of a validation procedure and a post-processor installation of a finite element code for industrial purposes. A first validation of the method was carried out thanks to experimental tests carried out by the laboratory of Professor Macha (Opole, Poland) on cruciform test pieces defined in Figure4. Ten random biaxial sequences of traction-compression were all different loading conditions.

a. Comparison of calculated and experimental durations

Table 3 mentions the calculated and experimental lifetimes corresponding to these ten sequences and expressed in terms of the number of sequences at the initiation of a crack. The calculations were carried out using Miner's law, that of Chaboche and that proposed by Tikri. Figure 5 allows a visual comparison to be made between the calculations carried out and the results recorded during the tests. Generally conservative forecasts are obtained by the proposed method. Average ratios of 2.02, 2.10 and 1.32 are observed between the real lifespans and those calculated respectively by the laws of Miner, Chaboche and Tikri. Taking into account the contribution of small cycles to the total damage by the nonlinear law explains the more conservative lifetimes than those obtained with Miner's law.

Table 3 – Lifetime predictions and experimental results.

Sequences	Expérimentale lifetime	Calculated lifetime		
		Miner	Chaboche	Tikri
1	3273	4142	1650	3642
2	287	189	160	241
3	398	652	416	452
4	875	505	369	605
5	1301	593	590	993
6	2468	971	910	1671
7	1664	672	848	1172
8	848	322	439	522
9	267	58	89	148
10	342	308	167	295

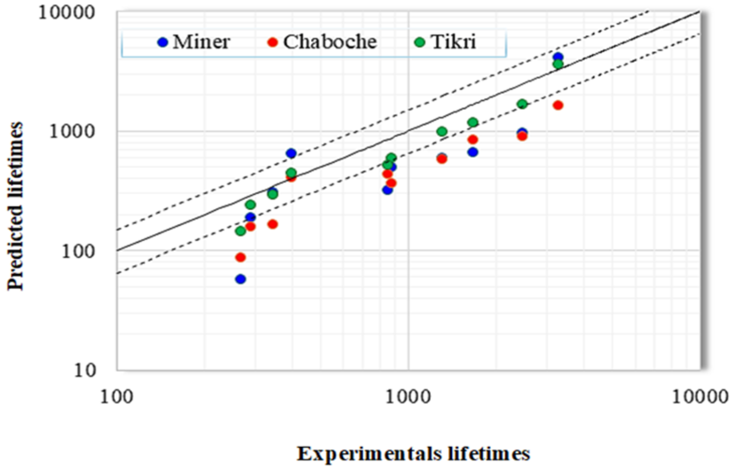

Figure 5– *Comparison of calculated and experimental lifetimes.*

b. Application to random biaxial sequences

The actual cracking orientations of the specimens and those obtained by numerical simulation are summarized in Table 4.

Those resulting from the simulation and data in parentheses correspond to lesser damage peaks, obtained on physical planes other than those of maximum damage. These plans nevertheless also constitute plans conducive to initiation due to their number and their associated damage.

Table 4 – Comparison of experimental and numerical cracking orientations.

Sequences	Experimental cracking angle φ (in degrees)	Angle γ from the simulation Digital (in degrees)
1	72.0 -73.5	30.0 (60.0) -30.0 (-60.0)
2	-62.0	30.0 (60.0) -30.0 (-60.0)
3	-73.5 -67.0	30.0 (60.0) -30.0 (-60.0)
4	71.0	30.0 (60.0) -30.0 (-60.0)
5	-25.0	-30.0 30.0
6	-72.5	-60.0 60.0
7	-71.0	-60.0 60.0
8	-70.0	-60.0 60.0
9	68.0 -70.5	65.0 -65.0
10	-65.0 -42.0	-65.0 65.0

8. Prediction of the Orientation of the Crack Initiation Plane

The damage calculations being carried out on all possible material planes, its distribution makes it possible to identify the critical plane(s), plane(s) where the cumulative damage during the sequence is maximum. Figure 5 presents the distributions corresponding to the two sequences 5 and 7. The double symmetry of these distributions with respect to the axes $\gamma = 90°$ and $\varphi = 90°$ is due to the particular stress states (biaxial with fixed principal directions) specific to these sequences. For sequence 5, four critical planes appear which correspond to values of φ equal to 30°, 60°, 120° and 150° and $\gamma = 90°$.

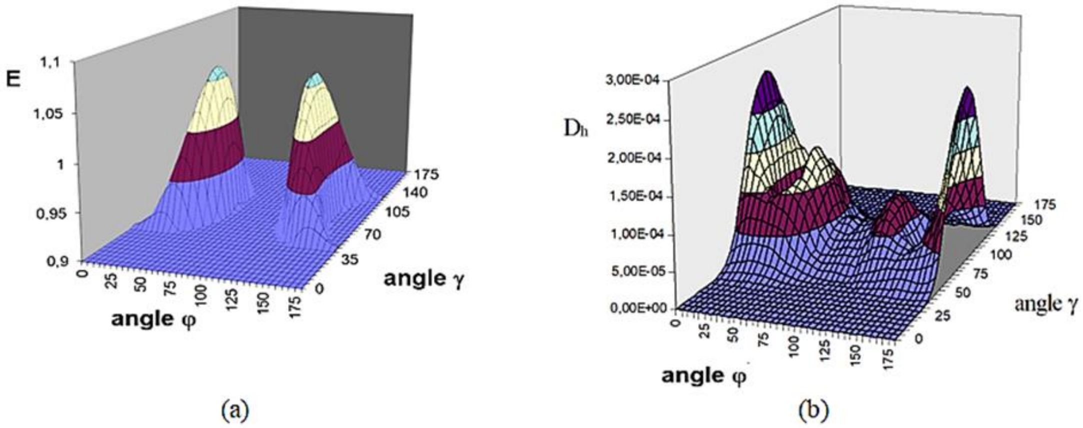

***Figure 6** - Distribution of damage by plane.*
(a) Sequence 5 - (b) Sequence 7.

The average difference between the actual rupture angle and the angle resulting from the numerical simulation is 20°, taking the plane of greatest damage as the critical plane. This average difference is reduced to 7.4° if we consider all the damage peaks, that is to say all the heavily damaged planes and whose orientation is much closer to that of the actual rupture plane.

Several different priming plans are sometimes detected experimentally. All information noted was reported. Concerning the results obtained by calculation, the main initiation planes appear as well as secondary planes in parentheses, the damage of which is less but which, because the material may contain defects or present certain anisotropy, can also be the site of a crack. The defects in question, such as inclusions for example, can modify the direction of propagation of a crack which then appears at a different angle when it emerges on the surface of the specimen. The fact is also that by doing this we compare the orientation of the theoretically predicted crack initiation plane with that observed after a certain propagation phase, which is sensitive to the barriers of the microstructure of the material. All calculations are carried out with an assumption of isotropy of the material which is in reality only an approximation. An average orientation deviation of 20° between the actual rupture plane and that predicted by the calculation is recorded over all ten biaxial sequences if we simply consider the critical plane. If we take into account the secondary initiation planes like those which appear for sequence 7 (figure 6) and which correspond to a cumulative damage less than that of the critical plane, the orientation difference is only of 7.4° [16].

9. Conclusion

A constraint approach for predicting the lifespan of a structure or component subjected to the most general case of multiaxial stresses of variable amplitude is established and based on the principle of counting and damage per plane. It carries out a systematic analysis of the critical plan type of all the material plans passing through the point of the structure where this analysis is carried out. Two important points guided the approach towards its final formalism. Firstly each facet or material plane undergoes constraints and therefore damage which are functions of its orientation, then the cyclical nature of the constraints applied to the plane can only be identified from these constraints, specific to itself, because a cycle stress on a given facet does not correspond, except in the particular case of a uniaxial stress state, to that identified on any other plane. A new fatigue life prediction method suitable for any kind of multiaxial variable amplitude stress states history is proposed. It is based upon a plane per plane damage distribution directly related to the stresses experienced by these material planes. The identification of multiaxial cycles is processed for any plane by considering the normal stress to this plane as the counting parameter. A multiaxial fatigue criterion extended from endurance to finite lives allows one to assess the life of the cycle with accounting for the six components of the stress states tensor. Linear or non-linear damage law are usable to express and make the cumulation of damage versus time. The procedure allows the assessment both of the crack initiation plane and the fatigue life of the material. A first validation of the life prediction method is realized by the way of fourteen biaxial random stress states histories issued from tests carried out on cruciform steel specimens. The average values of the ratio between experimental lives and expected ones indicates conservative assessment of 2.2 for Miner's damage rule and 3.4 for Lemaitre and Chaboche damage law.

All the results of the work undertaken on the critical plane type fatigue criteria can be integrated into the approach, whether it concerns the relative effects of the different components of the constraints occurring on the material level, the influence of the gradient, but also optimizations of calculation times and the way of homogeneously exploring all the possible directions of the plans. Classic damage laws can now be used in multiaxial fatigue. The necessary material data are three S-N curves in symmetrical alternating tension-compression and torsion and in repeated tension as well as specific tests characterizing the sensitivity of the material to gradient or to high average compressions if these two aspects are taken into account.

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