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## Measures to Improve the Bending Stiffness of Beams

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**Abstract** In engineering, often encounter bending members, these mechanical parts in the external environment and internal environment together, may lose the original mechanical properties of the parts mechanical properties and other chemical or other special properties, until can not fully return to the normal application phenomenon, mechanical parts bending deformation shorten the service life of the product, affect engineering safety. In this paper, we will analyze the deformation causes of bending beams under the external environment (under force), and discuss the safety of bending beams in terms of mechanics. The cross-section of the beam is subjected to normal and shear stresses. A reasonable assumption is made through the bending experiment, and then the positive stress calculation formula is obtained, and the bending positive stress strength condition  $\sigma_{\max} = M_{\max}/W_z \leq [\sigma]$  is obtained. To study the shear stress on the cross-section, some assumptions about the shear stress are made according to the specific shape of the cross-section, and then the local equilibrium method is applied to obtain an approximate formula. This paper introduces the formula of shear stress of equal section straight beam under symmetrical bending situation, from which it can be seen that the bending shear stress strength condition should also be considered (for short rough beams and thin wall beams). The bending moment of the beam is mainly affected by the normal stress, so according to the positive stress strength condition  $\sigma_{\max} = M_{\max}/W_z \leq [\sigma]$  it can be seen that the strength of the beam is related to the maximum bending moment caused by external force, the shape and size of the cross-section, and the material used. Therefore, to improve the bending strength of the beam, one way is to adopt a reasonable cross-sectional shape to increase the value of the bending cross-section coefficient  $W_z$  and make full use of the material; Another method is that the force situation of the beam should be reasonably arranged to reduce the maximum bending moment  $M_{\max}$ .

For the deformation of the beam caused by the bending moment under symmetrical bending. The deformation of the shear force on the slider beam is negligible, first derive the approximate differential equation of the deflection curve, summarize the superposition method to find the bending deformation method, and find the angle and deflection of the beam, according to the actual situation, it is necessary to limit the maximum rotation angle  $|\theta|_{\max}$  and the maximum angle  $|\omega|_{\max}$ , which must not exceed a certain value that has been specified, that is:  $|\theta|_{\max} \leq [\theta]$ ,  $|\omega|_{\max} \leq [\omega]$  This is the stiffness condition of the bending deformation of the beam. According to the approximate differential equation of the deflection curve and its integral, the bending moment and bending stiffness will affect the bending deformation, and the span of the beam, the load of the beam, the support situation, etc. will affect the magnitude of the bending moment; The bending stiffness is related to the elastic modulus of the material and the moment of inertia of the cross-section of the beam. Therefore, in order to improve the stiffness, the following measures can be adopted: reduce the span of the beam or change the support conditions of the beam (properly adjust the load action mode of the beam), improve the distribution of the load, and select a reasonable cross-section.

**Keywords** bending beam safety; bending strength; bending stiffness; positive stress strength conditions; shear stress strength conditions

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## 1. Introduction

In these engineering structures, we often encounter supporting members which may be deformed and damaged, such as train axle beam and large bridge crane wheel girder support and so on. Under the combined action of external environment and internal environment, these mechanical parts may lose their original mechanical properties and other chemical or other special properties, until they can not be completely restored to normal application. We call it the failure phenomenon caused by bending deformation of parts. The causes of bending deformation of mechanical parts include bending deformation caused by force, bending deformation caused by heat treatment and elastic-plastic deformation.

The bending deformation of mechanical parts shortens the service life of the product and affects the safety of the project. In order to ensure the safety of the project and improve the competitiveness of enterprises, it is necessary to study the influence reasons and improvement measures of mechanical parts. The bar will be subjected to a variety of external forces due to other external forces that exceed the angle perpendicular to the plane or intersection point of the bar axis or the duality of other external forces. The rod axis will suddenly twist from another straight line to the same curve, forming bending deformation.

This paper will analyze the causes of the deformation of the bending beam under the influence of the external environment (under the action of force), and discuss the safety of the bending beam in the aspect of mechanics.

## 2. Normal stress strength condition of curved beam

### 2.1 Knowledge preparation

Under the general mechanical conditions, the cross-sectional area of the beam plate in the design of concrete bridge engineering system is mainly affected by positive tensile stress and negative cutting shear stress respectively. The bending moment can be calculated by the continuous simplification of the normal stress to the centroid direction between the cross sections, and the cutting shear force can also be derived by the continuous simplification of the shear stress in its direction. If the shear value is set to zero and the bending moment value is set to constant, the bending stress state will become the last state of pure bending stress in the bending beam. At this time, there is only a pure bending positive stress in the direction of each bending cross section on the bending beam. The first part will focus on the case of pure bending normal stress existing in the upper and lower bending cross sections of the bending beam in this pure bending force state.

Bending experiments and hypotheses. When the section of one left and right symmetrical plane in the sorghum is taken as the beam, a vertical line and horizontal line are drawn on the surface of his section, and then an external force couple in the beam is acted on within the range of the two longitudinal symmetrical planes at the two ends of the sorghum. It is obtained that the stress state of the beam surface of the cross-section beam at this time is the force state of pure bending. The description of the force phenomenon obtained by the experiment is as follows: (1) the horizontal line is orthogonal to the vertical line, and the horizontal line will rotate relatively to a certain extent in the beam. (2) when the longitudinal line on the beam is bent, the upper longitudinal line will be shortened and the lower longitudinal line will be elongated.

According to the experimental phenomena, the assumptions of deformation and force in the beam are as follows: (1) before deformation, there is a plane cross section of the beam, but after deformation, the plane of the beam remains unchanged, and the plane of the beam is still orthogonal to the axis, and only the cross sections rotate relatively. It is called bending plane hypothesis: (2) there is no extrusion or tension between longitudinal "fibers". According to the plane hypothesis, the upper "fiber" of the beam is compressed and the lower "fiber" is stretched. According to the deformation continuity, the length of a layer of longitudinal "fiber" remains the neutral layer, and the intersection line between the neutral layer and the cross section is called the neutral axis. In summary, under the condition of pure bending, all the cross sections remain flat, only rotating relatively around the neutral axis, while the longitudinal "fibers" are in a unidirectional stress state.

### 2.2 Normal stress strength condition of curved beam

#### 2.2.1 Calculation formula



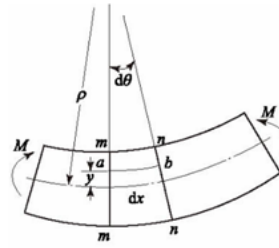


Figure 1:

Here, we need to further simplify or improve the finite element model, it can be further realized through the three-dimensional finite element calculation comprehensive simulation and analysis of the three-dimensional comprehensive calculation model composed of bending statics, deformation and physics, and then the formula of bending normal stress equation can be obtained. As shown in the figure on the left, a micro-segment of dx can be arbitrarily intercepted on the beam. (it is assumed that after the completion of the deformation, the minimum radius of curvature of the longitudinal segment on the neutral laminated beam is  $\rho$ , and the maximum relative displacement angle between the cross sections at both ends of the micro-section is  $d\theta$ ). Through the calculation of the formula, the longitudinal strain formula of line length ab can be deduced.

$$\epsilon_x = \frac{(\rho - y)d\theta - \rho d\theta}{\rho d\theta} = -\frac{y}{\rho}$$

According to the above formula, the proportional strain force of each point in the cross section should be square and proportional to the horizontal distance between the point and the neutral layer. The negative sign indicates that the longitudinal segment above the neutral layer ( $y > 0$ ) will be shortened ( $\epsilon_x < 0$ ) under the action of sinusoidal moment.  $\rho$

According to Hooke's law, the distribution of normal stress on the cross section can be calculated as follows:

$$\sigma = -\frac{Ey}{\rho}$$

At present, it is not certain that the barotropic stress on the cross section of the beam can only be calculated by using the above formula, because the relative position of the radius of curvature of the neutral layer  $\rho$  and the axis of the neutral layer is unknown. To get the above results, these are actually two unknown quantities about the stress value of the beam, and it may be necessary for us to make further use of the static relationship between the stress value of the beam and the stress value in the beam. As shown in the figure

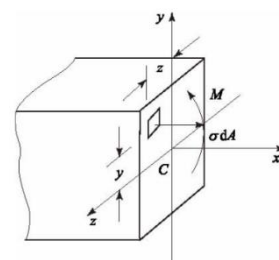


Figure 2:

The resultant force of the normal stress on the cross section forms the axial force, because the axial force of the beam is zero.

$$\int_A \sigma dA = -\int_A \frac{Ey}{\rho} dA = -\frac{E}{\rho} S_z = 0$$

In the formula, the integral  $S_z = \int_A y dA$  is called the static moment of the cross section with respect to the axis z. The above formula shows that the static moment  $S_z$  of the cross section facing the neutral axis is equal to zero,

that is, the neutral axis passes through the centroid of the section}, from which the position of the neutral axis can be determined.

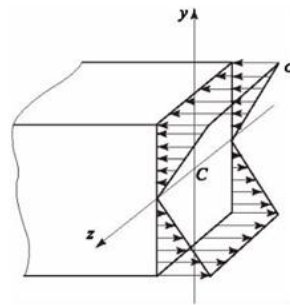


Figure 3:

### 2.2.2 Bending normal stress strength condition

In engineering design, in order to maintain the bending normal stress strength condition of bending beam, the maximum normal stress on the dangerous section of sorghum must be less than the allowable stress:

Different materials have different strength conditions, which can be divided into plastic materials and brittle materials, and the properties of materials are different. For bending, the expression of strength conditions should be analyzed according to the actual situation.

For some plastic materials, the tensile force is generally equal to its compressive strength. In this case, the section shape refers to the geometric section of the neutral axisymmetric deformation of the material, such as rectangle, I-shape, circle, cylinder and so on. In this part, the straight beam with equal cross section is studied, and its strength condition has one dangerous surface.

The maximum bending normal stress usually occurs at any point above the neutral axis line or the farthest end of the straight line bending on the cross-sectional body, and the bending tangent stress that may occur here may be only a near-zero value or very small. Therefore, it can be treated as a stress state of unidirectional bending, and the normal stress strength condition of unidirectional bending is established.

$$\sigma_{\max} = \left(\frac{M}{W_z}\right)_{\max} \leq [\sigma]$$

The above strength conditions are only applicable to the material with the same structure of allowable tensile stress  $[\sigma_+]$  and allowable compressive stress steel  $[\sigma_-]$ . If the tensile and compressive properties of the two materials are not necessarily the same, if they are hard and brittle materials such as cast iron and steel, the mechanical strength should be checked separately according to the requirements of tensile, tensile and compression properties.

The main design constraint measures are generally as follows: (1) to ensure that the shape distribution of the design section is symmetrical and reasonable. By making rational use of the symmetrical design geometry of the material cross section, it can be realized that the inertia cross section area of some smaller size materials can be used as little as possible in the calculation of the bending moment modulus of the larger size materials obtained from the design. Therefore, if the symmetrical design of the cross section of the material is placed in an appropriate position relatively far from the neutral axis of the material, the inertia rectangular area or the bending moment modulus of the section with larger geometric area can be obtained at the same time. In addition, when designing the cross-section shape of some materials, it is also emphasized that the geometric characteristics of the cross-section deformation of this part of the material must be fully and comprehensively considered in advance. For example, in the shape of the brittle material section (the tensile strength is lower than the compressive strength), the neutral shaft section shape is designed to be more suitable for the two tensile stress areas in the rigid material section. In this way, the designer hopes that the maximum strain of the cross-section stress of the two main stress zones under tension and compression can be guaranteed to exceed or slightly equal to its maximum allowable value at the same time. (2) both the form of beam constraint and the way of loading constraint can be designed reasonably and properly at the same time. It can fully ensure that to a large extent, to a large extent, the maximum tensile moment on the section can be significantly reduced.



Through the reasonable design to change the constraint loading constraint mode, it will also play a significant role in improving the beam strength.

### 3. Shear stress strength condition of bending beam

#### 3.1 Knowledge preparation

In engineering structure theory, it is considered that the main form of stress of some equal cross-section beams is vertical force or tends to bear transverse load, so the main influence factor of stress on the cross-section structure of the beam is transverse shear force. For example, the stress on the cross section of the beam in the beam should not only be sinusoidal stress, but also have internal shear stress. In order to study and analyze the various oblique shear stress distributions that actually exist in the cross-section structure of the beam, it is still necessary for researchers to further consider some other more important assumptions about the shear stress distribution according to the specific structural shape conditions of the cross-section beams. then, through the further application of a basic approximate method of local stress balance test, some formulas which are easy to be approximately applied are obtained. In the following, this paper only focuses on introducing the formula of oblique shear stress distribution on the shape of beam section, which is commonly used in straight beams with equal cross section width and equal cross section under the condition of symmetrical bending section.

#### 3.1.1 Bending Shear stress of rectangular Section Beams

As shown in the right or left part of the following figure, there is a rectangular section with a wide edge about  $b$  and a high perimeter about  $h$ , and there is another shear  $F_s$  along the upper and lower center of the section at the action of two axes perpendicular to the straight line  $y$ .

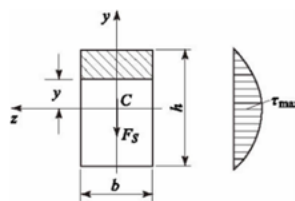


Figure 4:

It is assumed that the horizontal movement of each point and its corresponding shear stress to each stress source point should be parallel to each other and act similar to the shear force, and the shear stress should be uniformly distributed along the width direction of the plate section. The distance between the point and the neutral axis on the cross section curve can be calculated by this approximate calculation method as the approximate value of the shear stress at each point where the  $y$  axes intersect.

$$\tau = \frac{F_s S_z^*}{b I_z}$$

Counter-centroid section

$$S_z^* = b \left( \frac{h}{4} - y \right) \left( \frac{h}{4} + y \right) = \frac{b}{2} \left( \frac{h^2}{4} - y^2 \right)$$

$$\tau = \frac{3F_s}{3bh} \left( 1 - \frac{4y^2}{h^2} \right)$$

It can be seen intuitively that the bending shear stress of the rectangular beam shows a parabola distribution along the section.

Replace the above formula and  $I = bh^3/12$ , and get

$$\tau_{\max} = \frac{3F_s}{2bh} = 1.5 \frac{F_s}{A}$$

The maximum shear stress is about 1.5 times of the average shear stress. Accurate analytical experiments show that the error of this formula is very small when  $h/b \geq 2$ , but only about 10% when  $h/b=1$ .

#### 3.1.2 Bending shear stress of circular section



An analysis is made of the circular section beam in the left illustration below. On the plane of the two tangent mn points that are parallel to and intersecting the z axis and perpendicular to the neutral axis, the shear stresses at both ends come from the two parallel tangent directions perpendicular to the center line of the circle, and the tangent z stress directions between the internal points and planes are not necessarily the same. However, the bending shear stress on the beam point on the circular section still occurs only parallel to any neutral axial graph, and the bending shear stress on other points perpendicular to this point on the neutral axis diagram is also parallel and approximately equal to the shear force.

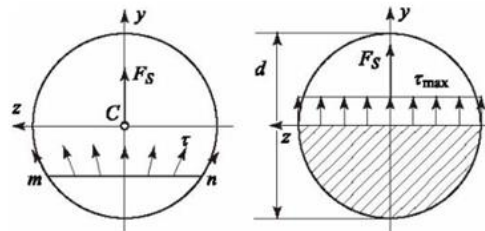


Figure 5:

In this way, the following formula can still be used to calculate the maximum bending shear stress on the section panel:

$$\tau_{\max} = \frac{F_s S_{z,\max}^*}{d I_z}$$

$$S_{z,\max}^* = \frac{\pi d^2}{8} \cdot \frac{3d}{2\pi} = \frac{d^3}{12}$$

As a result, the maximum bending shear stress of the circular section beam can be obtained.

$$\tau_{\max} = \frac{F_s S_{z,\max}^*}{d I_z} = \frac{4F_s}{3A}$$

Compared with the exact solution, the error of the above equation is about 4%.

### 3.2 Bending shear stress strength condition

The maximum bending shear stress strength usually occurs only at each point in the direction of the neutral axis, but if the normal stress strength here is zero, it is a pure shear stress state, and the corresponding strength condition of bending shear stress<sup>[3]</sup> is

$$\tau_{n-x} = \left( \frac{F_s S_{z,\max}^*}{d I_z} \right)_{\max} \leq [\tau]$$

In the formula, d represents the width of the rectangular section or the web thickness of the I-shaped section or the diameter of the circular section, respectively

For some truss beams with slender cross-section and thin-walled structures, the bending normal stress condition is also a very important reference factor. Usually, we only need to carry out the bending yield strength test and check the nuclear test in accordance with the need or the requirements of the bending normal stress first. However, for some beam-framed plates and thin-walled beams with relatively short and large cross-sections and over-thick cross-sections, the bending shear stress and strength conditions should be strictly considered. For some stress points of a beam that may have any one or a specific direction of force on a special cross-section structure, such as at the junction between the leading edge of the curved beam web and the beam flange surface in the I-section beam, the bending normal stress value and the bending shear stress value sometimes can simultaneously represent two corresponding values with the difference point of stress equal to that of the above two sections. When the beam body of a beam needs to bear many kinds of cross-section transverse load at the same time or continuously, it will also appear similar to this special stress condition. The above structural strength condition formula, which is based on the spontaneous establishment of the beam under unidirectional



normal stress, will completely or no longer exist all kinds of structural strength problems that may occur under the combined action of beam bending normal stress and beam bending shear stress.

#### 4. Measures for improving bending strength of Beams

One of the main factors controlling the strength and deformation of sorghum is the bending normal stress, so in order to improve the maximum bending strength of sorghum,  $\sigma_{\max}$  should be reduced first, so the maximum bending moment should be gradually reduced and the maximum bending section coefficient should be increased. On the other hand, the stress and deformation of the beam should be reasonably arranged to reduce the maximum bending moment  $M_{\max}$  shear stress accordingly. [4]

##### 4.1 Reasonable arrangement of the stress of the beam

To improve the longitudinal force distribution of the beam support, appropriate measures should be taken as far as possible to reduce the maximum transverse bending moment of the beam at both ends of the beam. For this reason, first of all, we require that the longitudinal beam supports at both ends of the beam should be arranged as reasonably as possible. A simply supported beam working stably under uniformly distributed load is taken as a counterexample,  $M_{\max}=ql^2/8=0.125ql^2$ . If we move the supports at both ends of the beam to the same inward direction to the max value, the maximum anti-bending moment force will gradually decrease, that is,  $M_{\max}=ql^2/40=0.025ql^2$ , both of which are only about 1/5 of the original load coefficient on the former, that is to say, the load coefficient can at least be increased by nearly 4 times. Gantry crane generally uses girder, boiler column plate and other components as support and structure, so that the horizontal position between the upper and lower horizontal support points can be slightly moved to the middle point, which can effectively achieve the lifting load effect which can reduce the maximum load above  $M_{\max}$  level. Secondly, the reasonable placement of load can also achieve the effect of reducing its maximum bending moment. For example, the gear placement needs to rely on the bearing, which will make the force  $F$  of the gear to the shaft lean closely against the support. In some cases, the bending moment when the concentrated force is applied to the midpoint of the shaft is much larger than that at the midpoint where the concentrated force is not in the axis. In addition, in some specific cases where the distribution conditions are fully allowed and exist, special attention should be paid to and try its best to avoid dispersing some of the larger concentrated forces into other relatively small concentrated forces, or at least changing into distributed loads.

##### 4.2 Reasonable section of beam

In order to make more reasonable, full and reasonable use of these material resources, we should first try to remove all these materials to a position far away from the neutral axis as far as possible

The beam with circular cross section may gather a lot of other materials near this position away from the neutral axis, so that it can not play its role reasonably and fully. In order to facilitate the design so that the rectangular material can directly shift the cross section to a position farther away from the neutral axis of the section, we can first try to directly convert the solid circular section into a hollow circular section. For this kind of rectangular cross-section material, the hollow rectangular material near the neutral axis of the cross-section can be directly transplanted to the edge of the upper and lower end of the cross-section material, so that it is directly designed with I-shaped section. The use of groove cross-section materials or box-shaped cross-section materials is also an approximate cross-section method of using the same material cross-section. The above contents are only based on the systematic discussion and analysis of these technical problems from the theoretical point of view of the electrostatic load characteristics of cross-section materials or the cross-section design of flexural buckling strength. However, it is precisely because the actual use of the section is sometimes more extensive, specific, complex and complex, when we actually discuss some special and reasonable section shapes related to its material in the section, more attention should be paid to emphasizing that the section takes into account some special and reasonable properties of other special material sections used with it. Tensile resistance or compared with various large cross-section materials (such as carbon steel) with basically equal compressive strength, it should be more suitable for small cross-sections with paired sections or neutral and axisymmetric relationships (such as circular, rectangular and I-shaped, etc.). This means that the complete equality can be achieved at the same time so that there is a complete equal stress range between the maximum static tensile stress and the maximum compressive stress at the upper and lower edges of the cross section. at the same time, they are easier to approach the allowable stress. For those brittle materials (such as cast iron) whose tensile impact resistance is



different and their compressive load strength is not very different, the cross-section shape of the side of the neutral axis upward near the tension point should be adopted as far as possible.

#### 4.3 The concept of equal strength beam

The strength condition of equal section beam is  $\sigma_{\max} = M_{\max}/W_z \leq [\sigma]$ , which causes waste at  $M \leq M_{\max}$ . In order to save and use the section material effectively and reduce the weight of the section, the section size can be changed directly so that the  $W_z(x)$  value changes with the change of the section moment coefficient. When the radius  $M$  is larger, it is suitable to use the section material with larger section size. Smaller than the cross section, it is suitable to use the variable cross section with smaller cross section, so that the variable cross section changes upward along the axis, and the maximum normal stress of the material applied at each section point is almost the same, which can be basically equal to the allowable stress of the material. According to this, it can be said that the variable cross section beam structure in this design is the most basic reasonable and feasible, which is called equal size strength beam. Engineering application of equal cross-section strength spring beam: the fish-bellied spring beam spring commonly used in factory building damping and the folding plate spring used in automobile damping are based on the concentrated force in the middle part of the simply supported beam section, and the middle width of the section remains unchanged. The height is designed with the change of the position of the section.

#### 5. Stiffness condition of curved beam

Now we have mainly done some research on the deformation of the beam. The main purpose of this study is to study the shear bending deformation of slender beams caused by bending moment under the constraint of symmetrical bending deformation. It can be proved by approximate experiments that the effect of shear constraint on the shear bending deformation caused by the shear of slender beams is almost negligible.

##### 5.2 Knowledge preparation

Approximate differential equation of torsion curve [5].

One of the main structural features of the beam deformation curve is to turn the beam axis into a deflection curve, which is generally called the variable deflection beam curve. When the curve is symmetrically bent, the plane of the flexural force curve is parallel to or coincides with the bending plane under external force, which refers to a subflat curve. Before deformation, the transverse direction of the beam axis is parallel to two axes, and the vertical direction and oblique direction are two axes perpendicular to  $y$  respectively. The centroid shifts along another transverse line on the axis of the vertical plane  $y$ , which is the point of deflection on the cross section. The deflection function equation is another kind of function about the position equation of the center of the section, which is called the deflection curve equation.

$$\omega = f(x)$$

It can be guided according to the plane hypothesis that the cross section should still be such a plane after the end of the deformation, even though it is perpendicular to the axis after the termination of the deformation.

Under the assumption that the small circle is deformed, there are

$$\theta \approx \tan \theta = \frac{df(x)}{dx} = \omega'(x)$$

In the actual engineering calculation, the angle deviation of the beam is generally not more than 0.0175 radvalue or more than  $1^\circ$ , so the above formula can fully meet the design requirements of general engineering. It should be further pointed out that although the axial deformation of the beam will also occur in the  $x$ -ray direction after the beam axis is bent into a curve, the axial bending deformation amplitude of the beam curve should be very small compared with the bending deflection coefficient under the condition of small deformation. In general, it can be omitted or ignored. In the study of the distribution and form of bending normal tensile stress on the cross section of beam, a bending deformation formula expressed by neutral layer curvature under the condition of pure bending stress has been successfully obtained.

$$\frac{1}{\rho} = \frac{M}{EI_z}$$

If the direct influence of the shear force on the bending deformation coefficient can be ignored, the above formula is also suitable for the non-straight bending of the general section. If the direct influence of the shear





force on the bending deformation coefficient can be ignored, the above formula is also suitable for the impure bending of the general section

Under the condition of small circle deformation, the rotation angle of the beam is likely to be small, so the curvature  $1/\rho$  can be approximated to bring the above formula into the pure bending state.

$$\frac{1}{\rho} \approx w''(x)$$

By using the bending deformation formula expressed by the curvature of the neutral layer, the approximate differential equation of the deflection curve can be obtained.

$$w''(x) = \frac{M}{EI_z}$$

the integral method is used to solve the deflection curve and rotation equation

By dividing the approximate differential equation of the torsion curve into two times, the rotation angle equation and the deflection equation can be obtained.

$$\theta(x) = \int \frac{M}{EI} dx + C$$

$$w(x) = \int \left( \int \frac{M}{EI} dx \right) dx + Cx + D$$

Terms C and D in the above formula are set as integral constants, which can be determined by analysis by considering and making use of the displacement constants, boundary conditions and their variation constants on the beam section itself. For example, at the fixed end of the beam, the deflection coefficient and rotation angle coefficient on the cross section are about equal to the zero constant, but at the beam dumping chain and support, the deflection on the cross section are all zero. If the bending moment equations of the two beams and columns are a piecewise function, then we can make a piecewise integral of the number of equations, and we know that more forms of logarithmic integral constants may appear at the same time. In order to facilitate the calculation and determination of these mechanical constants, in addition to making full use of their displacement boundary conditions, it should also be noted that the continuity conditions between Yao degree direction and rotation angle direction at the plane intersection between them and adjacent beams and columns on both sides should be used as far as possible

As a result, the rotation angle equation of the cantilever corner beam acting on the free arm end under the concentrated couple potential can be obtained.

$$\theta(x) = \frac{m}{EI} x$$

The maximum deflection and rotation angle of the beam occur at the free end section B, that is  $x$ , respectively.

$$\theta_{\max} = \frac{ml}{EI}, \quad w_{\max} = \frac{ml^2}{2EI}$$

Calculating bending deformation by superposition method [6].

As mentioned in the previous article, when the known conditions for determining the deformation value of a small beam are known, when the internal stress and deformation value of the known beam does not necessarily exceed the proportional limit of the beam material itself, an approximate linear ordinary differential equation of the deflection slope curve equation is also such an approximate linear ordinary differential equation, so the superposition method can not be used to measure the deformation of beam columns. If the beam acts alone, the maximum total deflection coefficient and the minimum rotation angle of the section on any cross section, equal to the separate action of each section load is the sum of the minimum total torsion deflection system and the maximum rotation angle coefficient when the load is applied perpendicular to the cross section.

### 5.3 Stiffness condition of curved beam



After the rotation angle and deflection of the beam are calculated, according to the actual situation, it is necessary to limit the maximum rotation angle  $|\theta|_{\max}$  and maximum deflection  $|\omega|_{\max}$ , which must not exceed a specified value, that is:

$$|\theta|_{\max} \leq [\theta], \quad |\omega|_{\max} \leq [\omega]$$

This is the stiffness condition of the bending deformation of the beam. The above equations  $[\theta]$  and  $[\omega]$  are called allowable deflection and allowable rotation angle, respectively.

The standard of allowable deformation value of sorghum varies greatly in different engineering application fields.

For example, for a bridge crane beam whose span (length) is about 1 meter, its maximum allowable vertical deflection is  $[\omega]=1/750 \sim 1/500$ .

In civil engineering construction, the allowable deflection of beam is  $[\omega]=1/800 \sim 1/200$ .

In the general purpose shaft, the allowable deflection of the beam is  $[\omega]=3/10000 \sim 5/10000$ .

At thawing bearings or mounting gears, the allowable rotation angle of the shaft is  $[\theta]=0.001 \text{ rad}$ .

## 6. Measures for improving bending stiffness of Beams

According to the approximate equation of the flexure surface curve, the solution of the differential equation and its integral value can be easily obtained. The large bending moment and the maximum bending stiffness of the material will indirectly affect the bending energy and bending deformation of the material. Because the span of the beam, the load of the beam, the degree of change of the support and other conditions will also indirectly affect the value of the bending moment of the material. At the same time, the bending stiffness is also related to the bending elastic modulus of the material surface and the bending inertia moment applied on the cross section size of the material perpendicular to the beam axis. In order to further effectively improve the section stiffness, it can be considered that the following measures are mainly adopted.

### 6.1 Reduce the span of the beam or change the support conditions of the beam.

The span of the beam has a great influence on the deflection of the beam compared with other factors. In order to reduce the deflection, it is necessary to find a way to reduce the length of the beam. For example, if the simply supported beam under uniformly distributed load is changed into an extension beam under uniformly distributed load, the deformation of the beam can be greatly reduced. When the actual length of the beam body can no longer be reduced for hours, in this extreme case, it may be necessary to add two intermediate supports to continuously process complex workpieces of quite long width on the lathe production line, in order to properly reduce the maximum deflection of deformation caused by various other complex factors such as cutting impact deformation, and to achieve the expected goal of properly improving the machining and assembly accuracy of the machine tool. Any intermediate bracket can be added directly between the chuck seat and the rear frame support. By changing the position of the support or increasing the support, the stiffness of the beam can be doubled.

### 6.2 Improve the distribution of load

Under the condition of allowable load, the load action mode of the beam can be adjusted dynamically or passively, and the bending moment of the trabecula can be reduced suitably and greatly. so as to achieve the expected goal of significantly reducing the bending and deformation severity of the trabecular structure. For example, if the action at the midpoint of the inner span of a simply supported beam converts the concentrated force into a uniformly distributed load, the deflection of the midpoint of the beam can be reasonably reduced by nearly half.

### 6.3 Select reasonable cross section.

From the point of view of improving the bending stiffness of the beam, the reasonable section should be to obtain a larger moment of inertia with a smaller cross-section area. Therefore, I-beam and channel steel are often used as bending beams in engineering, different forms of thin-walled hollow sections are often used in the frames of some machines, and steel plates are often welded into box-shaped sections of crane beams. It should be noted that because the elastic modulus  $E$  of all kinds of steel is very close, so the use of high-strength steel can not effectively improve the stiffness of the members.

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