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Research Article

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Parametric Study in a Layered Bottom Water Drive Reservoir with an Impermeable Interface

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Abstract This paper studies the influence of parameters like dimensionless wellbore radius (r_{wD}), dimensionless pay thickness (h_D), square root of permeability ratio($\sqrt{(k/k_x)}$ and vertical well location(z_{WD}) on dimensionless pressure (P_D) and derivative (P_D') behaviour of horizontal wells in a two layered reservoir with a bottom water drive. Models used in the study were developed based on the mathematical solution to the flow of fluid in a layered system with a constant pressure boundary at the bottom (under the conditions of no crossflow interface). Log-log plots of dimensionless pressure and derivative versus dimensionless time (t_D) for the wells in each layer were used for the analysis.

Results for well in layer one indicates that P_D and P_D' increased with larger h_D and steady state was attained faster for thin reservoirs hence production can be maximized at intermediate flow times. Decreasing r_{WD} produced increased pressure response for radial and intermediate flow periods and stabilized at constant values as steady state sets in at late times as a result of constant pressure boundary effects. Pressure and derivative were insensitive to changes in $\sqrt{(k/k_x)}$ and z_{WD}. Horizontal well in layer two exhibits same pressure behaviour as well one with varying $h_{D.}$ at early time but converges to a single straight line at late times. Increasing pressure response was observed with smaller r_{WD} to late times while derivative was observed to be insensitive. Pressure and derivative were insensitive to changes in $\sqrt{(k/k_x)}$ at early time, but at intermediate and late time pressure increases with larger values. The result indicates that well productivity increases when the total permeability of the system is high compared to the horizontal permeability. Changes in Y_D and X_D produced negligible pressure and derivative response for both wells in layer one and two. The information from this study can aid in effective pressure maintenance and improved production optimization of the wells.

Keywords Layered Reservoir, Constant pressure boundary, Dimensionless Pressure, Dimensionless pressure derivative, Horizontal Well

1. Introduction

Pressure data are without any doubt, among the most important, regarding reservoir engineering because they play a key role in all reservoir exploitation stages [1, 2]. It reveals vital information about the condition of the well and reservoir. Changes in the behaviour of the pressure distribution of a well can be used to deduce a lot of pertinent information relevant in effective well / reservoir management.

Many research work done in the past in the petroleum industry has been for single system using vertical wells. Emerging trends now involves horizontal wells in multi-boundary and layered system with the focus on effective exploitation and production of oil and gas from the reservoir at reduced cost. Various mathematical methods have been used to analyse pressure transients of horizontal wells. For single reservoir systems, subject to bottom water drive studies have shown that the effect of the boundary on the pressure response of wells and the type of flow regimes depends on the length of the well and the distance to the nearest boundary. For a reservoir subject to edged water and a bottom water drive mechanism, permeability ratio, wellbore radius and height of reservoir can influence production. Anisotropy can cause the occurrence of the beginning of the radial and steady state flow periods to be earlier [3, 4].

The loglog representation gives a visual impression that has been greatly enhanced by the introduction of the pressure derivative, it can also be used to determine the time at which pseudo-steady state flow begins for a bounded reservoir and the onset of steady state for a reservoir with constant pressure boundary. The applications of pressure derivative procedure to define formation parameters and initial reservoir pressure has shown that there were specific equalities that exist between pressure changes and its derivative and these can be used to analyse data when producing time is short. [5, 6].

Recovery from a layered reservoir can be optimized by making changes in some important parameters in the system. Certain well, fluid and reservoir properties can affect the pressure and pressure derivative response given by the system and hence the productivity. When the interface between the layers is permeable (cross flow) or impermeable (no cross flow), the pressure response and time of emergence and duration of the flow periods that would manifest in the system differs.

Recent research focused on the formulation of mathematical model equations describing the flow of fluid in the different layers and on methods of well test analysis for the layered system. When the layered reservoir has an underlying aquifer (bottom water drive) its unique pressure and derivative distribution can provide vital information about the behaviour of the well and reservoir. Eventual fluid breakthrough can be predicted from the pressure distribution and also conditions that will favour long periods of clean oil (water free) production. Layered reservoirs with impermeable interface also referred to as commingled systems has one outstanding advantage, it is cost effective since fluid from different layers can be produced to a common wellbore [7,8].

Information about the pressure behaviour of layered reservoir subject to a bottom water drive system is scarce, few literature exist as research is still ongoing [9]. The effects in parameters like pay thickness(h_D), well radius (r_{WD}) and permeability ratios(k/k_h) have on layer pressure and derivative behaviour , flow periods and hence the productivity of the system is not documented. This paper intends to bridge that gap. The aim of this study is to investigate the effect of these parameters on the pressure transient behaviour of horizontal wells in a two layered reservoir subject to a bottom water drive when the interface is sealed (no crossflow layered system).

2. Methodology

The mathematical models used in this study were derived and presented in [10]. Schematic of model shown in figure 1.A in Appendix consists of a two layered reservoir with two horizontal wells each situated in the top and bottom layer of the reservoir with an interface that is impermeable. Mathematical models for the dimensionless pressure and derivative of the horizontal wells were formulated based on the Source and Greens function method, the Product Rule and the law of Superposition as suggested by [11].

Only the case of no crossflow (impermeable) interface was investigated, all equations were solved numerically [12]. Employing varying pay thicknesses, well radii, permeability ratios, well width and flow points, changes in dimensionless pressure and pressure derivatives were used for the investigation.

3. Results and Discussions

In this study the bottom layer is modelled as a water drive reservoir and the top layer as a bounded reservoir. Study was carried out using dimensionless pressure and derivative equations (equations 10-12 in Appendix) and numerical data (Table 1) for an isotropic system. Results were computed for well 1 and well 2

Table1: Wellbore /Reservoir Parameters (isotropic layered reservoir)

Parameters	Symbols & units	Value
Well length	L ,ft	1 000
Height	h , ft	200
Well standoff	Z_w ,ft	100
Reservoir length	X_e ,ft	10 000
Reservoir width	Y_e , ft	6 000
Elevation allowance	Elv, ft	20
Layer permeability ratio	$K_2 : K_1$	10

3.1 Influence of Dimensionless pay thickness (h_D)

The effect of changes in reservoir pay thickness (h_D) was investigated for each of the horizontal wells in layer one and layer two using values in the range of $0.03125 \leq h_D \leq 1$ as shown in figure 1. For well one, it was observed that pressure and derivative values increased with increasing h_D indicating higher productivity, early time radial flow period commences late but had a short duration, no intermediate flow period was noticed. At late times the pressure stabilises at a constant value while the derivative converges to a single line tending to zero, this is as a result of the effect of the constant pressure boundary at the bottom of the reservoir. Smaller h_D (thin reservoirs) exhibits longer radial flow durations than thick reservoir

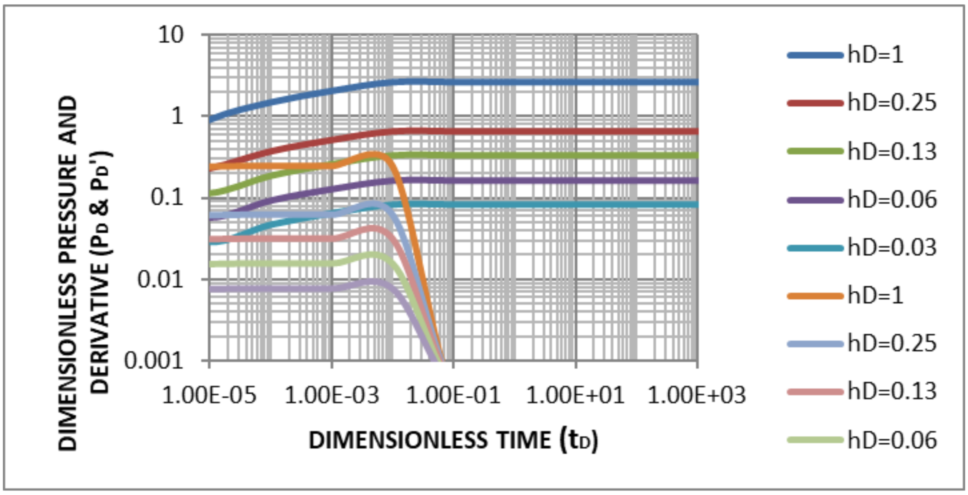

Figure 1: Influence of Dimensionless pay thickness h_D(well one)

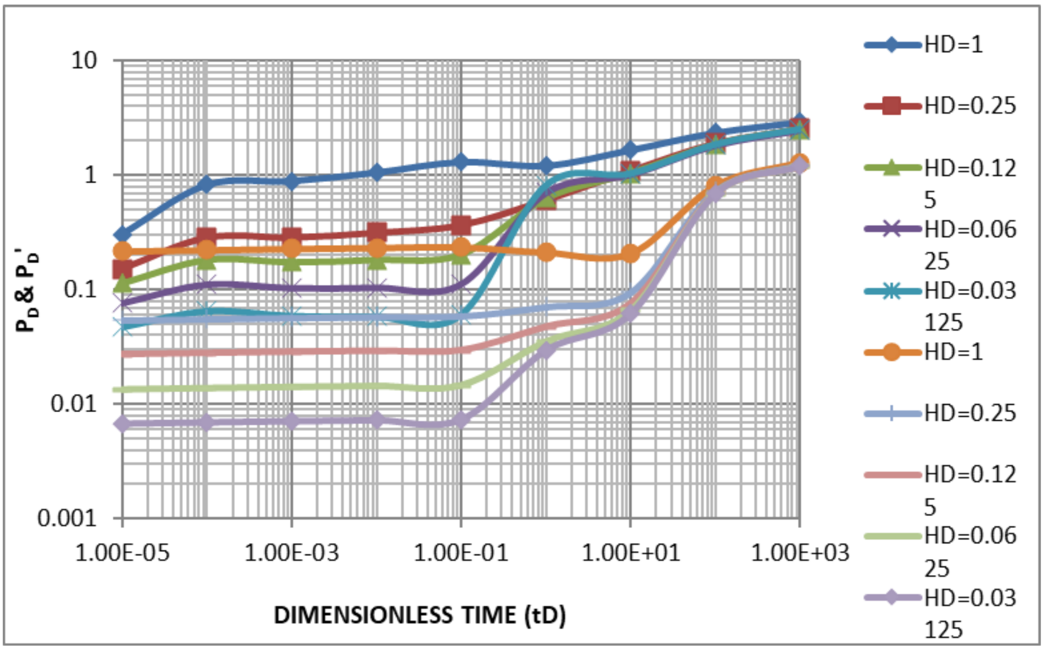

Figure 2: Influence of Dimensionless pay thickness h_D (well 2)

For layer two, it was observed that the pressure and derivative trends conformed to that of a horizontal well in a bounded reservoir. At early time, pressure and derivative response increases slightly as h_D values becomes larger, at intermediate flow periods the system experiences a sharp increase in pressure drop before the values converge to a single straight line at late times indicating that at long times the pressure response becomes insensitive to changes in h_D. Results are shown in figure 2

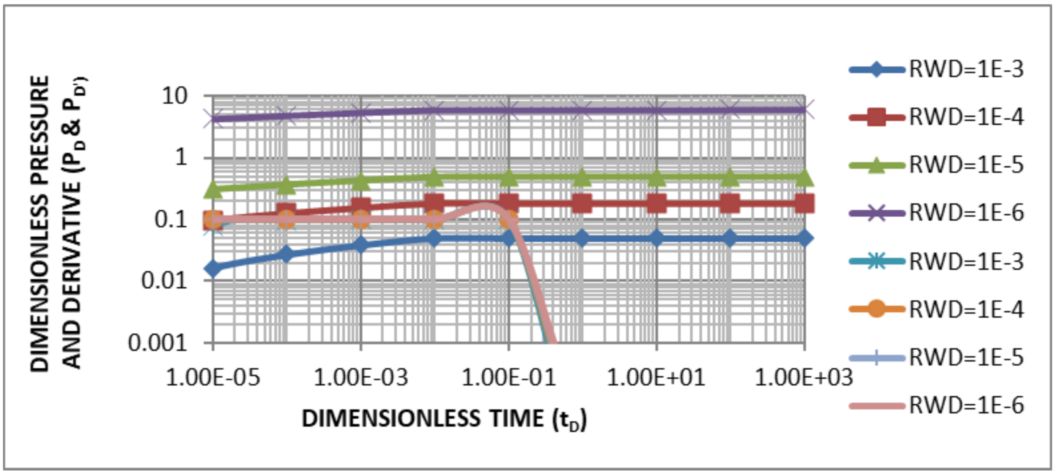

Figure 3: Influence of dimensionless wellbore radius R_{WD} (Well1)

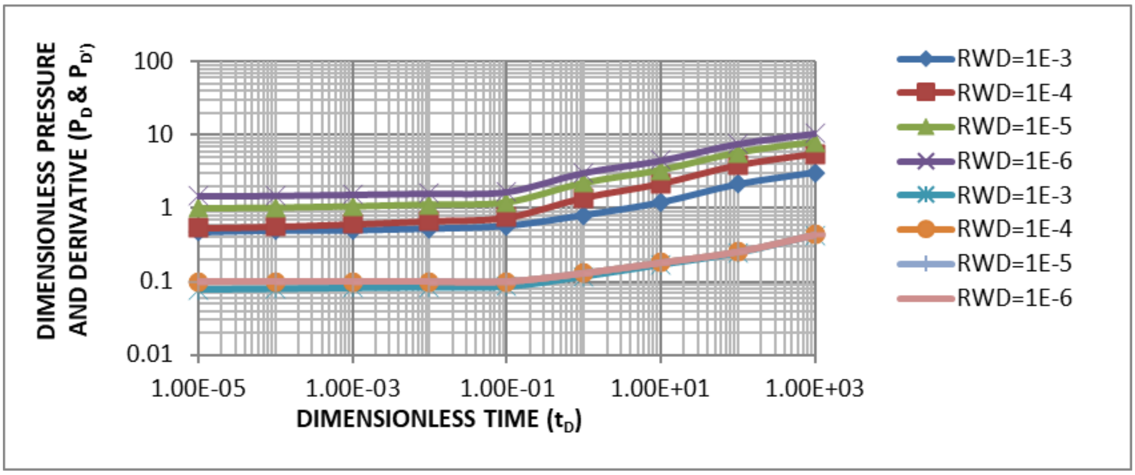

Figure 4: Influence of dimensionless wellbore radius R_{WD} (Well2)

3.2 Influence of Dimensionless radius R_{WD}

For well one, using values of R_{WD} in the range of $1E\text{-}06 \leq R_{WD} \leq 1E\text{-}03$, the influence of changes in wellbore radius was studied. At early time a gradual increase in pressure response was observed, at middle and late times values stabilised as the effect of constant pressure boundary became dominant as shown in figure 3. Smaller R_{WD} produce larger pressure response. Hence, it can be used to produce well optimally. Same observation was made for wells in single system [5].

In well 2, using same values, the pressure response also increased with smaller well radius from early time to late times. The derivative displayed negligible increase at early time and at late times as the values converge to a single line. This can be seen from figure 4.

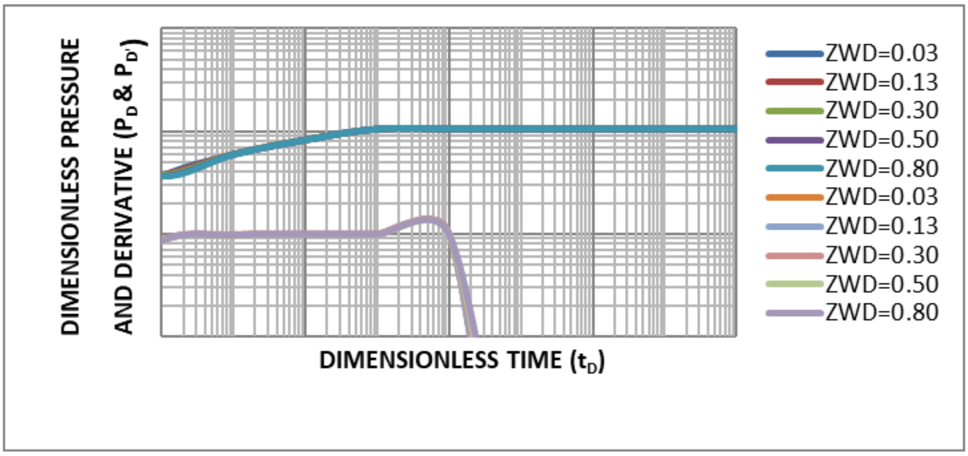

Figure 5: Influence of dimensionless well standoff Z_{WD} (well1)

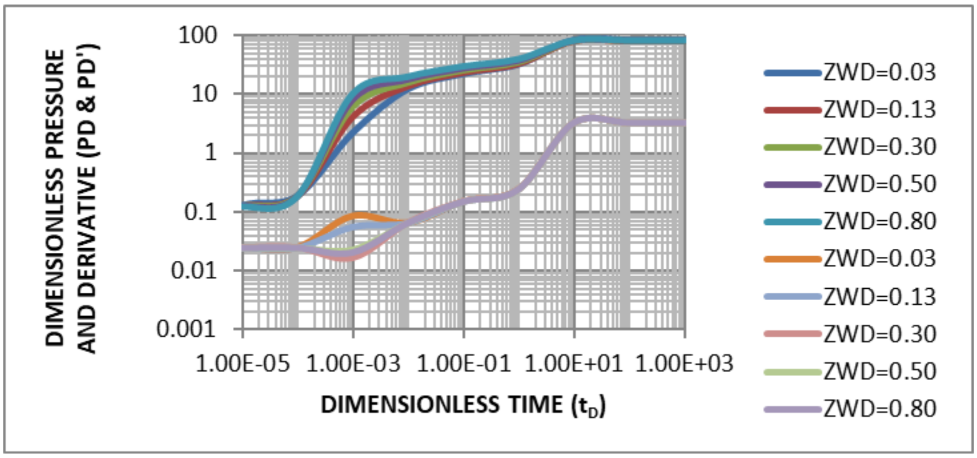

Figure 6: Influence of dimensionless well standoff Z_{WD} (well2)

3.3 Influence of well standoff **(Z_{WD})**

Values of well standoff (Z_{WD}) in the range of $0.025 \leq Z_{WD} \leq 0.8$ were used for this study. Figure 5 shows the influence of Z_{WD} for well one. It is observed that at early time there is no effect on pressure and derivative response this is depicted by a single line that continues to the onset of steady state. Derivative shows no change also at late time. Well 2 has its result displayed in figure 6, the only change observed is in the early time flow period, intermediate and late time periods shows no change with change in Z_{WD}

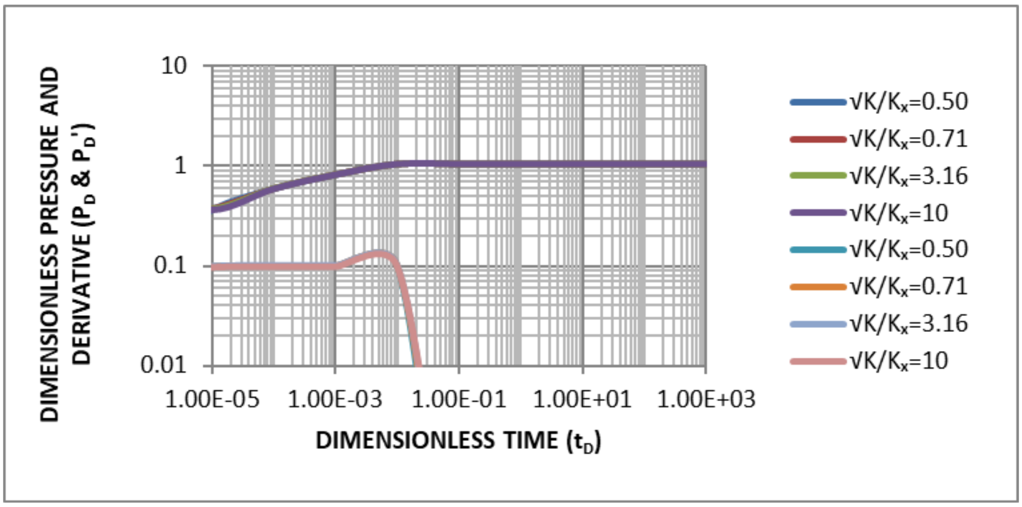

Figure 7: Influence of √(k/kx) (well1)

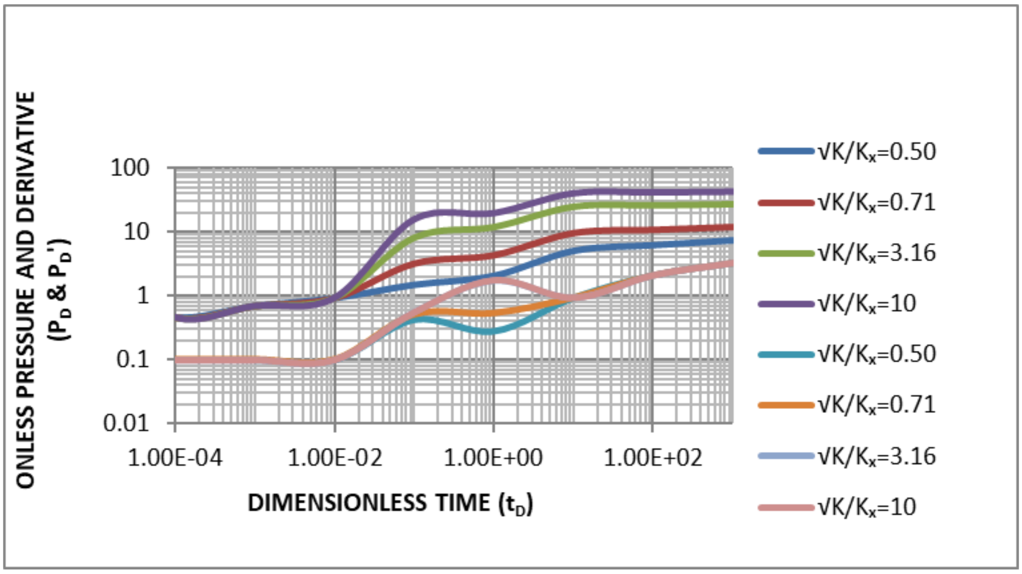

Figure 8: Influence of √(k/kx) (well 2)

3.4 Influence of Permeability ratio √(k/kx)

Well 1 pressure and derivative response shows the Influence of √(k/k_x) in the range of 0.5≤√(k/kx) ≤10 in figure 7. It was observed that it had no effect on pressure and derivative response.

Well 2, pressure and derivative are insensitive to change in √(k/k_x) at early time. At intermediate and late time pressure increases with larger values of √(k/k_x). The derivative showed only slight increase in value with increasing √(k/k_x) during the transition period, and at late time converges to a single straight line as shown in figure 8. The result indicates that well productivity increases when the total permeability of the system is high compared to the horizontal permeability.

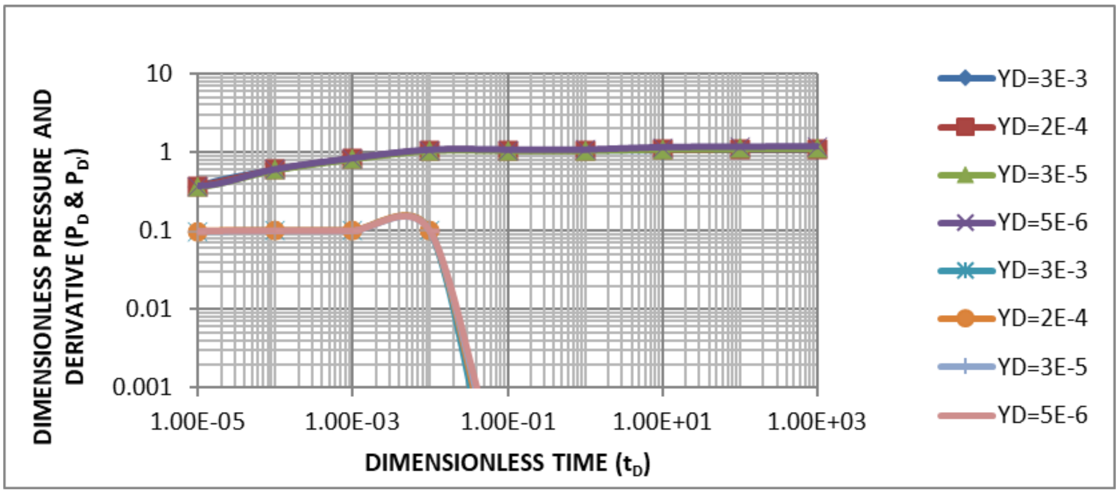

Figure 9: Influence of Y_D (well1)

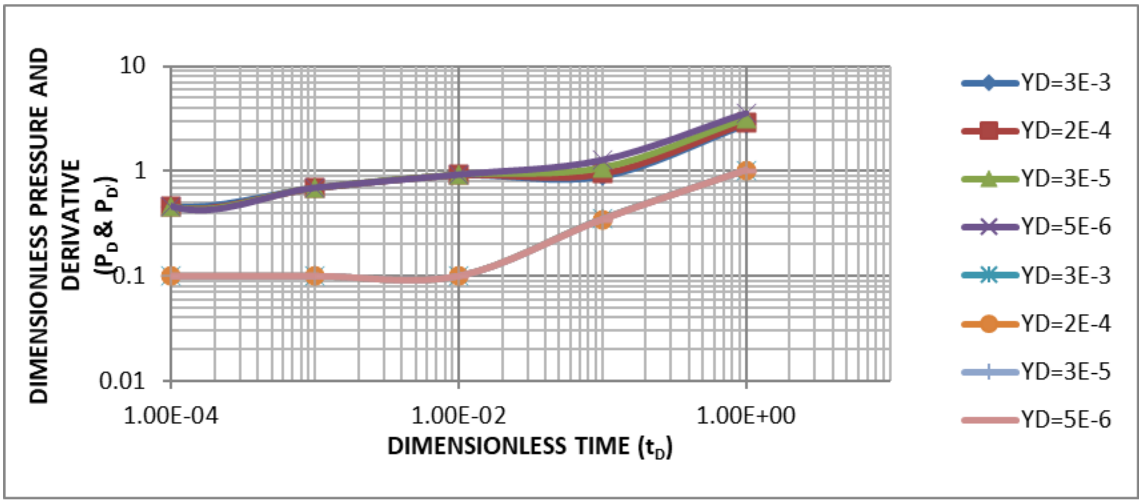

Figure 10: Influence of dimensionless well width Y_D (well2).

3.5 Influence of Y_D

Changes in Y_D produced no change for well 1 pressure and derivative response as shown in figure 9, while in well 2, at late time pressure increases with decreasing Y_D, the derivative is insensitive to changes in Y_D. as shown in Figure 10

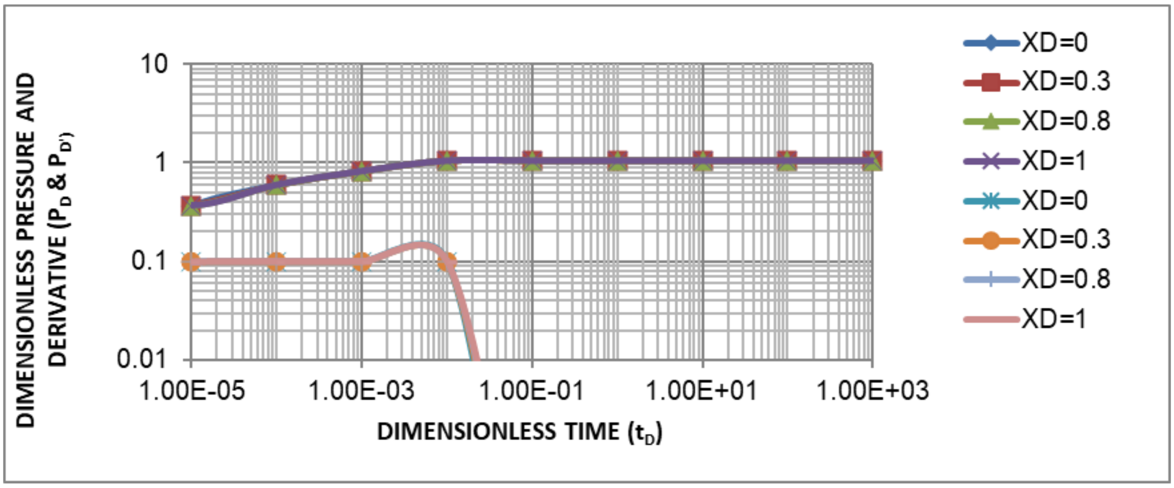

Figure 11: Influence of dimensionless X_D (well1)

3.6 Influence of X_D

Well 1 - No change is observed in pressure and derivative response for well 1 as a result of changes in X_D as shown in figure 11

4. Conclusion

Layered reservoir system with impermeable interface do not experience crossflow, the layers are independent and no fluid communication occurs across the interface. Pressure and derivative response to changes in well and reservoir parameters differs for each layer. Well completion methods may also be different for the layers. From the investigation carried out in this work the following conclusions were drawn:

Horizontal well in layer one experienced P_D and P_D' increased with larger h_D, thin reservoirs (smaller h_D) attained steady state faster. Well two exhibits same pressure behaviour at early time as well one but converges to a single straight line at late times (insensitive). This is the usual trend for reservoirs with constant pressure boundary. Production can be maximised at intermediate flow times. Small R_{WD} produced increased pressure response as observed for system in well one for radial and intermediate flow periods and stabilised at constant values as steady state sets in at late times. Pressure derivative was insensitive. Well two observed increasing pressure response with smaller $R_{WD,}$ a sharp increase was observed at late times, derivative observed to be insensitive. Hence for optimal production, smaller R_{WD} is recommended.

For well one, P_D and P_D ‘ showed negligible changes for varying Z_{WD}, Y_D and $\sqrt{(k/k_x)}$ while for Z_{WD}, well two showed changes only in transition period but at early and late time was insensitive. No effect was observed on P_D at early time but increased with larger $\sqrt{(k/k_x)}$ at late times, P_D’ gave only slight changes during the transition period. When the permeability ratio is large it produces larger pressure drop in well two, hence there is potential for higher productivity. Results from this study will be relevant in effective well planning and optimal production of layered systems with bottom water drive using horizontal wells.

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Appendix

Dimensionless Parameters

$$i_D = \frac{2i}{L}\sqrt{\frac{k}{k_i}} \tag{1a}$$

(where i = x, y, z)

Dimensionless reservoir pay thickness

$$h_D = \frac{2h}{L}\sqrt{\frac{k}{k_z}} = \frac{1}{L_D} \tag{2a}$$

$$R_{eD} = \frac{r_w}{L}\left(\sqrt{\frac{k}{k_z}} + \sqrt{\frac{k}{k_y}}\right) \tag{3a}$$

Dimensionless time

$$t_D = \frac{0.0002637Kt}{\varphi\mu C_t\left(\frac{L}{2}\right)^2} \tag{4a}$$

The flow periods durations [8] were converted to their dimensionless form

Theory

The equation governing the flow of a slightly compressible fluid in a porous media is the diffusivity equation in real time in dimensionless form is

$$\frac{\partial^2 P_D}{\partial_{x^2 D}} + \frac{\partial^2 P_D}{\partial_{y^2 D}} + \frac{\partial^2 P_D}{\partial_{z^2 D}} = \frac{\partial P_D}{\partial t_D} \tag{5a}$$

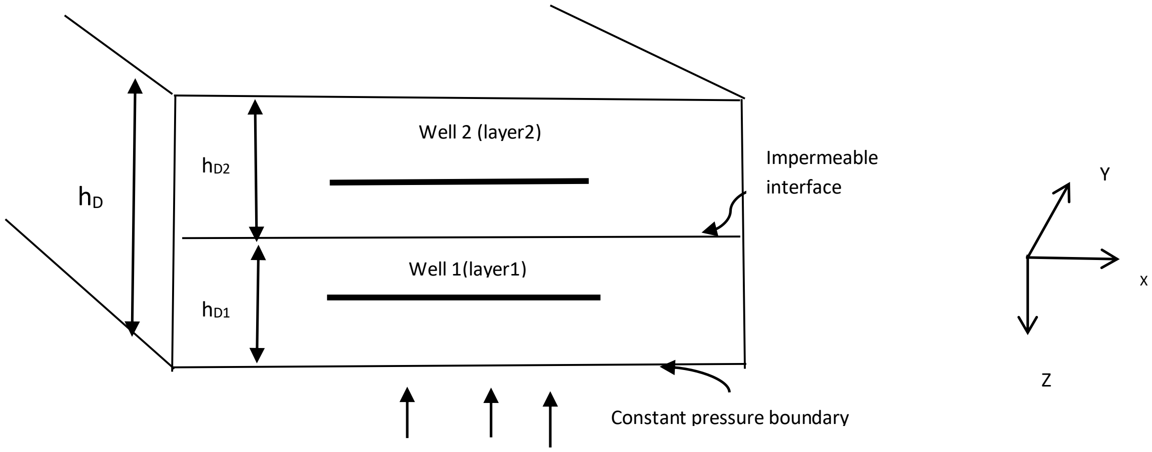

Figure 1A: Schematic of the model for a two layered reservoir with horizontal wells

The Instantaneous Source Function [8] are the solution to equation 6 for the corresponding source reservoir. The pressure drop caused by production from a continuous source is expressed as

$$\Delta p(x,y,z,t) = \frac{1}{\varphi C_t}\int_0^t q_l.S(x,y,z).\partial t \tag{6a}$$

Where q_L represents flow rate per unit length of the source, s(x, y, z, t) represents the instantaneous source function (ISF) for the particular reservoir and well configuration[11]

Sources in a horizontal well are three dimensional in nature, and can be visualised as the product of the one dimensional sources in the three principal axis using Neumann Product rule. The dimensionless instantaneous source function is expressed as

$$S(X_D, Y_D, Z_D, \tau) = S(X_D, \tau).S(Y_D, \tau).S(Z_D, \tau) \tag{7a}$$

$$P_D\left(X_D, Y_D, Z_D, \tau\right) = 2\pi h_D E_j \int_0^{t_D} S\left(X_D, \tau\right).S\left(Y_D, \tau\right).S\left(Z_D, \tau\right)\partial\tau \tag{8a}$$

For a layered system Equation 8 has a factor E_j (modification factor) that accounts for the dual nature of the interface as a result of cross flow between the layers. For no crossflow systems E_j equals 1

The full dimensionless pressure (P_{Dj}) for each layer is developed as

$$P_{Dj} = P_{D1} + P_{D2} + P_{D3} + \ldots\ldots P_{Dn} \tag{9a}$$

where j depicts the layer and n is the last flow period

Dimensionless pressure P_{D1} expression for layer 1 (No crossflow) is [10]

$$P_{D1} = \left[\begin{array}{l}
\left[\dfrac{\alpha}{8L_D}\sqrt{\dfrac{K}{K_y}}Ei\left(-\dfrac{r_{WD}{}^2}{4\tau_D}\right)\right] + \\
\left[\sqrt{\pi}\displaystyle\int_{t_{De1}}^{t_{De2}}\left\{\begin{array}{l}
erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}} + X_{D1}}{2\sqrt{\tau_D}}\right) + erf\left(\dfrac{\sqrt{\dfrac{K}{K_X}} - X_{D1}}{2\sqrt{\tau_D}}\right)\left(1 + 2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\pi y_{wD1}}{Y_{eD}}\cos\dfrac{n\pi Y_{D1}}{Y_{eD}}\right). \\
\dfrac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(z_{D1}-z_{wD1})^2}{4\tau_D}}
\end{array}\right\}\partial\tau\right] + \\
\left[\sqrt{\pi}\displaystyle\int_{t_{De2}}^{t_{De3}}\left\{\begin{array}{l}
\left(1 + \dfrac{4X_{eD}}{\pi}\displaystyle\sum_{m=1}^{\infty}\exp^{\left[--\frac{m^2\pi^2\tau_D}{X_{eD}{}^2}\right]}\sin\dfrac{m\pi x_f}{2X_{eD}}\cos\dfrac{m\pi x_{wD1}}{X_{eD}}\cos\dfrac{m\pi x_{D1}}{X_{eD}}\right). \\
\left(1 + 2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\pi y_{wD1}}{Y_{eD}}\cos\dfrac{n\pi Y_{D1}}{Y_{eD}}\right).. \\
\dfrac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(z_{D1}-z_{wD1})^2}{4\tau_D}}
\end{array}\right\}\partial\tau\right] + \\
\left[\dfrac{2\pi}{X_{eD}Y_{eD}}\displaystyle\int_{t_{De3}}^{t_D}\left\{\begin{array}{l}
\left(1 + \dfrac{4X_{eD}}{\pi}\displaystyle\sum_{m=1}^{\infty}\exp^{\left[--\frac{m^2\pi^2\tau_D}{X_{eD}{}^2}\right]}\sin\dfrac{m\pi x_f}{2X_{eD}}\cos\dfrac{m\pi x_{wD1}}{X_{eD}}\cos\dfrac{m\pi x_{D1}}{X_{eD}}\right). \\
\left(1 + 2\displaystyle\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{Y_{eD}{}^2}}\cos\dfrac{n\pi y_{wD1}}{Y_{eD}}\cos\dfrac{n\pi Y_{D1}}{Y_{eD}}\right). \\
\left(\displaystyle\sum_{i=i}^{\infty}\exp^{\left[--\frac{(2i-1)^2\pi^2\tau_D}{4h_D{}^2}\right]}\sin\dfrac{(2i-1)\pi z_{wD1}}{2h_D}\sin\dfrac{(2i-1)\pi z_{D1}}{2h_D}\right)
\end{array}\right\}\partial\tau\right]
\end{array}\right] \tag{10}$$

Dimensionless pressure P_{D2} expression for layer2 (No crossflow) is

$$\left[\begin{array}{l}\left[\frac{\alpha}{8L_D}\sqrt{\frac{K}{K_y}}Ei\left(-\frac{r_{WD}{}^2}{4\tau_D}\right)\right]+\\ \left[\frac{\sqrt{\pi}}{2}\int_{t_{De1}}^{t_{De2}}\left\{\begin{array}{l}erf\left(\frac{\sqrt{\frac{K}{K_X}}+X_{D2}}{2\sqrt{\tau_D}}\right)+erf\left(\frac{\sqrt{\frac{K}{K_X}}-X_{D2}}{2\sqrt{\tau_D}}\right).\\ \left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{h_D{}^2}}\cos\frac{n\pi y_{wD2}}{h_{D2}}\cos\frac{n\pi y_{D2}}{h_{D2}}\right).\\ \frac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(z_{D2}-z_{WD2})^2}{4\tau_D}}\end{array}\right\}\partial\tau\right]+\\ \left[\frac{\sqrt{\pi}}{2}\int_{t_{De2}}^{t_{De3}}\left\{\begin{array}{l}\left(1+\frac{4X_{eD}}{\pi}\sum_{m=1}^{\infty}\exp^{\left[--\frac{m^2\pi^2\tau_D}{X_{eD}{}^2}\right]}\sin\frac{m\pi x_f}{2X_{eD}}\cos\frac{m\pi x_{wD2}}{X_{eD}}\cos\frac{m\pi x_{D2}}{X_{eD}}\right).\\ \left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{h_D{}^2}}\cos\frac{n\pi y_{wD2}}{h_{D2}}\cos\frac{n\pi y_{D2}}{h_{D2}}\right).\frac{1}{2\sqrt{\tau_D}}\exp^{-\frac{(Z_{D2}-Z_{WD2})^2}{4\tau_D}}\end{array}\right\}.\partial\tau\right]+\\ \left[\frac{2\pi}{X_{eD}Y_{eD}}\int_{t_{De3}}^{t_D}\left\{\begin{array}{l}\left(1+\frac{4X_{eD}}{\pi}\sum_{m=1}^{\infty}\exp^{\left[--\frac{m^2\pi^2\tau_D}{X_{eD}{}^2}\right]}\sin\frac{m\pi x_f}{2X_{eD}}\cos\frac{m\pi x_{wD2}}{X_{eD}}\cos\frac{m\pi x_{D2}}{X_{eD}}\right).\\ \left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{Y_{eD}{}^2}}\cos\frac{n\pi y_{wD2}}{Y_{eD}}\cos\frac{n\pi Y_{D2}}{Y_{eD}}\right).\\ \left(1+2\sum_{n=1}^{\infty}\exp^{-\frac{n^2\pi^2\tau_D}{h_D{}^2}}\cos\frac{n\pi z_{wD2}}{h_D}\cos\frac{n\pi z_{D2}}{h_D}\right)\end{array}\right\}\partial\tau\right]\end{array}\right] \quad (11)$$

The derivatives of the equations were derived by differentiating Equations (10) and (11) using The pressure derivative expression given by Equation(12a)

$$P_D' = \frac{\partial P_D}{\partial Int_D} \quad (12a)$$