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Nonlinear Three-Degree-Of-Freedom Vehicle Handling Stability Analysis

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Abstract Aiming at the handling stability of the vehicle, this paper established a linear two-degree-of-freedom vehicle simulation model and a three-degree-of-freedom vehicle model with nonlinear wheel side stiffness. The system was analyzed in time domain. The two-degree-of-freedom vehicle model considered two degrees of freedom, lateral velocity and yaw velocity. On the basis of the two-degree-of-freedom vehicle model, the forward speed of the vehicle is added as the third degree of freedom. The steady-state response curve under different side deflection characteristics was calculated, the influence of side deflection stiffness on vehicle handling stability was analyzed, and the linear two-degree-of-freedom vehicle simulation model and the three-degree-of-freedom vehicle model with nonlinear wheel side deflection stiffness were analyzed and compared. Finally, it is concluded that the stability and maneuverability of the vehicle become better with the increase of the lateral stiffness of the front wheel (absolute value). The increase of the lateral stiffness value under a certain linear change is conducive to improving the handling stability of the vehicle. The feasibility of a three-degree-of-freedom vehicle model with nonlinear wheel lateral stiffness is demonstrated.

Keywords yaw angular velocity; Side deflection Angle of center of mass; Handling stability; Lateral stiffness

1. Preface

The handling stability of the vehicle refers to the ability of the vehicle to follow the direction given by the driver through the steering system and steering wheels without excessive tension and fatigue, and to resist interference and maintain stable driving when encountering external interference. It is an important evaluation index of the active safety of the vehicle.

This paper mainly analyzes the handling stability of the vehicle in the process of driving through the analysis of yaw velocity and centroid side deflection Angle. The yaw velocity of a vehicle and the side deflection Angle of the center of mass are two important indicators to describe the handling stability of a vehicle [1]. MATLAB/Simulink software is the main software used in this paper to model and simulate the handling stability of linear two-degree-of-freedom and three-degree-of-freedom vehicles. Under the condition of considering the nonlinear transformation of tire lateral stiffness, The three-degree-of-freedom vehicle model is compared with the two-degree-of-freedom model considering the forward speed, lateral speed and yaw velocity of the vehicle center of mass.

2. Model building

2.1 Two-degree-of-freedom vehicle model assumptions

Car driving involves a complex motion process. In order to facilitate analysis, the paper simplifies car motion into a linear two-degree-of-freedom model, as shown in Figure 1.

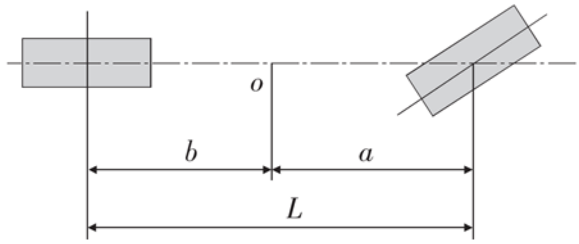

Figure 1: Two-degree-of-freedom vehicle model

During the analysis of the two-degree-of-freedom model, it is assumed that:

The influence of the steering system on the handling stability is ignored, and the front wheel Angle is directly used as the input; Ignoring the impact of the suspension system, the car only moves parallel to the ground; Assume that the speed u of the car is constant; tire sideward characteristics are in the linear range; The driving force is not large, ignoring the impact of the ground tangential force on the tire side characteristics; Ignore air resistance; Ignore the changes in tire characteristics caused by changes in load on the left and right wheel tires and the role of tire righting torque.

Finally, a linear 2-DOF vehicle handling stability model containing only lateral motion along the y axis and yaw motion around the z axis is obtained [2].

2.2 Two-degree-of-freedom vehicle model establishment

According to Figure 1, after making reasonable assumptions about the model, it can be seen that the sum of the resultant force along the Y-axis and the torque around the center of mass of the external force on the two-degree-of-freedom vehicle is as follows:

$$\begin{cases} m(\dot{v}+u\omega_{\mathrm{r}}) = F_{Y1}\cos\delta + F_{Y2} \\ I_Z\,\dot{\omega}_{\mathrm{r}} = aF_{Y1}\cos\delta - bF_{Y2} \end{cases} \tag{1}$$

Where: *m* is the mass of the car (kg); *v* is the longitudinal speed of the car (m/s); *u* is the forward speed of the car (m/s); ω_{r} is the yaw velocity of the car (rad/s); I_Z is the moment of inertia of the car (kg·m^2); δ is the front wheel Angle (°); F_{Y1} and F_{Y2} are the lateral reaction force of the ground facing the front and rear wheels, that is, the lateral bias force. *a* and *b* are the distances (m) from the front and rear axles of the vehicle to the center of mass.

According to Figure 1 and formula (1), we can obtain the differential equation of linear two degrees of freedom of vehicle considering and ignoring the front wheel Angle. If the front wheel Angle δ is considered, the two-degree-of-freedom differential equation of vehicle motion considering the front wheel Angle is obtained:

$$\begin{cases} m(\dot{v}+u\omega_{\mathrm{r}}) = (k_1\cos\delta + k_2)\beta + \dfrac{1}{u}(ak_1\cos\delta - bk_2)\omega_{\mathrm{r}} - k_1\delta\cos\delta \\ I_z\,\dot{\omega}_{\mathrm{r}} = (ak_1\cos\delta - bk_2)\beta + \dfrac{1}{u}(a^2k_1\cos\delta + b^2k_2)\omega_{\mathrm{r}} - ak_1\delta\cos\delta \end{cases} \tag{2}$$

In the formula: k_1、 k_2 are the total lateral stiffness (N/rad) of the front and rear tires of the vehicle respectively. β is the lateral deflection Angle (°) of the center of mass of the car.

According to equation (2), when the front wheel Angle δ is 0°, that is, $cos\,\delta$, the linear two-degree-of-freedom differential equation of vehicle motion ignoring the front wheel Angle can be obtained:

$$\begin{cases} m(\dot{v}+u\omega_{\mathrm{r}}) = (k_1 + k_2)\beta + \dfrac{1}{u}(ak_1 - bk_2)\omega_{\mathrm{r}} - k_1\delta \\ I_z\,\dot{\omega}_{\mathrm{r}} = (ak_1 - bk_2)\beta + \dfrac{1}{u}(a^2k_1 + b^2k_2)\omega_{\mathrm{r}} - ak_1\delta \end{cases} \tag{3}$$

2.3 Assumption of three-degree-of-freedom vehicle model

Due to the oversimplification of the above two degrees of freedom model, the application scope of the model is greatly limited. When the lateral acceleration of the vehicle is greater than 0.4g and the acceleration steering, the two-degree-of-freedom model can not simulate the actual state of the vehicle properly. In the actual driving process, the longitudinal speed of the vehicle changes at any time, and the changing speed will directly or indirectly affect the sideward characteristics and handling stability of the tire. Therefore, the longitudinal speed and its change should be considered in the study of vehicle handling stability [3].

Finally, a three-degree-of-freedom vehicle nonlinear model considering longitudinal velocity, lateral velocity and yaw velocity of vehicle centroid is established, as shown in figure 2.

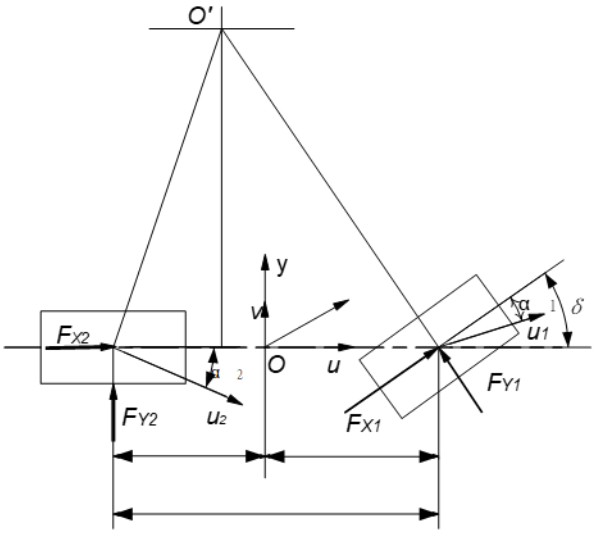

Figure 2: Linear three-degree-of-freedom vehicle nonlinear model

The assumptions made in establishing the three-degree-of-freedom vehicle model are as follows:

1) Ignore the influence of steering system, and directly use the front wheel Angle as input;
2) Ignoring the role of the suspension, the carriage has no vertical, pitch and side movement;
3) Assuming that the wheel load transfer of the vehicle does not occur, the wheels of the front and rear axles of the car are replaced by a wheel located in the middle of the axle, and the entire vehicle is simplified into a "two-wheel motorcycle model";
4) Ignore air resistance.

In this way, the vehicle is reduced to a three-degree-of-freedom model with longitudinal, lateral, and yaw motions [4].

2.4 Establishment of three-degree-of-freedom vehicle model

According to the force situation, the vehicle dynamics equation is as follows:

$$\begin{cases} \sum F_X = F_{X1}\cos\delta - F_{Y1}\sin\delta + F_{X2} = m(\dot{u} - v\omega_r) \\ \sum F_Y = F_{X1}\sin\delta + F_{Y1}\cos\delta + F_{Y2} = m(\dot{v} + u\omega_r) \\ \sum M_z = a(F_{X1}\sin\delta + F_{Y1}\cos\delta) - bF_{Y2} = I_z\dot{\omega}_r \end{cases} \tag{4}$$

Formula (4) is the equation of the vehicle moving along the forward direction, sideways and yaw in turn. In the formula, F_{X1} and F_{X2} are the lateral reaction force of the ground facing the front and rear wheels.

In order to increase the scope of application of the model, considering the situation that the side deflection characteristics are in the nonlinear region, this paper calculates the tire side force through the tire magic formula. The magic formula is as follows:

$$F = D\sin(C\arctan(B\alpha - E(B\alpha - \arctan B\alpha))) \tag{5}$$

In the above formula, F is the tire lateral force, α is the tire side deflection Angle, B is the stiffness factor, C curve shape factor, D peak factor, and E is the curve curvature factor. Parameter values are selected as shown in Table 1 [5].

$$\begin{cases} \alpha_1 = \arctan\left(\dfrac{v+a\omega}{u}\right)-\delta \\ \alpha_2 = \arctan\left(\dfrac{v-b\omega}{u}\right) \end{cases} \tag{6}$$

In the above formula: α_1 is the front wheel side Angle, α_2 is the rear wheel side Angle.

Table 1: Magic formula parameters

Tyre	**B**	**C**	**D**	**E**
front wheel	6.7651	1.3	6436.8	-1.999
rear-wheel	9.0021	1.3	5430	-1.999

If the vehicle is rear-wheel drive, the tangential force of the front and rear wheels is as follows:

$$\begin{cases} F_{X1} = -\dfrac{a}{a+b}mgf \\ F_{X2} = \dfrac{T_{tq}i}{r}-\dfrac{b}{a+b}mgf \end{cases} \tag{7}$$

Where, T_{tq} is the driving torque (N·m), f is the wheel rolling resistance coefficient, i is the transmission ratio of the transmission system, and r is the wheel rolling radius (m).

3. Model simulation process

This time, an SUV with good market sales is selected as the simulation object, and the linear 2-DOF vehicle model is modeled and simulated by MATLAB/Simulink. The selected vehicle structure parameters are shown in Table 2 [6].

Table 2: Vehicle structure parameters

Name	**Unit**	**Value**
Vehicle mass m	kg	1680
Distance from front wheel to center of mass a	m	1.482
Distance from rear wheel to center of mass b	m	1.531
Yaw moment of inertia I_z	kg·m^2	3916
Total front wheel lateral stiffness k_1	N/rad	-63427
Total rear wheel lateral stiffness k_2	N/rad	-110168

The fitting factors that need to be used in the use of the magic formula have been calculated using the fitting factors for each parameter in Table 1, as shown in Table 3 [7].

Table 3: Vehicle structure parameters

name of parameter	**a_1**	**a_2**	**a_3**	**a_4**	**a_5**	**a_6**
parameter values	-22.1	1011	1078	1.82	0.208	0
name of parameter	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
parameter values	-0.354	0.707	0.028	0	14.8	0.022

To facilitate the calculation, $i = 5.67$, r=0.316m, T_{tq}=125.53N·m, $f = 0.02$ are designed. In the process of model simulation, R2021a version of MATLAB is used. During the simulation, the Normal mode is used, the simulation step is set to "Fied-step", the sampling time is 0.01s, and the total duration is 5s. A two-degree-of-freedom vehicle model considering the front wheel Angle and a three-degree-of-freedom vehicle model with tire lateral stiffness in the nonlinear range were established. Through the input of the same parameters, the output side deflection Angle of the center of mass and yaw Angle velocity of the two models of two degrees of freedom are compared. In addition, the effects of vehicle speed, front wheel Angle and side deflection characteristics on vehicle handling stability are analyzed on a two-degree-of-freedom vehicle model considering front wheel Angle.

Simulink modeling was carried out on a two-degree-of-freedom vehicle model considering the front wheel Angle, as shown in Figure 3.

Figure 4 shows the Simulink modeling of a three-degree-of-freedom automobile model with tire side stiffness in the nonlinear range.

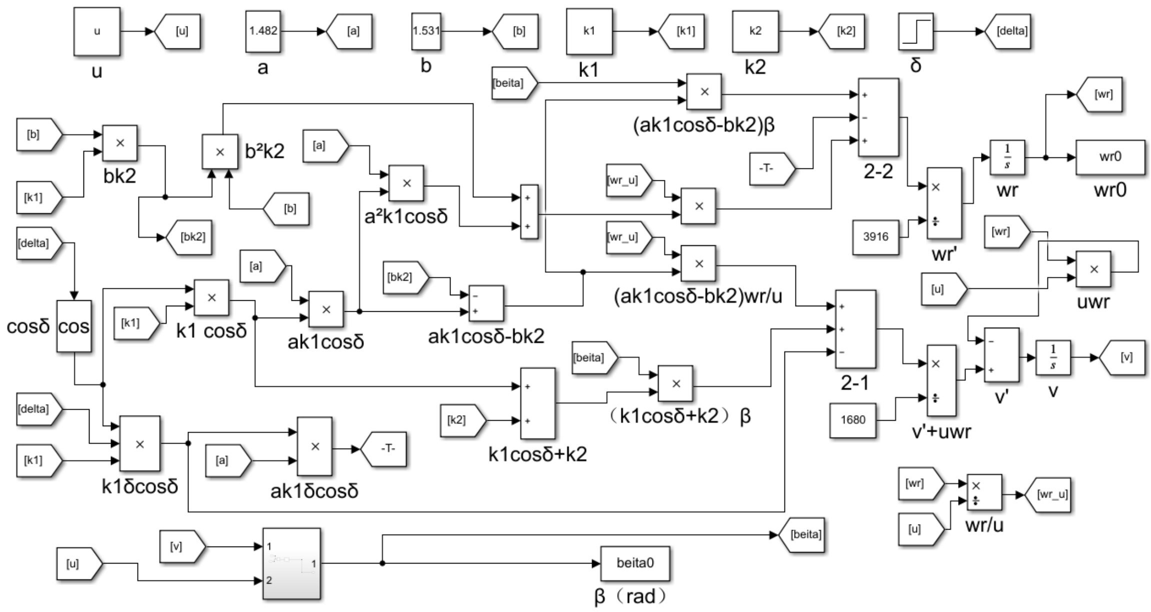

Figure 3: Two-degree-of-freedom Simulink model considering side deflection Angle

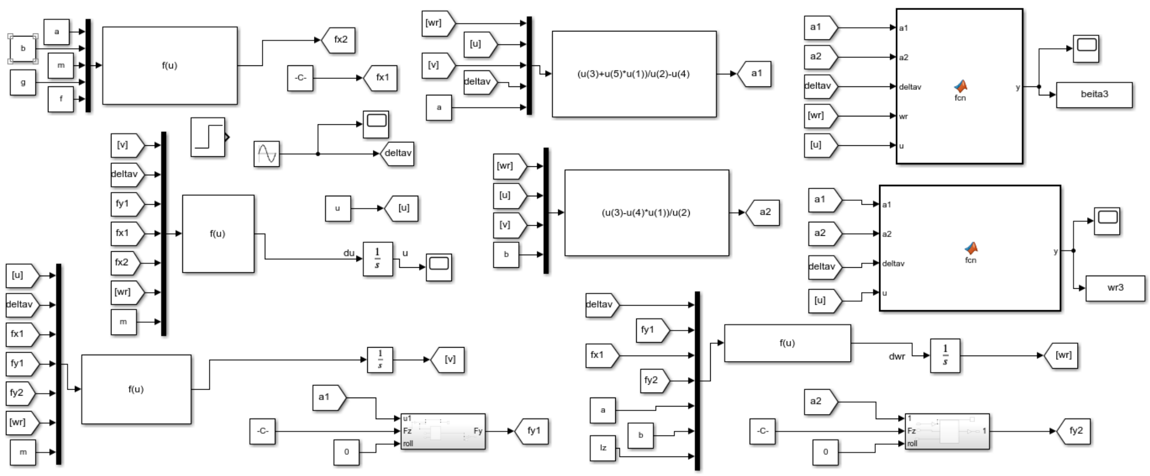

Figure 4: Three-degree-of-freedom vehicle model with tire lateral stiffness in the nonlinear range

In the two-degree-of-freedom vehicle model considering the front wheel Angle, the side stiffness of the front wheel is -73160N/rad, -74160N/rad, -75160N/rad, -76160N/rad, and -77160N/rad, respectively. The remaining structural parameters of the car remain unchanged, and the car travels at a speed of 30m/s. When the simulation time is 1s, a step signal is given to the front wheel, respectively making the front wheel Angle change from 0° to 15°, and keeping the Angle unchanged. At this time, the yaw velocity response curve is shown in Figure 5, and part of the yaw velocity response data are shown in Table 4. The response curve of centroid side deflection Angle is shown in Figure 6, and the partial data of centroid side deflection Angle response are shown in Table 5.

In the three-degree-of-freedom model of automobile, the vehicle speed is 20m/s, and the sine function $\delta = 15^{\circ}\sin(2\pi t)$ of the front wheel Angle is given to consider the linear transformation and nonlinear transformation of the wheel side deflection characteristics respectively, and the other parameters of the automobile are unchanged. The difference between the two is compared. At this time, the yaw angular velocity response curve is shown in figure 7, and the side deflection Angle response curve of the center of mass is shown in figure 8.

A sine function of $\delta = 15^{\circ}\sin(2\pi t)$ is given to the front wheel Angle, and the vehicle speed is taken as 20m/s. The two-degree-of-freedom vehicle model with front wheel Angle and the three-degree-of-freedom vehicle model with tire lateral stiffness in the nonlinear range are considered respectively, and the other parameters of the vehicle are unchanged. At this time, the yaw angular velocity response curve is shown in figure 9, and the side deflection Angle response curve of the center of mass is shown in figure 10.

Table 4: Vehicle yaw velocity ω_r data

	serial number	1	2	3	4	5	6	7	8	9	10
yaw velocity	**Time/s**	0.50	1.00	1.10	1.20	1.30	1.35	1.38	1.39	1.40	1.41
	-73160N/rad	0.000	0.000	0.537	0.941	1.229	1.335	1.388	1.405	1.420	1.435
	-74160N/rad	0.000	0.000	0.537	0.939	1.223	1.327	1.379	1.395	1.410	1.424
	-75160N/rad	0.000	0.000	0.537	0.937	1.218	1.319	1.370	1.385	1.399	1.413
	-76160N/rad	0.000	0.000	0.536	0.935	1.212	1.312	1.361	1.375	1.389	1.402
	-77160N/rad	0.000	0.000	0.536	0.933	1.207	1.304	1.352	1.366	1.379	1.392
	serial number	11	12	13	14	15	16	17	18	19	20
Tyaw velocity	**Time/s**	1.45	1.50	1.60	1.70	1.80	2.00	2.30	2.50	3.00	4.00
	-73160N/rad	1.487	1.538	1.600	1.626	1.626	1.591	1.522	1.488	1.455	1.457
	-73160N/rad	1.474	1.521	1.578	1.598	1.594	1.552	1.480	1.447	1.417	1.421
	-74160N/rad	1.461	1.506	1.557	1.572	1.563	1.515	1.440	1.408	1.382	1.388
	-75160N/rad	1.448	1.490	1.536	1.546	1.533	1.480	1.403	1.372	1.350	1.357
	-76160N/rad	1.435	1.475	1.515	1.520	1.504	1.446	1.367	1.338	1.321	1.328
	-77160N/rad	1.410	1.445	1.476	1.472	1.448	1.382	1.304	1.278	1.268	1.276

Table 5: Data of vehicle centroid side declination Angle β

	serial number	1	2	3	4	5	6	7	8	9	10
side slip angle	**Time/s**	0.50	1.00	1.02	1.04	1.06	1.08	1.10	1.30	1.50	1.60
	-73160N/rad	0.000	0.000	0.005	0.008	0.008	0.006	0.003	-0.094	-0.220	-0.274
	-73160N/rad	0.000	0.000	0.005	0.008	0.008	0.006	0.003	-0.094	-0.218	-0.270
	-74160N/rad	0.000	0.000	0.005	0.008	0.008	0.006	0.003	-0.093	-0.215	-0.266
	-75160N/rad	0.000	0.000	0.005	0.008	0.008	0.006	0.003	-0.092	-0.212	-0.262
	-76160N/rad	0.000	0.000	0.005	0.008	0.008	0.006	0.003	-0.092	-0.210	-0.258
	-77160N/rad	0.000	0.000	0.005	0.008	0.008	0.006	0.003	-0.090	-0.205	-0.250
	serial number	11	12	13	14	15	16	17	18	19	20
side slip angle	**Time/s**	1.70	1.80	1.90	2.00	2.10	2.20	2.30	2.50	3.00	4.00
	-73160N/rad	-0.318	-0.353	-0.379	-0.398	-0.412	-0.420	-0.426	-0.429	-0.424	-0.419
	-73160N/rad	-0.313	-0.346	-0.370	-0.388	-0.400	-0.407	-0.411	-0.413	-0.407	-0.402
	-74160N/rad	-0.307	-0.338	-0.361	-0.377	-0.388	-0.394	-0.397	-0.398	-0.390	-0.387
	-75160N/rad	-0.301	-0.331	-0.353	-0.368	-0.377	-0.382	-0.384	-0.383	-0.375	-0.372
	-76160N/rad	-0.296	-0.324	-0.345	-0.358	-0.366	-0.370	-0.371	-0.370	-0.361	-0.359
	-77160N/rad	-0.286	-0.311	-0.329	-0.339	-0.345	-0.348	-0.348	-0.344	-0.335	-0.334

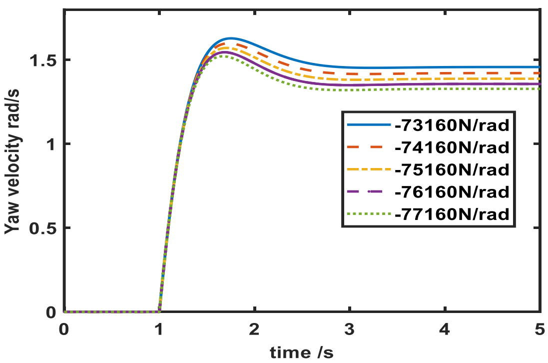

Figure 5: yaw velocity curve of vehicle under linear change of lateral stiffness

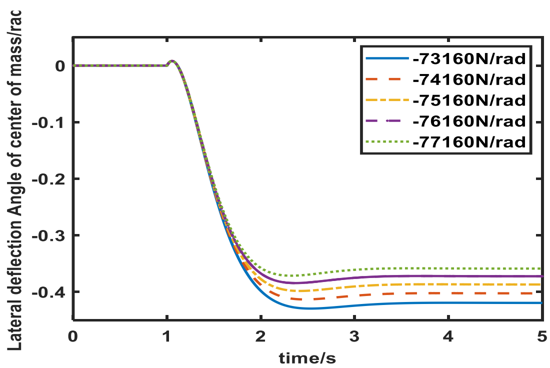

Figure 6: lateral deflection Angle curve of vehicle center of mass under linear change of lateral stiffness

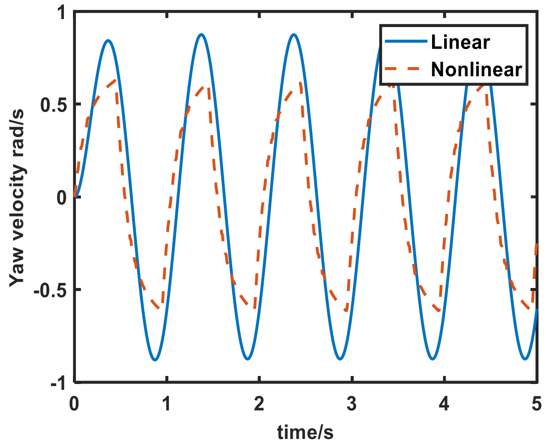

Figure 7: yaw angular velocity curve of three-degree-of-freedom vehicle

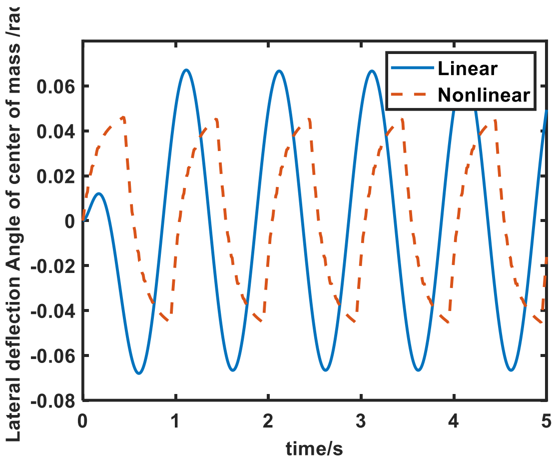

Figure 8: Lateral declination curve of three-degree-of-freedom vehicle centroid

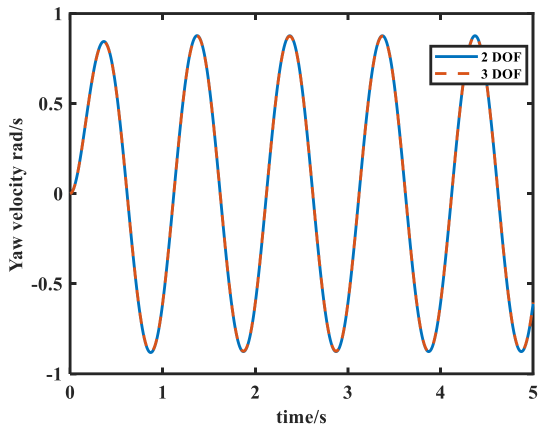

Figure 9: yaw velocity curves of two-degree-of-freedom and nonlinear three-degree-of-freedom models

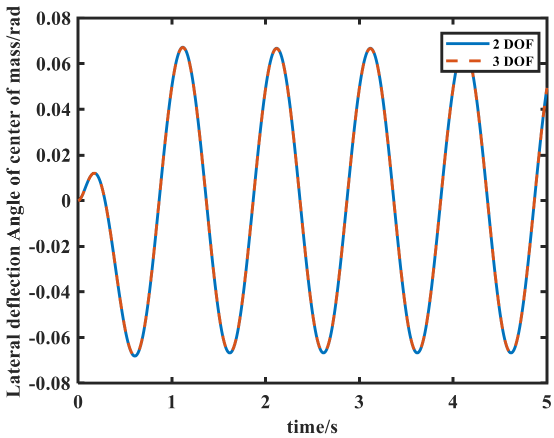

Figure 10: Lateral declination curve of vehicle center of mass in 2-DOF and nonlinear 3-DOF models

According to Fig. 5 and Fig. 6, it can be observed that with the increase of the lateral stiffness (absolute value) of the front wheel, the overshot of the yaw velocity slightly decreases and is basically the same, and the time for stabilizing is also gradually shortened. It can be concluded that the greater the lateral stiffness (absolute value), the better the handling stability of the vehicle.

With the increase of the side stiffness of the front wheel, the smaller the side deflection Angle of the vehicle center of mass, the shorter the time for the vehicle to enter the steady state. When the lateral rigidity is -77160N/rad, the lateral deflection Angle of the vehicle entering the steady state is the smallest, and the absolute value of the lateral deflection Angle is 0.334rad. When the lateral rigidity is -73160N/rad, the lateral deflection Angle of the car entering the steady state is the largest, and the absolute value of the lateral deflection Angle is 0.424rad. When the lateral stiffness is smaller, the lateral deflection Angle of the center of mass entering the steady state is larger, and the control stability is worse.

As can be seen from FIG.7, in the case of a three-degree-of-freedom model with linear transformation and nonlinear transformation taking into account wheel side deflection characteristics, it can be seen that the yaw velocity period and frequency are basically the same, with only a slight difference in peak value. The peak yaw velocity of the linear three-degree-of-freedom model with side-deflection characteristics is 0.8753N/rad, and the peak yaw velocity of the nonlinear three-degree-of-freedom model with side-deflection characteristics is 0.6168N/rad.

As can be seen from Fig. 8, in the case of a three-degree-of-freedom model with linear transformation and nonlinear transformation taking into account the side-deflection characteristics of wheels, it can be seen that the side-deflection curve of the linear three-degree-of-freedom model is 0.32s ahead of that of the nonlinear three-degree-of-freedom model, and the period and frequency are basically the same. In terms of peak value, the peak value of lateral deflection Angle is 0.0667rad for the linear three-degree-of-freedom model with lateral deflection characteristics, and the peak value of yaw velocity is 0.0454rad for the nonlinear three-degree-of-freedom model with lateral deflection characteristics.

It can be seen from Fig. 9 and Fig. 10 that the two-degree-of-freedom vehicle model considering the front wheel Angle is basically consistent with the three-degree-of-freedom vehicle model whose tire side stiffness is in the linear range, and the resulting yaw velocity basically coincides with the side deflection Angle of the center of mass. Therefore, the situation of wheel side deflection characteristics in the linear and nonlinear transformation of the three-degree-of-freedom model is basically the same as that considering the front wheel Angle of the two-degree-of-freedom model and the nonlinear transformation of the three-degree-of model.

Summary of experiment 4: It can be concluded from the simulation data that with the increase of the side deflection stiffness (absolute value) of the front wheel of the vehicle, its yaw velocity and side deflection Angle of the center of mass will be smaller, the time to enter the steady state will be shorter, and the stability and maneuverability of the vehicle will be improved. The increase of the lateral stiffness value in a certain linear change can help to improve the handling stability of the vehicle.

For the three-degree-of-freedom model constructed in this paper, considering that the wheel yaw characteristics are under linear transformation and nonlinear transformation, the yaw velocity period, evaluation rate and phase of the two are basically the same, only the peak value is different. The three-degree-of-freedom model with the wheel yaw characteristics under linear transformation has a higher peak value than the three-degree-of-freedom model with the wheel yaw characteristics under nonlinear transformation.

In terms of the side deflection Angle of the center of mass, the period, evaluation rate and peak value of the side deflection Angle of the center of mass of the two are basically the same, and the peak value is different. The three-degree-of-freedom model with wheel side deflection characteristics under linear transformation has a higher peak value than that of the three-degree-of-freedom model with wheel side deflection characteristics under nonlinear transformation, but the curve lag is 0.32s.

Since the three-degree-of-freedom model with linear and nonlinear transformation of wheel side deflection characteristics is basically the same as the two-degree-of-freedom model with front wheel Angle and the three-degree-of-freedom model with nonlinear transformation, the feasibility of the three-degree-of-freedom model with linear transformation of wheel side deflection characteristics is demonstrated.

4. Conclusion

Aiming at the control stability of the two-degree-of-freedom vehicle model, MATLAB/Simulink was used for modeling and simulation. The front wheel Angle was used as input to obtain the variation curves of the vehicle yaw velocity and the side deflection Angle of the center of mass, and the resulting curves were analyzed.

By controlling variables and analyzing the effects of vehicle speed, front wheel steering angle, and lateral stiffness on the handling stability of a car, simulation data shows that the greater the lateral stiffness of the car, the shorter the overshoot of the yaw rate and the time to enter steady state, and the smaller the lateral deviation angle of the center of mass, indicating better handling stability of the car. The lower the vehicle speed, the smaller the front wheel steering angle, and the greater the lateral stiffness, the better the handling stability of the car.

Regarding the handling stability of a three degree of freedom car model with nonlinear wheel lateral stiffness, a comparison was made with a two degree of freedom car model considering front wheel angle and a three degree of freedom car model with linear wheel lateral stiffness. It was found that the three degree of freedom car model with non-linear tire lateral stiffness was more consistent with the two degree of freedom car model when the vehicle speed was 20m/s and the front wheel angle was around 15 °.

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