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Reconstructibility analysis and state estimation of Boolean control networks based on output and flip control

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Abstract In this paper, we study the state estimation problem of BCNs. First, a new concept of output-dependent reconstructible for some special output sequences is proposed. and impose a flip control technique on the set of unreconstructible states. Then, a sufficient condition for all the states in the unreconstructible set of states to reach the reconstructible set of states is given. Third, an algorithm to find the required control sequence is designed so that all states in the unreconstructible state set reach the reconstructible state set. Finally, the corresponding observer is designed to estimate the state of the system. An example is given to illustrate the feasibility of the proposed methods.

Keywords Boolean control networks; reconstructible; co-controller design; state estimates

Introduction

The Boolean network (BN) model, originally proposed by Kauffman and others [1], is a discrete system based on directed graphs. Boolean network model has been widely used in many fields including cell differentiation, immune response, biological evolutionary neural network and gene regulation. The Boolean network with inputs and outputs is regarded as Boolean control network (BCN). A BCN can be seen as a family of BNs, each of them associated with a specific value of the input variables.

In recent years, Cheng et al. have proposed a mathematical tool called Semi-tensor product (STP) [2-3], which can convert the dynamics of BNs into a model that is similar to the standard discrete-time state space model. In this way, logic-based problems can be transformed into algebraic problems. Based on the STP approach, some other theoretical issues of BCNs are studied such as controllability and observability [7, 8, 9, 10], reconstructibility [4, 5, 6,], output tracking [11, 12], perturbation decoupling problem [13, 14], optimal control [15, 16, 17], and other results [18, 19, 23]. But many of these works assume that the state is known or measurable. However, most control systems, including BCNs and others, are unknown or immeasurable. Therefore, the reconstructibility analysis of BCNs and the design of observer are particularly important, which will be the topic of this paper.

State estimation is one of the hot topics in control theory and plays an important role in many fields, such as state feedback stabilization and fault diagnosis based on state estimation. Besides, the reconstructibility is the property that the finial state can be uniquely determined for BNs or BCNs, and the reconstructibility condition of BNs or BCNs is necessary for the existence of state observers and firstly proposed in [6]. Nowadays, many observer design methods have been proposed to uniquely estimate the system state of BCNs. Shift-register observers and multi-state observers were first proposed in the paper [6]. Subsequently, the paper [12] explains the relationship between observability and reconstructibility and proposes a class of Longaberger observers. Most of these papers rely solely on observers to estimate the system state, there is not much paper related to designing controllers. For BCNs, the system state and control input are multiplicatively coupled by the semi-tensor product of matrices. So, the observability and reconstructibility of BCNs are control input-dependent [6]. Therefore, it is necessary to design controllers only to estimate the system state.

Flip control is a new control mechanism with little intervention in the system [21, 22, 23]. It works by changing the value of certain nodes in the BCN from 1 to 0 or from 0 to 1, simulating turning on or off genes in a biological system. Due to its ease of manipulation, many researchers have used flip control to study problems. For example, the paper [21] also investigated attractor controllability of BNs by flipping a subset of nodes in multiple attractor states. [22] studied the attractor stability of BNs by flipping some state nodes in the attractor once after the network passes through a transient period in the attractor. Cheng et al. provided controllability and stability criteria for BCNs by flipping a subset of nodes in some initial states instead of flipping the nodes in the attractor after the network passes through a transient period. Recently, Zhang et al. applied the flipped mechanism to switch Boolean control networks (SBCNs) for stability and set stabilization, where a subset of initial state nodes is considered to be flipped once [23]. In addition, the weak stability of BNs with flipped sequences has been studied in the paper [24]. So far, many researchers have applied state flip control to the stability and controllability of BNs (BCNs). However, there are not many directions to study state estimation of BCNs using flip control techniques.

This paper focuses on the study of Boolean control network output dependence reconstructibility, which is mainly summarized as follows:

1) By studying specific outputs, a new concept of output-dependent reconstructible state sets is proposed to estimates the system state.

2) For the set of output-dependent unreconstructible states, the flip control technique is introduced, then a sufficient condition is proposed for the set of output-dependent unreconstructible states to reach a reconstructible state.

3) An algorithm is designed for all states in the output-dependent unreconstructible state set, and a sequence of common joint control pairs is obtained. Then, the state of the system is finally uniquely estimated by the designed controller and observer.

Preliminaries and Problem Formulation

Preliminaries

$\mathcal{D} := \{0,1\}$, $\mathcal{D}^n = \underbrace{\mathcal{D}\times\mathcal{D}\times\cdots\times\mathcal{D}}_{n}$. $|S|$ is the cardinality of the set S. $\mathbb{Z}_+$ is the set of nonnegative integers, and $\mathcal{M}_{m\times n}$ is a set of $m\times n$ matrices. δ_n^i denotes the i-th column of the $n\times n$ identity matrix $A\in\mathbb{R}^{m\times n}$. $\Delta_n := \{\delta_n^i \mid i=1,\ldots,n\}$, $\Delta := \Delta_2$. $\mathcal{L}_{m\times n}: m\times n$ denotes the set of $n\times m$ logical matrices. $(C)_{ij}$ denotes the i-th row and j-th column element of C. $Col(M)$ is the set of columns of M. $Col_i(M)$ is the i-th column of the matrix M. For $A\in\mathbb{R}^{m\times n}$ and $B\in\mathbb{R}^{p\times q}$, the semi-tensor product (STP) of two matrices[2] is defined as $A\ltimes B = (A\otimes I_{l/n})\cdot(B\otimes I_{l/p})$, where $l = lcm\{n,p\}$ is the least common multiple of n and p, and '$\otimes$' is the Kronecker product. Let $X\in\mathcal{D}^{2^n}$, then $X\ltimes X = \Phi_n\ltimes X$, where $\Phi_n = \left[\delta_{2^n}^1\otimes\delta_{2^n}^1\ \ \delta_{2^n}^2\otimes\delta_{2^n}^2\ \cdots\ \delta_{2^n}^{2^n}\otimes\delta_{2^n}^{2^n}\right]$ is the power-reducing matrix.

System Formulation

A BCN with n states, m inputs and p outputs are represented as

$$\begin{cases} X_i(t+1) = f_i\left(X(t),U(t)\right), i=1,2,\cdots,n, \\ Y_j(t) = h_j\left(X(t)\right), \quad j=1,2,\cdots,p, \end{cases} \tag{1}$$

where $X(t) = \left(X_1(t),\cdots,X_n(t)\right)^T\in\mathcal{D}^n$, $U(t) = \left(U_1(t),\cdots,U_m(t)\right)^T\in\mathcal{D}^m$, $Y(t) = \left(Y_1(t),\cdots,Y_p(t)\right)^T\in\mathcal{D}^p$, are the state, input and output vectors, respectively. $f_i:\mathcal{D}^{n+m}\to\mathcal{D}$ and $h_j:\mathcal{D}^{n+m}\to\mathcal{D}$, are logical functions.

The vector form of Boolean variables X_i, Y_j and U_k can be simply expressed as $x_i=\begin{bmatrix}X_i & \neg X_i\end{bmatrix}^T\in\Delta_2$, $y_j=\begin{bmatrix}Y_j & \neg Y_j\end{bmatrix}^T\in\Delta_2$ and $u_k=\begin{bmatrix}U_k & \neg U_k\end{bmatrix}^T\in\Delta_2$, respectively. Let $x(t)=\ltimes_{i=1}^{n} x_i(t)\in\Delta_{2^n}$, $u(t)=\ltimes_{i=1}^{m} u_i(t)\in\Delta_{2^m}$, $y(t)=\ltimes_{i=1}^{p} y_i(t)\in\Delta_{2^p}$, the algebraic form of BCN (1) is

$$\begin{cases} x(t+1)=Lu(t)x(t), \\ y(t)=Hx(t), \end{cases} \tag{2}$$

where $L\in L_{2^n\times 2^{n+m}}$ and $H\in L_{2^p\times 2^n}$ are the logical matrices.

Matrix Representation of Flipping control

Definition 1([20]): For a flip set $A=\{A_1,A_2,\cdots,A_r\}\subseteq[1,n]$, the flip function with respect to A is defined as

$$\bar{X}_A=\phi_A^{\mapsto}(X)=\left(X_1,\cdots,\bar{X}_{A_1},\cdots,\bar{X}_{A_2},\cdots,\bar{X}_{A_r},\cdots,X_n\right),$$

which can transform the logical state X to $\bar{X}_A$. The flip matrix of $\phi_A^{\mapsto}(X)$ is denoted by H_A and satisfies $col_j(H_A)=\delta_{2^n}^i$, $j\in[1,2^n]$, if $x=\delta_{2^n}^j \xrightarrow{\phi_a^{\mapsto}} \bar{x}_A=\delta_{2^n}^i$.

Definition 2([20]): Let $B=\{B_1,B_2,\cdots,B_s\}\subseteq[1,n]$. For $A\subseteq B$, the combinatorial flip matrix with respect to B is defined as

$$(C_B)_{ij}=\begin{cases} 1,\ \exists\, a\in\Theta_B \Rightarrow X \sim \delta_{2^n}^j \xrightarrow{\phi_A^{\leftrightarrow}} \bar{X}_A \sim \delta_{2^n}^i, \\ 0,\ \text{otherwise.} \end{cases}$$

Equivalently, if there exists a subset $A\in\Theta_B$, then $(H_A)_{ij}=1$ holds. In order to represent the above in mathematical notations, we can derive $C_B=\sum_{A\in\Theta_B} H_A$ and $C_B\in \mathcal{B}_{2^n\times 2^n}$.

Here, two sets A and B are briefly explained. For a given state, A is the actual flip set, and every element of A corresponding to a node in the given state must be flipped. However, B is a combinatorial flip set, and a subset of B is chosen to represent as coming to be flipped.

Main results

State analysis of Boolean control network

Definition 3: For the Boolean control network (2), the output is $y=Hx=\delta_{2^p}^i$, $i\in[1,2^p]$. Then the set of output-dependent state estimates can be defined as

$$H_c=\left\{\delta_{2^n}^j \,\middle|\, H_{ij}=1,\ j=1,2,\cdots,2^n\right\} \tag{3}$$

From equation (3), it is clear that the $H_1\cup H_2\cup\cdots\cup H_{2^p}=\Delta_{2^n}$.

Definition 4: Consider a Boolean control network (2), if there is a column in the output that is different from the other columns, then the state corresponding to that column is the true state, and we know that the output depends on the set of state estimates, which is satisfied $|H_c|=1$, then the state of the BCN (2) is the output-dependent reconstructible.

Corollary 1: Consider the Boolean control network (2), for which each column in the output H is different, and it can be shown that the output-dependent set of state estimates in Definition 4, are satisfied $|H_c|=1$, then the BCN (2) is output-dependent globally reconstructible.

In Corollary 1, the cardinal number of given H_c is 1, i.e., $|H_c|=1$, and it is clear that the state of the system estimated by $y=\delta_{2^p}^i$ is unique, otherwise it is not. Therefore, the proof of Corollary 1 is straightforward and obvious. It should be noted that if $|H_c|=0$, then $y=\delta_{2^p}^i$ does not exist, so the case of $|H_c|=0$ is not considered in this paper.

Definition 5: For a BCN (2) the set of output state estimates satisfies $\left|H_c\right| \neq 1$, $c = 1,2,\cdots,2^p$, then the state of the BCN (2) is output-dependent reconstructible.

In Definition 4 the BCN can satisfy $\left|H_c\right| = 1$ by relying on the set of output state estimates, then the BCN (2) is output-dependent reconstructible. However, in Definition 5, it is known that there exists a set of output-dependent state estimates to satisfy $\left|H_c\right| \neq 1$. At this time, it is necessary to add state flip control to flip the dependent output unreconstructible state to the output dependent reconstructible state set to realize the state estimation of the Boolean control network.

Example 1: Consider the equations of a BCN with $n=3$, $m=1$, $p=2$ where the logic matrix is of the form $L = \delta_8\left[2,1,1,5,5,2,1,7,1,2,1,5,5,1,1,7\right]$, $H = \delta_4\left[1,1,2,2,4,3,1,3\right]$.

By equation (3) it can be concluded that can be known $y = \delta_4^1$, $H_1 = \left\{\delta_8^1, \delta_8^2, \delta_8^7\right\}$; $y = \delta_4^2$, $H_2 = \left\{\delta_8^3, \delta_8^4\right\}$; $y = \delta_4^3$, $H_3 = \left\{\delta_8^6, \delta_8^8\right\}$; $y = \delta_4^4$, $H_4 = \left\{\delta_8^5\right\}$; i.e. $\left|H_1\right| = \left|H_2\right| = \left|H_3\right| \neq \left|H_4\right|$, by definition 4 it can be known that for Example 1 for the output $y = \delta_4^4$ is output-dependent reconstructible. In summary, considering the BCN (2), it is known that part of the states are output-dependent reconstructible and part of the states are output-dependent unreconstructible, while the set of state estimates for output-dependent unreconstructible states can be flipped into an output-dependent reconstructible set of states by adding a flipped control.

Co-controller design

In summary, in the case of Definition 5, the system state cannot be estimated from the output, which requires other control techniques to estimate the system state. The design steps are as follows:

1) In the case of Definition 5, determine the set of output-dependent reconstructible states by output, using the representation output-dependent reconstructible state set, simply referred to as the reconstructible state set. Determine the set of dependent output unreconstructible state estimates, simply referred to as the unreconstructible state set.

2) For the set of unreconstructible state estimates, the design of the sequence of joint control pairs makes it possible to flip the states in the set of unreconstructible state estimates to the set of reconstructible state estimates.

3) Unreconstructible state estimation sets all states to design common joint control pair sequences to realize state estimation for BCNs.

For a flip transition of $\delta_{2^n}^j$ from to $\delta_{2^n}^i$, it can be represented by a sequence of joint control pairs

$$\Lambda^k_{\left\{\delta_{2^n}^j \Rightarrow \delta_{2^n}^i\right\}} := \left\{\left(\phi_{A_0}^{\mapsto}, u_0\right), \left(\phi_{A_1}^{\mapsto}, u_1\right), \left(\phi_{A_{k-1}}^{\mapsto}, u_{k-1}\right)\right\}$$

denoted simply as Λ^k, $A_j \subseteq B$ is a flip set, $u_j \in \Delta_{2^m}$ is a control input, $j \in \left[0, k-1\right]$. Hence a transformation path can be obtained:

$$P = \left\{x_0 = \delta_{2^n}^j \xrightarrow{\phi_{a_0}^{\mapsto}, u_1} x_1 = \delta_{2^n}^{p_1} \xrightarrow{\phi_{a_1}^{\mapsto}, u_2} \cdots \xrightarrow{\phi_{a_k}^{\mapsto}, u_k} x_t = \delta_{2^n}^i\right\}$$

Definition 6([24]): Given a subset $B \subseteq \left[1, n\right]$, the matrix $\tilde{L} \in \mathcal{L}_{2^n \times 2^n}$ is called the flip transfer matrix of BCN(2) under flip control of B, where $\tilde{L} = MC_B$, $M := \sum_{q=1}^{2^m} M_q \in \mathcal{L}_{2^m \times 2^m}$. According to Definition 2, $\tilde{L}$ is called the flip control transfer matrix based on B.

Lemma 1([24]): $B := \left\{B_1, B_2, \cdots, B_s\right\} \subseteq \left[1, n\right]$ and the flip control transfer matrix $\tilde{L}$ in Definition 5 , then there is $\left[\left(\tilde{L}\right)^k\right]_{ij} = x\left(k, \delta_{2^n}^j, \delta_{2^n}^i\right)$, and then $\left(\tilde{L}^k\right)_{ij}$ is a joint control pair Λ_{ij}^k under which one can go from state $d_{2^n}^j$ to state $d_{2^n}^i$ after k flip-inputs.

Lemma 2([24]): there exists a feasible path P from the initial state $\delta_{2^n}^j$ to $\delta_{2^n}^i$. The length of the path is represented by k_j. Considering that there are 2^n states in the state space of the BCN (2), there must be some loops in the path, and removing the loops in the path, the length of k_j is less than or equal to $2^n - 1$, $k \leq 2^n - 1$.

According to Lemma1, the reachability $\left(\tilde{L}\right)_{ij} > 0$ between any two states in BCN(2) under B flip control, based on the computation of the matrix $\tilde{L}$ and its powers, implies that there exists at least one flip set H_A and a control transfer matrix M_q such that $\delta_{2^n}^i = M_q H_A \delta_{2^n}^j$. Assume that the dependence outputs the unreconstructible state estimation set H_c, $H_c = \left\{\delta_{2^n}^1, \delta_{2^n}^2, \cdots, \delta_{2^n}^\gamma\right\}$, where $\gamma < 2^n$.

Theorem 1: For a given subset $B \subseteq [1,n]$ and initial state $x_0 = \delta_{2^n}^j$, destination state $x_d = \delta_{2^n}^i$, $\delta_{2^n}^i \in \mathbf{R}$ in the set of reconstructible states, if $\left(\tilde{L}^k\right)_{ij} > 0$, then BCN (2) is reachable under B flip control to the set of reconstructible states $\mathbf{R}$.

Proof: When given a subset $B \subseteq [1,n]$, it follows from Lemma 2 that any initial state $x_0 = \delta_{2^n}^j \in H_c$ can evolve to a reconstructible states $\delta_{2^n}^i$, $\delta_{2^n}^i \in \mathbf{R}$, after k steps, i.e., $\delta_{2^n}^i = \underbrace{M_q H_A \cdots M_q H_A}_{k} \delta_{2^n}^j$, $x\left(k, \Lambda_{ij}^k, \delta_{2^n}^j\right) = \delta_{2^n}^i$.

Consider any other initial state if $\delta_{2^n}^\lambda \in H_c$ is and $\delta_{2^n}^j \neq \delta_{2^n}^\lambda$ similarly there exists a sequence of joint control pairs Λ_{ij}^k that reach the initial state $d_{2^n}^\lambda$ to reach the reconstructible state $x_d = \delta_{2^n}^i$ by reaching it to the reconstructible state $x_d = \delta_{2^n}^i$ at step k. Similarly, when $|H_c| \neq 1$, all states in H_c are able to reach the reconstructible state $x_d = \delta_{2^n}^i$ under the joint control sequence Λ_{ij}^k, i.e., when, $t \geq k$, $x\left(k, \Lambda_{ij}^k, \delta_{2^n}^j\right) = \delta_{2^n}^i$. In other words, assuming that there exists any initial state $d_{2^n}^j \in H_c$ that cannot reach the set of reconstructible states $\mathbf{R}$ under the sequence of joint control pairs, then we get $\left(\tilde{L}^k\right)_{ij} = 0,\ j \in [1, \sigma]$, implying $|H_c| \neq 1$, which contradicts $\left(\tilde{L}^k\right)_{ij} > 0$, so the assumption is not true. Proof complete.

Finding Common Flip Sets and Control Inputs

Since the current state cannot be determined through the outputs, it is necessary to design the joint control input Λ_k to transfer all states in the unreconstructible state set to the reconstructible state set. Next, algorithms are proposed to compute the desired joint control inputs, which can lead any initial state of the network to a given state. Assume that the desired flipping set is B and $|B| = \theta$, with $A_1,\ A_2,\ \cdots,\ A_{2^\theta - 1}$ for all subsets of B. The case of the empty set and full set is not considered in this paper. Based on the flip control transfer matrix $\tilde{L}$, it can be determined that there is a flip transition from $\delta_{2^n}^j$ to $\delta_{2^n}^i$. A flip transition path:

$$P = \left\{x_0 = \delta_{2^n}^j \to x_1 = \delta_{2^n}^{p_1} \to \cdots x_k = \delta_{2^n}^i\right\},$$

where x_p is located at the $k-p$ step reachable from x_k. For convenience, suppose that $\delta_{2^n}^{p_0} = \delta_{2^n}^j$ and $\delta_{2^n}^{p_k} = \delta_{2^n}^i$. After finding the path from $\delta_{2^n}^j$ to $\delta_{2^n}^i$, there may be several different feasible sequences of joint control pairs. In using of flip control, it is only desirable to find at least 1 sequence of joint control pairs that can make all the states in the set of unreconstructible states reach the states in the set of reconstructible states. Here, assume the set $S_i = \left\{d_{2^n}^1, d_{2^n}^2, \cdots, d_{2^n}^\sigma\right\}$, $\sigma < 2^n$. Algorithm 1 is proposed below to compute the sequence of joint control pairs for each state in the set of unreconstructible states as well as at each step using a search algorithm.

Algorithm 1: Finding joint control pair sequences
input : H_c, $\mathbf{R}$, set $S_j^k = \varnothing$
Output: S_j^k
line 1: for $j = 1 : \lvert H_c \rvert$
line 2: for $k = 1 : 2^n - 1$
line 3 : $\delta_{2^n}^1 = x_0 \to M_q$ â H_{A_t} â $x_0 = x_1 \to \cdots \to x_d = \delta_{2^n}^i$

line 4 : Save the flip-function $\phi_A^{\mapsto}(X)$ in line 3 and the control transfer matrix M_q to form a sequence of joint control pairs $\Lambda^k_{\{\delta^j_{2^n} \Rightarrow \delta^i_{2^n}\}} := \{(\phi^{\mapsto}_{A_{r_1}}, u_0), (\phi^{\mapsto}_{A_{r_2}}, u_1), (\phi^{\mapsto}_{A_{r_{k-1}}}, u_{k-1})\}$ stored in the set S^k_j.
line 5: for end
line 6: for end

In Algorithm 1, a search algorithm is used in line3 to compute the sequence of joint control pairs for all state computations in the unreconstructible state set, where M_{q_t} is the control input, H_A denotes the algebraic matrix of the flip function $\phi_A^{\mapsto}(X)$, and A_{r_t} is the set of flips required for the flip, $r \in [1, 2^{\theta} - 1]$. Where S^k_j superscript k denotes k steps, where S^k_j, and subscript j denotes the individual state values in the set of unreconstructible states j. For example, S^4_3, denotes the set of all control sequences for the state at time $t = 4$. Output the set of joint control pair sequences S^k_j, which is represented as $S^1_1, S^2_1, \cdots S^k_1; S^1_2, S^2_2, \cdots S^k_2; \cdots; S^k_j$.
From Algorithm 1, all pairs of control sequences are obtained for each state in the unreconstructible state set that arrives at a reconstructable state. For the output-dependent unreconstructible state estimation set H_c, and $|H_c| \neq 1$, not all sequences of joint control pairs are able to transfer all the states in the unreconstructible state set to the reconstructible state set, which in turn determines the system specific state. Therefore, for the set of unreconstructible state estimates, a common sequence of joint control pairs must be found. Algorithm 2 is proposed below to compute the common control sequence. Its common control sequence pair $\tilde{\Lambda}^k$ is denoted to.

Algorithm 2: Finding common control sequence pairs

Input: $S^1_1, S^1_2, \cdots, S^1_\sigma; \cdots; S^2_1, S^2_2, \cdots, S^2_\sigma; \cdots; S^k_1, S^k_2, \cdots S^k_\sigma$, $S^k = \varnothing$

Output: $\tilde{\Lambda}^k$

line 1: Initialize $k = 1$ and $k \le 2^n - 1$.

line 2: Selection set S^k_j, and find the intersection S^k

line 3: Assign the common control sequence in the intersection S^k from step 2 to $\tilde{\Lambda}^k$, stop the algorithm.

line 4: If the intersection set $S^k \neq \varnothing$ in line 2.

line 5: $k = k + 1$.

In Algorithm 2, line 2, when $k = 1$, selects the set of joint control pair sequences $S^1_1, S^1_2, \cdots, S^1_\sigma$ that can reach the set of reconstructible states in one step. line 3 will find the common joint control pair sequence.

Luenberger-like observer design

Next, observers are invoked to observe the states in the reconstructible state set. for the purpose of state estimation. The following form of Luenberger-like observer is obtained from the paper [25]:

$$\begin{cases} \hat{x}(0) = H^T y(t), \ t = 0, \\ \hat{x}(t) = \Phi_n^T L \hat{x}(t-1) H^T y(t), \\ \quad = \Phi_n^T \left(I_{2^n} \otimes H^T \right) L \hat{x}(t-1) u(t-1) y(t), \ t > 0. \end{cases} \tag{4}$$

From equation (4) it is clear that it is not of the same form as BCN(2). As a result, (4) can be rewritten to make the following class of Luenberger-like observer

$$\begin{cases} \hat{x}(t) = H^T y(t), \ t = 0, \\ \hat{x}(t+1) = \Phi_n^T Lu(t)\hat{x}(t) \cdot H^T y(t+1), \ t \geq 1, \end{cases} \tag{5}$$

where the state estimate at $t \geq 1$ can be obtained by using the pseudo-commutative nature of the STP operation to obtain a simplified state estimate of: $\Phi_n^T \left(I_{2^n} \otimes H^T\right) Lu(t)\hat{x}(t)y(t+1)$.

Lemma 3([26]): Let A and B be matrices with the same dimension $m \times n$, then the Hadamard product of A and B is denoted as $A \circ B$, defined as $A \circ B = H_m^T (A \otimes B) H_n$, where $H_n = diag\left(\delta_n^1, \delta_n^2, \cdots, \delta_n^n\right)$.

The known input and output trajectories are $\{y(0), u(0), y(1), u(1), \cdots, y(t), u(t)\}$, Then all possible state values compatible with the input and output trajectories are contained in $\hat{x}(t+1)$. Since Boolean control network systems are time-invariant, system state reconstructability implies that the input and output trajectories at the same moment can uniquely determine the state values at the last moment of the system. A shift register observer based directly on input and output trajectories of length $r+1$ has been proposed in the paper [6]. Thus state estimation at a given moment naturally requires immediate information about the outputs. Using Lemma 3, the following equation is obtained,

$$\begin{aligned} \hat{x}(t+1) &= Lu(t)\hat{x}(t) \circ H^T y(t+1) \\ &= H_n^T \left(Lu(t)\hat{x}(t) \otimes H^T y(t+1)\right) H_1 \\ &= H_n^T \left(Lu(t)\hat{x}(t) H^T y(t+1)\right) \\ &= \Phi_n^T Lu(t)\hat{x}(t) H^T y(t+1) \\ &= \Phi_n^T \left(I_{2^n} \otimes H^T\right) Lu(t)\hat{x}(t) y(t+1). \end{aligned}$$

Simulation Example

In this section, the theoretical results obtained are demonstrated using a simple BCN, reconsidering the Boolean control network of Example 1.

By equation (3) it follows that $y = \delta_4^1$ can be known as $H_1 = \left\{\delta_8^1, \delta_8^2, \delta_8^7\right\}$; $y = \delta_4^2$, $H_2 = \left\{\delta_8^3, \delta_8^4\right\}$; $y = \delta_4^3$, $H_3 = \left\{\delta_8^6, \delta_8^8\right\}$; $y = \delta_4^4$, $H_4 = \left\{\delta_8^5\right\}$, knowing, $|H_1| = |H_2| = |H_3| \neq |H_4|$, From Definition 4 it is clear that the pair of BCNs (4) is output-dependent reconstructible for output $y = \delta_4^4$. It can be shown that the state δ_8^5 can be reconstructed with the set of states $\mathbf{R} = \left\{\delta_8^5\right\}$ by output confirmation.

Next, the joint control sequence will be designed by the algorithm to transfer all states in the output-dependent unreconstructible state set to the reconstructible state set. If the output is $y = \delta_4^1$ know $H_1 = \left\{\delta_8^1, \delta_8^2, \delta_8^7\right\}$, a partial sequence of joint control pairs of all states in the unreconstructible state set arriving at the reconstructible state set can be derived by Algorithm 1, and due to space constraints. Due to space limitations, only some of the joint control sequence pairs are listed: $\Lambda^3 = \left\{\left(\phi_{\{1\}}^{\mapsto}, u_2\right), \left(\phi_{\{3\}}^{\mapsto}, u_1\right), \left(\phi_{\{3\}}^{\mapsto}, u_2\right)\right\}$. A partial common control sequence can be derived by Algorithm 2: $\tilde{\Lambda}^3 = \left\{\left(\phi_{\{1\}}^{\mapsto}, u_2\right), \left(\phi_{\{3\}}^{\mapsto}, u_1\right), \left(\phi_{\{3\}}^{\mapsto}, u_2\right)\right\}$. For all states in the unreconstructible state set H_c, they can be transferred to the reconstructible state set. The specific transfer path is shown below:

$$\begin{aligned} &\delta_8^1 \xrightarrow{H_{\{1\}}, u_2} \delta_8^5 \xrightarrow{H_{\{3\}}, u_1} \delta_8^1 \xrightarrow{H_{\{3\}}, u_1} \delta_8^5, \\ &\delta_8^2 \xrightarrow{H_{\{1\}}, u_2} \delta_8^1 \xrightarrow{H_{\{3\}}, u_1} \delta_8^1 \xrightarrow{H_{\{3\}}, u_1} \delta_8^5, \\ &\delta_8^7 \xrightarrow{H_{\{1\}}, u_2} \delta_8^1 \xrightarrow{H_{\{3\}}, u_1} \delta_8^1 \xrightarrow{H_{\{3\}}, u_1} \delta_8^5, \end{aligned}$$

where $H_{\{1\}}$ is the algebraic matrix of the flip-function $\phi_A^{\mapsto}(X)$ for the state transfer process, the lower right corner $\{1\}$ denotes a $A = \{1\}$, and u_1 or u_2 denotes a sequence of joint control inputs, which will not be described in the following.

For the set of unreconstructible states $H_1 = \left\{\delta_8^1, \delta_8^2, \delta_8^7\right\}$, all state transfer diagrams are shown in Fig. 1.

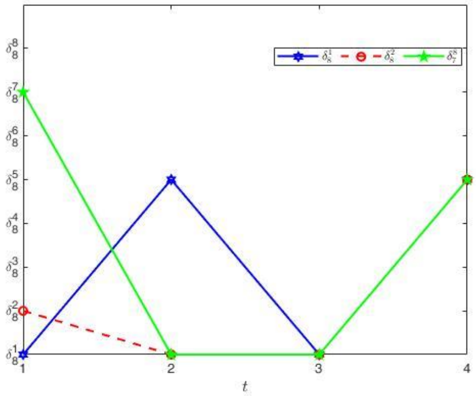

Figure 1: State transfer diagram

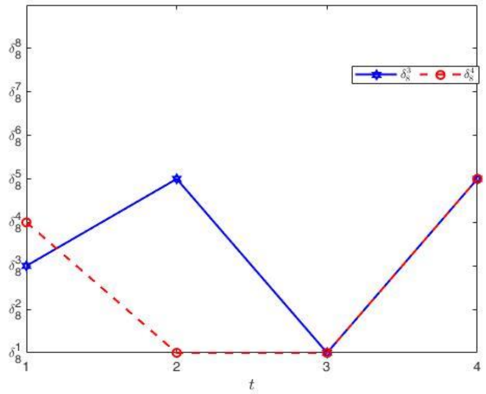

Figure 2: State transfer diagram

From Fig. 1, it can be seen that there exists a sequence of public joint control pairs $\tilde{\Lambda}^3 = \{(\phi^{\mapsto}_{\{1\}}, u_2), (\phi^{\mapsto}_{\{3\}}, u_1), (\phi^{\mapsto}_{\{3\}}, u_2)\}$, which is able to reach in the reconstructible state for all states in the unreconstructible state set $H_1 = \{\delta_8^1, \delta_8^2, \delta_8^7\}$. And the existent sequence of public joint control pairs is not unique.

Similarly, it follows that for the set of irreducible states $H_2 = \{\delta_8^3, \delta_8^4\}$, there exists a common joint control sequence pair $\tilde{\Lambda}^3 = \{(\phi^{\mapsto}_{\{3\}}, u_1), (\phi^{\mapsto}_{\{3\}}, u_1), (\phi^{\mapsto}_{\{1\}}, u_1)\}$ by Algorithm 1 and Algorithm. The transfer diagram is shown in Fig 2.

Similarly, it follows that for the set of irreducible states $H_3 = \{\delta_8^6, \delta_8^8\}$, there exists a common joint control sequence pair $\tilde{\Lambda}^3 = \{(\phi^{\mapsto}_{\{3\}}, u_1), (\phi^{\mapsto}_{\{3\}}, u_1), (\phi^{\mapsto}_{\{1\}}, u_1)\}$ by Algorithm 1 and Algorithm. The transfer diagram is shown in Fig 3.

In summary, for the irreducible state set H_c of the Boolean control network (2), a sequence of joint control pairs can be designed to reach the irreducible state set to the reconstructible state set. In turn, the state estimation of the Boolean control network is realized.

Next the observer is applied to observe the states in the set of Boolean control network dependent output reconstructible states. The state of its observation system is shown in Fig. 4.

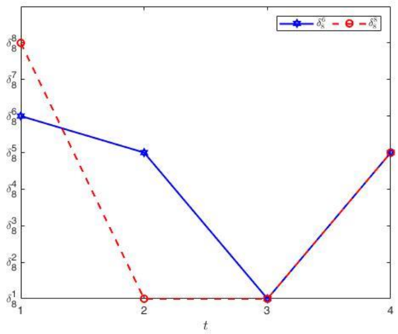

Figure 3: State transfer diagram

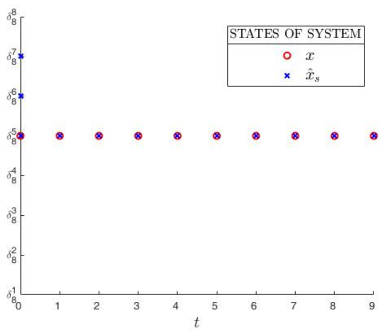

Figure 4: State estimation diagram

It can be concluded from the state estimation diagram in Fig. 4 that the system state can be observed by applying the Luenberger-like observer.

Conclusion

This chapter discusses output-based reconstructability analysis and state estimation for BCNs. Special output sequences are utilized to determine part of the system state and the reconstructible state set is given. Since the states corresponding to some outputs of the system are not unique, the system state cannot be determined. Therefore, it is necessary to design a joint flipping control pair sequence, so that the state of the unreconstructible state set can reach the reconstructible state set, and then estimate the state of the system. Then, Then the specific state of the system is observed by the Luenberger-like observer. Finally, an example is used to validate the necessity of the research results obtained.

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