



Applied Mathematical Modeling of Monitoring the Station's Movement using Total Station

Sobhy Abdel^a, Monam Younes^b

^aPublic Works Engineering Department, Faculty of Engineering, Tanta University, 31733, Tanta city, Egypt.
Email: sobhy_younes@f-eng.tanta.edu.eg

^bHigher Future Institute of Engineering and Technology in Mansoura, Mansoura City, Egypt.
ORCID: <https://orcid.org/0000-0002-9767-4006>

Abstract The paper concerns the practical application of Total Station in the case of ground deformation monitoring. The use of an electronic total station during instrumental observations allows you to speed up the survey process and automate the processing of results. For this, it is necessary to carry out a mathematical assessment of measurement errors and check the accuracy of results to meet the tolerances of regulatory documents. Paper presents the existing technology for monitoring the earth's surface using traditional surveying equipment. A diagram of observations at the station using an electronic total station is presented and accuracy requirements are given. A mathematical model for calculating errors at the station was developed. On its basis, mathematical modeling of the survey conditions with an electronic total station was investigated and a statistical analysis of factors affecting the accuracy of monitoring the earth's surface was carried out. As a result of the research, the range of surveying with an electronic total station at surveying observation stations was determined, in which the errors in determining the lengths of intervals and elevations between benchmarks of observation stations satisfy the requirements of accuracy. Recommendations are given for surveying conditions using electronic total stations of various accuracy classes.

Keywords Applied mathematical modeling, errors, electronic total station, deformations, observation station.

1. Introduction

One of the main aspects of the development of any country is the introduction of modern high-performance technologies into the production process. For all surveying stations, the task of geodetic support of observations of the earth's surface deformations is urgent. In this case, such technologies can be represented by electronic tacheometry, laser scanning, satellite imaging methods, successfully used in the fields of mining, oil and gas industries, energy, construction, architecture, etc. [1]. Choose the measurement method depends upon the accuracy requirements for the survey [2]. Using an inappropriate method to monitor deformations in terms of accuracy has a significant influence on the magnitudes and directions of specific deformations (movements) [3]. According to Engineer Manual (2018), surveying accuracy specifications are intended to ensure that a certain amount of movement is detected under normal operating conditions [4].

The use of total stations to monitor the movement of structures has been reported with good results by many authors, such as [5-8]. With the development of electronic theodolites, total stations (TS) appeared, and later with automated robotic total stations (RTS), which allowed the new generation of automated measurements, found a wide area of use [9]. The monitoring level using RTS was reached with a sampling interval of 5-10 Hz and monitoring of the moving reflectors. Because of these advantages, it is widely used in many surveying and other engineering projects [10-13]. The methodology for observing the displacement of the earth's surface and



deformations of undermined objects was presented. Measurements carried out are characterized by high accuracy, but at the same time a significant amount of work at the station. In this regard, using electronic observation methods, which ensure rapid recording and processing of results, was preferred. High-precision electronic tacheometers have such functions. However, an assessment of the errors of electronic tacheometry is required to make a decision on the possibility of including it in the process of monitoring the earth's surface.

The measurement intervals can be adjusted according to the requirements of the monitoring program, and the measurement intervals of high-risk areas can be adjusted according to priority, providing more frequent monitoring of these areas. Monitoring surveys make use of two data sources, namely data measured by the RTS (e.g. distances and angles) and data external to the total station (e.g. meteorological sensor). External data to the comprehensive station are supplementary data (i.e. atmospheric condition measurements, i.e. ambient temperature and atmospheric pressure, for atmospheric corrections) [14]. Furthermore, the introduction of supportive automated monitoring data processing software such as the Leica GeoMoS (Geodetic Monitoring Software) and APSWin (Automatic Polar System for Windows) has enhanced the use of total stations, as automated measurements can be carried out at a predetermined schedule [15].

The purpose of the study is to develop recommendations for surveying profile lines using electronic total stations based on evaluating the accuracy of observations. For this, it is necessary to solve the following problems:

- Creation of possible survey schemes using electronic tacheometry;
- Development of a mathematical modeling for formulas for the accumulation of errors when determining distances and elevations;
- Repeated modeling of survey conditions at the station and identification of optimal geometric and accuracy observation parameters;
- Analysis and statistical processing of modeling results;
- Recommended conditions for observing with an electronic total station were presented.

2. Methodology

Design specifications are carried out in collaboration with a geotechnical engineer (i.e. expected magnitude of movement, parameters to be measured, type and size of deformation to be monitored, purposes of various instruments, equipment locations, required accuracy/precision, checks using different surveying methods and equipment) have a significant influence on the selection of slope monitoring equipment [14]. [16], emphasized that the selection of survey monitoring equipment depends on the economic value, the level of confidence required in the results, ease of interface (i.e. compatibility with other monitoring techniques), adaptability of GIS, environmental conditions, survey budget and necessary survey training, for optimal use. Equipment required for prism monitoring surveys includes a robotic total station (RTS), a total station shelter, equipment to measure weather conditions, pole beacons (for transmitted and reference signals) and a prism for monitoring points [17]. A typical monitoring station should consist of at least two profile lines. The lengths of the profile lines are determined depending on the slope angle of the formation and the presence of old workings.

In accordance with the traditional observation methodology, the complete series of works at the station consists of leveling benchmarks, measuring the distances between them along profile lines and photographing cracks on the earth's surface, recording the time of their appearance and the magnitude of their opening. When performing observations with an electronic total station, the survey scheme is presented in (Figure 1). The station where the instrument is positioned is located at a certain distance to the side of the profile line (T, T1) or within its alignment (T2). Before starting observations, orientation is performed to the initial reference point T0 (setting the directional angle α_0 and the coordinates of the initial point T). Then sequential observation of benchmarks is carried out by determining distances S_i , horizontal angles β_i and vertical angles δ_i . Based on the measurement results, the coordinates of the reference points P1, P2, ..., Pn are calculated; from them the lengths of intervals ($L_i, i+1$) and excesses ($H_i, i+1$) are calculated.

In previous studies, the requirements for the accuracy of measurements the observation station are presented: variations in the length of the intervals between benchmarks should not exceed 1:10,000; the different in the amounts of errors between forward and reverse observations (in mm) should not be more than $15 \text{ mm} \sqrt{L}$, where



L is the observation length in one direction, km [18]. The specified requirements for measuring lengths correspond to the requirements for polygonometry of the 1st category, and for leveling - the average value of the different between the III and IV classes of leveling, therefore, the mean square error of the III leveling class, which is 5 mm, is used as an acceptable value [18].

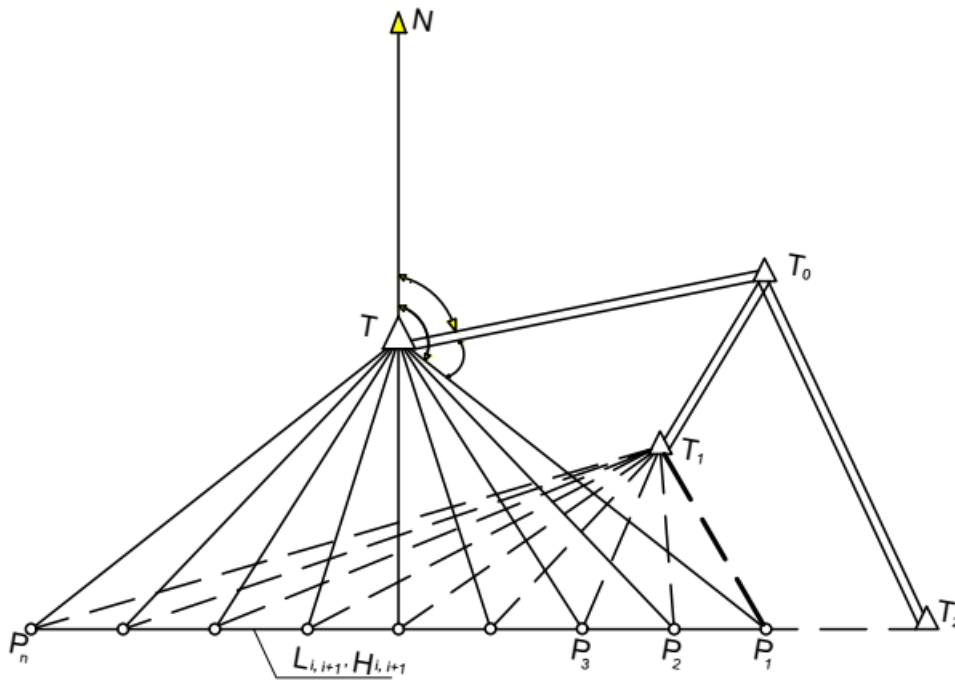


Figure 1: Diagram of observing a profile line with an electronic total station.

To assess the accuracy, a mathematical modeling was developed by analyzing the errors in the lengths of intervals and its variations, defined as the Root Mean Square errors of the measured values. Root mean square errors (RMS) of coordinates are calculated using the formulas [19]:

$$m_{x_i}^2 = m_{x_{ct}}^2 + (\cos(\alpha_0 + \beta_i) \cos \delta_i)^2 m_s^2 + (S_i \sin \delta_i \cos(\alpha_0 + \beta_i))^2 \frac{m_\delta^2}{\rho^2} + (S_i \cos \delta_i \sin(\alpha_0 + \beta_i))^2 \frac{m_\alpha^2 + m_\beta^2}{\rho^2};$$

$$m_{y_i}^2 = m_{y_{ct}}^2 + (\sin(\alpha_0 + \beta_i) \cos \delta_i)^2 m_s^2 + (S_i \sin \delta_i \sin(\alpha_0 + \beta_i))^2 \frac{m_\delta^2}{\rho^2} + (S_i \cos \delta_i \cos(\alpha_0 + \beta_i))^2 \frac{m_\alpha^2 + m_\beta^2}{\rho^2};$$

$$m_{z_i}^2 = m_{z_{ct}}^2 + (\sin \delta_i)^2 m_s^2 + (S_i \cos \delta_i)^2 \frac{m_\delta^2}{\rho^2} + m_i^2 + m_v^2,$$

where m_x, m_y, m_z - RMS for determining the coordinates of a point; $m_{x_{ct}}, m_{y_{ct}}, m_{z_{ct}}$, RMS for coordinates of the initial point; m_s — RMS for length measurement; $m_\alpha, m_\beta, m_\delta$ — RMS of the directional angle of the initial direction, horizontal and vertical angles; m_i, m_v — RMS for measuring instrument height and sighting.



Based on the RMS coordinates, errors in determining lengths and elevations between adjacent reference points P_i and P_{i+1} are calculated by formulas [19]:

$$m_L^2 = \left\{ \frac{x_i - x_{i+1}}{L} \right\}^2 (m_{x_i}^2 + m_{x_{i+1}}^2) + \left\{ \frac{y_i - y_{i+1}}{L} \right\}^2 (m_{y_i}^2 + m_{y_{i+1}}^2) + \left\{ \frac{z_i - z_{i+1}}{L} \right\}^2 (m_{z_i}^2 + m_{z_{i+1}}^2);$$

$$m_H^2 = m_{z_i}^2 + m_{z_{i+1}}^2,$$

where m_L - is the RMS for determining the distance between benchmarks; m_H — RMS for determining the excesses between benchmarks; x_i, y_i, z_i — coordinates of benchmark i .

The optimal monitoring parameters are determined based on the results of repeated modeling of survey conditions, which include station geometry, observation errors and technical characteristics of the equipment used. Based on the above mathematical models of error accumulation, software tools were developed to automate calculations. To determine the best conditions, 5,000 different types of monitoring stations were simulated. The survey scheme assumes that the electronic total station is located opposite the middle of the profile line (see Fig. 1). The shortest distance from the point of the tool to the profile line during the modeling process varies from 5 to 100 m (in increments of 5 m). The interval between benchmarks is assumed to be 5; 10; 15; 20 and 25 m. In order to take into account changes in the station geometry, calculations are performed for an even and odd number of benchmarks (50 and 51), with different vertical survey angles ($0^\circ; 5^\circ; 10^\circ; 15^\circ; 20^\circ$). In addition, the technical characteristics of the equipment used are taken into account: angle measurement accuracy - $1''; 2''; 3''; 5''; 7''$; distance measurement accuracy (with reflector) ± 2 mm/km. The speed of the automated measurement and recording process is proportional to the sampling rate of the measurement process [11], [12], and [20].

3. Results and Discussion

Based on the analysis of the calculation results, the following conclusions were drawn:

- RMS lengths of intervals and elevations between benchmarks increase with increasing distance from the tool to the profile line;
- Increasing the interval between benchmarks has some effect on the values of the RMS of distances and elevations;
- The use of electronic total station with angular accuracy of $1''$, $2''$, $3''$ can significantly reduce errors in determining distances and elevations at distances >40 m;
- The inclination angle within 20° has an insignificant effect on the RMS of distances and elevations.

Figure 2 shows the dependence of the error in determining the interval length on the distance to the tachometer station point. The curve describes changes in errors for profile lines with different intervals between reference points; the straight red line indicates the acceptable observation accuracy. Analysis of the results of figure 2 proves that the RMS of lengths decreases as it close to station of observation point and increases for more intervals between benchmarks.



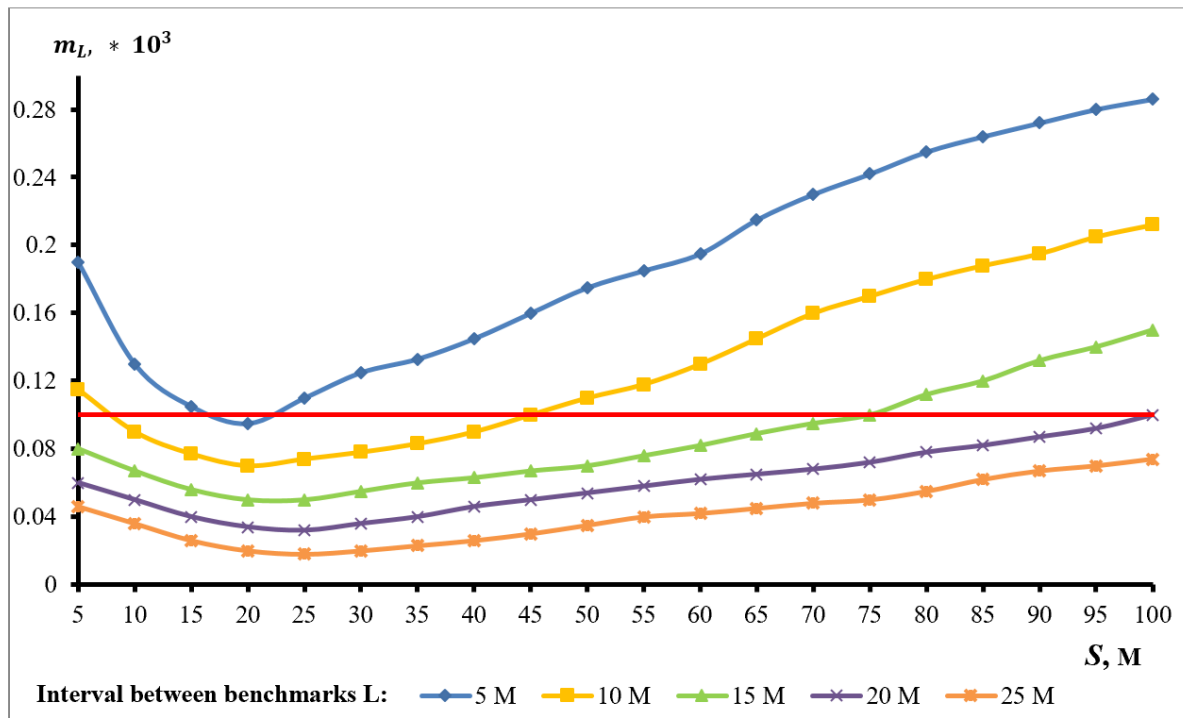


Figure 2: The dependent of the errors m_L of the lengths on the distance S to the point of the instrument.

in Figure 3 and 4 show the distribution of errors along the central part of the observation station (fragment of 11 benchmarks, the interval between benchmarks is 20 m, the observing from a distance of 20 m, the angular accuracy of the electronic total station is 5"), here the electronic total station is located opposite the middle of the line and is indicated by a triangle. Obviously, the observing range does not cover all the profile line benchmarks within the acceptable accuracy of determining distances; At the same time, the RMS of the variations were obtained in accordance with the tolerances. From these figures, it is observed that increasing the distance between total station and the observed target increases the errors in measurements

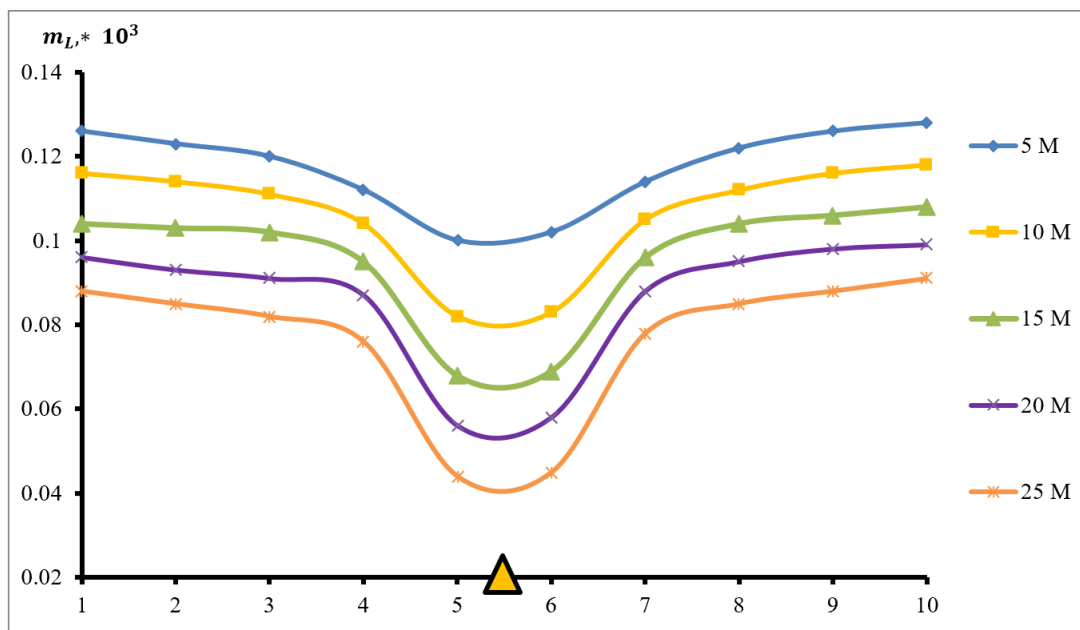


Figure 3: The relation between relative length errors along the station and the interval L between reference points.

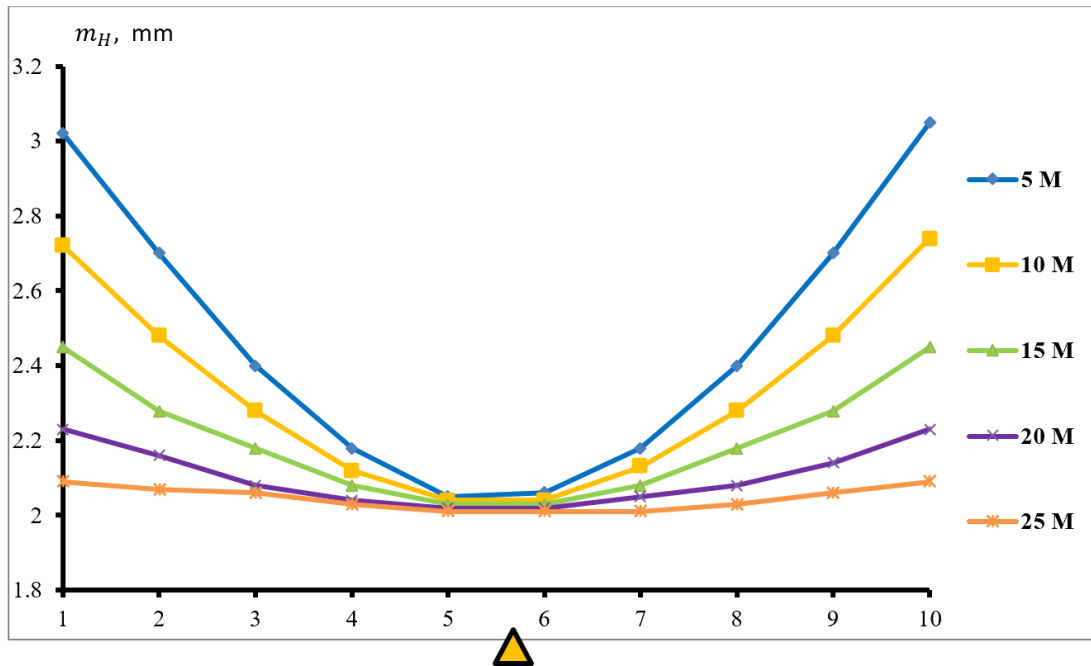


Figure 4: Distribution of elevation errors of the station depending on the interval L between benchmarks.

Figure 5 shows the dependence of the errors in interval lengths on the accuracy of measuring angles with various electronic tacheometers. The use of electronic tacheometers with an angular accuracy of 5" or less may cause errors that do not correspond to the allowed tolerances.

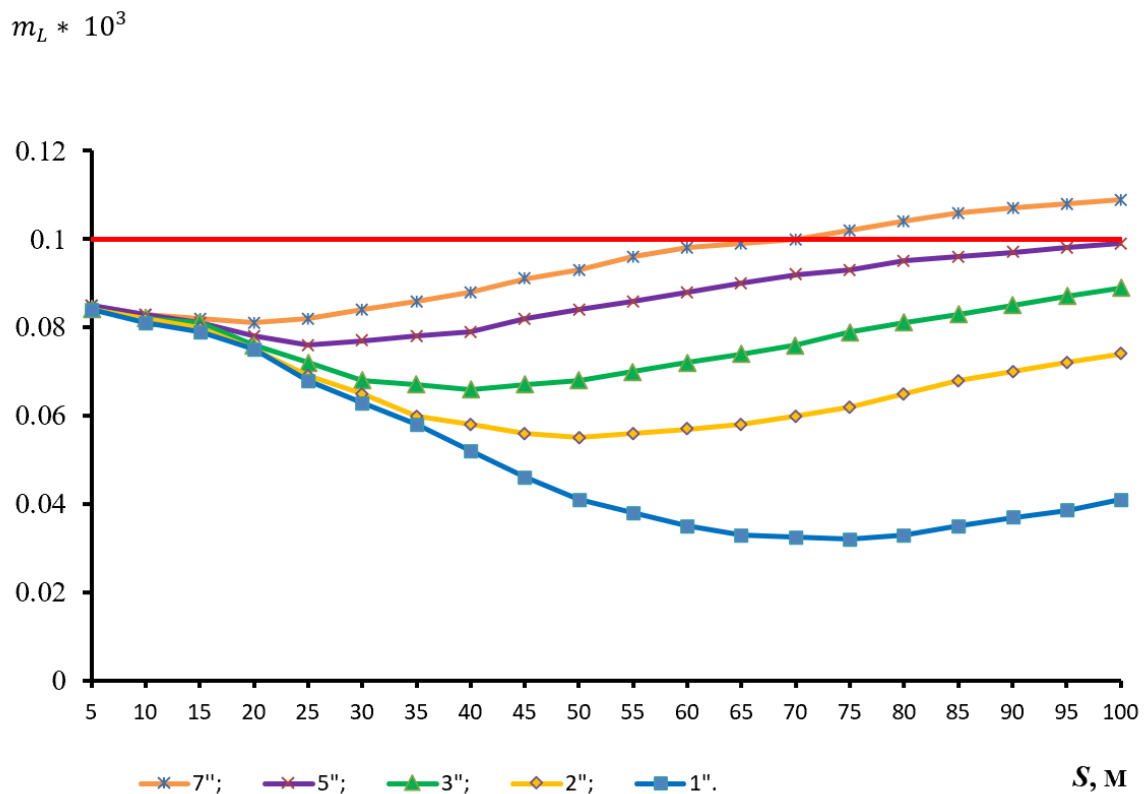


Figure 5: The dependence of the errors in interval lengths on the accuracy of measuring angles.

Based on the observed data, a statistical analysis was performed in the form of a multiple regression calculation for errors and factors influencing their variation. The influence of each factor was assessed on the basis of more samples values of RMS lengths and elevations. The coefficients for calculating multiple regressions are presented in table 1. They allow us to conclude that the distance from which the observing is carried out, as well as the angular accuracy of the electronic total station, has a huge impact on errors. The angle of inclination, on the contrary, has virtually no effect on the accuracy of observations.

Table 1: Multiple regression coefficients

Factors	K_{m_L}	K_{m_H}
Distance to profile line	0.892	0.654
Interval between benchmarks	0.167	0.824
Total station angular accuracy	0.762	0.954
Observing angle	-0.012	0.023

Based on the obtained results and taking into account the above statistical analysis data, the conditions for using an electronic total station were determined under which the errors in distances and elevations between benchmarks meet the requirements of regulatory documents. Recommended observing conditions are given in table. 2.

Table 2: Recommended conditions for monitoring the earth's surface with an electronic total station

L, M	1"		2"		3"		5"		7"	
	N_P	S	N_P	S	N_P	S	N_P	S	N_P	S
5	3	15-40	5	15-25	3	15-40	3	15-20	Not recommended	
	6	45-100	3	30-55	3	15-40	5	15-20		
10	5	20-50	5	25-60	5	15-40	3	15-20	5	10-30
	8	55-100	9	65-100	7	45-65	5	25-35	3	10-30
15	7	20-70	6	20-45	6	25-60	6	15-20	5	5-10
	12	75-100	11	50-100	11	65-75	5	25-55	4	15-45
20	21	5-25	21	5-100	15	65-100	7	55-80	5	45-65
	23	30-65	23	5-100	23	40-60	14	30-50	10	30-40
25	22	70-100	23	5-100	25	5-35	21	5-25	15	5-25
	41	5-100	43	60-85	30	5-100	12	85-95	5	70-85
	43	5-100	41	5-55	29	5-100	17	5-80	12	5-65

Table 2 includes recommendations for five total stations with varying angular accuracies (1"–7"). Each of them can take a profile line with an interval L equal to 5; 10; 15; 20 or 25 m. For all these options, the number of benchmarks N_P , taken with the required accuracy, and the distances S, at which this survey should be carried out, are presented. In some cases, observing is not recommended

4. Conclusions

In conclusion, this paper has given the step-by-step procedures to consider developing a reliable and cost-effective geodetic monitoring system for the monitoring of station’s movements. As a result of the research, the range of surveying with an electronic total station at surveying observation stations was determined, in which the errors in determining the lengths of intervals and elevations between benchmarks of observation stations satisfy the requirements of accuracy.

Carrying out instrumental observations in accordance with developed recommendations allows you to minimize time costs and the amount of work in the monitoring process and switch to the use of electronic equipment without compromising accuracy

Acknowledgement

The authors want to thank the staff members of Surveying Laboratory at high Future institute for engineering and technology in Mansoura for helping in the experimental works and field observations collection data.

Conflict of interest

The authors declare that they have no conflict of interest.

Funding

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. The authors state that they did not get any funding for this research and it is self-funded.

Data availability

All relevant data are within the paper.

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