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Research Article

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Consideration of Failure Time in Linear Poroelastic Constitutive Model for Formation Failure during Drilling Operation

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Abstract The incorporation of failure time would be a vital tool for petroleum and drilling engineers to precisely predict what happens in the subsurface during production, improved oil recovery and drilling operations. As a result, it is critical that we create a model that combines the linear poroelastic constitutive model with the rock failure criterion while taking failure time into account. To forecast the well pressure at which sanding and formation failure occurs, the stress equations at the wellbore wall must be compared to the failure criterion. This study proposed a model that consists of linear poroelastic constitutive model and rock failure criterion with the consideration of failure time. Various poroelastic models have been developed in the past years; however, this study focuses on the failure time forecast at which these rock species will fail. A new model which involves two time-dependent variables, the characteristics time, and the failure time was developed while utilizing the cohesive strength of the formation with respect to wellbore pressure.

Keywords Failure time; Poroelastic constitutive model; Formation failure; Drilling operation

1. Introduction

In a static scenario, the stresses present in the formation are equal and thus uniform in all directions. However, additional loads are imposed on the formation through drilling operations and other operations such as mud movement during drilling. These not only disrupt the formation, but also shorten the time it takes for it to fail under the given force. Several poroelastic models have been created in recent years. However, this study focuses on the predicted failure period for certain rock species [1].

The understanding of stress and how to analyze it in a solid that is typically loaded is essential to all work related to rock mechanics and geomechanics. A body's internal stress is typically distributed unevenly. It is a tensor quantity with six distinct components that act and are sustained at any place inside the body rather than merely a single value (i.e., a scalar) [2]. Finding the stresses associated with surfaces oriented in three orthogonal directions—that is, the faces of an infinitesimal cube—is essential to provide a thorough description of the stress state at a given location. This cube has shear stress and normal stress operating on each face.

The weight of the strata above causes vertical compressive stress in underground formations, whereas lateral restrictions that confine them create horizontal stresses [3]. Before a borehole is drilled, the rock mass is in an equilibrium state caused by these in situ tensions, which will be destroyed during excavation. The rock next to the drilled borehole takes up the weight that the removed rock was carrying in order to restore equilibrium. The in-situ stresses are subsequently altered as a result of the stress concentration that is created around the well.

Failure in the formation could occur if no support pressure is injected into the borehole. Consequently, in order to prevent rock failure in the field, equilibrium must be maintained. This is typically accomplished by applying a support pressure, which is supplied by a pressurized fluid known as "mud." Interestingly, the borehole wall is where the greatest concentration of stress occurs in a linear elastic material [4]. Consequently, it is anticipated that borehole failure will start there. Therefore, stresses at the borehole wall are the ones that should be tested against a failure criterion for wellbore instability analysis. In the sections that follow, these stresses are calculated for deviated, vertical, and horizontal wellbores.

To develop a model to forecast the events that occur in the subsurface during production or hydrocarbon recovery, one must first understand the failure criterion and constitutive model [5]. Rocks are heterogeneous, solid materials; hence the linear elastic constitutive model is widely employed to predict their stress state. The way rocks respond to stresses or mechanical breakdown is determined by the empty space required for oil production from a reservoir. The linear elastic constitutive model requires fewer parameter and value inputs. The model calculates the critical wellbore pressure, below which sand production is expected. The Mogi-Coulomb failure criterion is used in conjunction with the linear poroelastic constitutive model to create a model that takes failure time into account [6]. The applied stress is the main parameter that governs the failure time; any change in applied stress results in an immediate deformation. The time-dependent effects can be classified into two types: consolidation and creep.

Table 1 shows the summary of the empirical review of some of the selected literatures.

Table 1: Some of the selected Literatures

S/N	Author, Year	Aim of the Study	Methods Adopted	Outcome of the Study	Gaps Identified
1.	Ewy et al.	To develop a model for openhole stability and sanding predictions by 3D extrapolation from Hole-Collapse tests	Experimental and analytical method (Modified Lade criterion)	Developed equations for mud weight required to prevent hole instability during drilling and also wellbore pressure for sanding onset during production.	It was especially designed for horizontal wells and open hole completion
2.	Xianjie	To develop numerical and analytical models for sanding onset prediction.	Experimental, analytical and numerical methods	The author developed sand prediction models assuming shear failure and tensile stress induced sanding respectively.	
3.	Nouri, Vaziri, Kuru, and Rafqul	To predict the onset and rate of sanding.	Model experiment (HCS) and numerical method (Mohr Coulomb)	Developed a numerical model to predict the onset and severity of sanding and conducted a hollow cylinder sample experiment to validate the numerical model.	Published literatures have shown that Mohr coulomb is too conservative in estimating sanding onset.
4.	Al-Ajmi, and Zimmerman	To develop an analytical model for stability analysis of vertical boreholes.	Mogi-Coulomb criterion	Presented easy equations for computing critical mud weight required to maintain wellbore stability	Mogi-Coulomb criterion does not consider the non-linear behaviour of failure criterion
5.	Isehunwa and Farotade	To determine sand failure mechanism and sanding parameters in	Field observations technique	Bean size, water cut and GOR contributed to sanding in the Niger Delta	Field observation data may be subjected to recording instrument and

		Niger Delta oil reservoirs		individually or in combination	personnel
6.	Mingyan,	to develop a simple yet accurate analytical model capable of predicting the rock behaviour around the borehole and conduct experimental works to validate the developed analytical solutions	HCS experiment; analytical solutions using Mohr Coulomb, Drucker- Prager and Hoek-Brown failure criteria; and analytical solution using elastoplastic model with strain softening / hardening effect	Developed a set of closed-form solutions within elastoplastic framework with strain softening to calculate the stress and displacement distribution around a circular opening drilled in finite and isotropic medium.	The author clearly pointed out that prediction of onset of plasticity using Mohr-Coulomb seems a bit conservative, although it also depends on stress level.
7.	Alakbari, Mohyaldinn, Ayoub, Muhsan, and Hussein	To develop a robust fuzzy logic-based model for predicting the critical total drawdown in sand production in oil and gas wells.	Fuzzy-logic approach (Statistical analysis)	Developed a fuzzy logic model for predicting the critical total drawdown (CTD) in sand formations in oil and gas wells.	The approach does not consider the non-linear behavior of rock failure criterion

Methods

The Mohr-Coulomb failure criterion is the most straightforward and easily referred to standard for rock failure. Fjaer et al.[6]) gave the minimum critical well flowing pressure using the Mohr-Coulomb criterion for vertical boreholes with isotropic horizontal stresses in the scenario where ($\sigma_\Theta \geq \sigma_z \geq \sigma_r$). Figure 1 shows the graphic representation of the three main stressors that were previously mentioned:

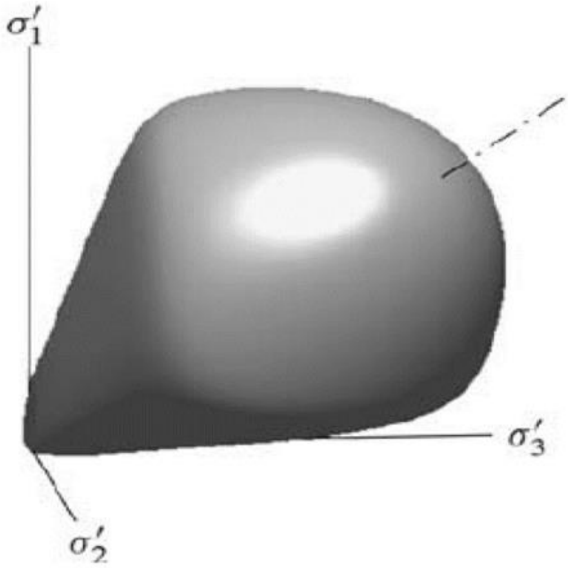

Figure 1: Schematic illustration of a failure surface in the principal stress space. The line point towards represents the hydrostatic axis (Fjaer et al. [6]).

The idea of "safe" and "unsafe" situations, which allow for a broad application of geometrical and hydrogeological complexity under a variety of failure modes, was utilized by Mufundirwa et al. [8]. An example of this idea is provided in Figure 2.

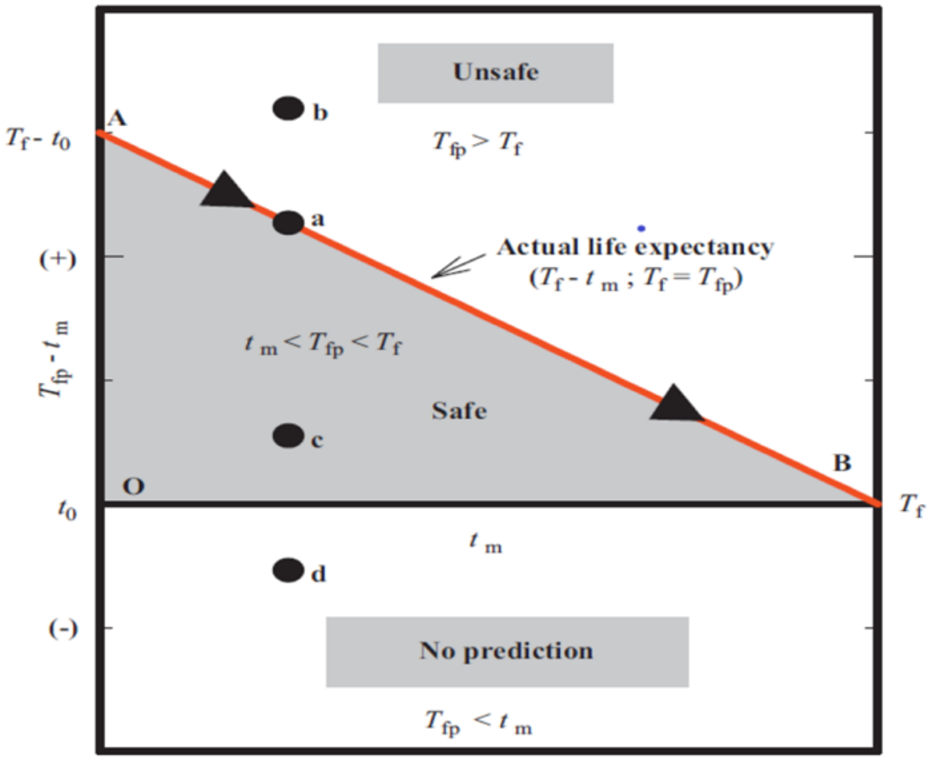

Figure 2: A simple conceptual model representing predicted life expectancy as a function of tm (time at instant of predicting) as failure-time approaches (Mufundirwa et al. [8]).

Under creep conditions, rocks deform at a strain rate that varies over time. To find out how much stress a formation can withstand and how long it will take for it to deform, a model is implemented. Figure 3 displays the workflow that was created to complete this investigation. In this study, the stress at the borehole is determined using a linear poroelastic constitutive model [9]. A rock failure criterion is necessary in addition to the constitutive law, which is utilized to determine the stress state surrounding the well.

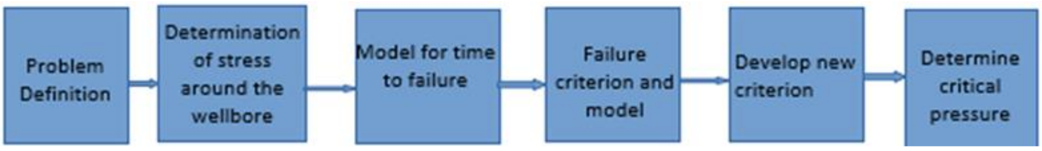

Figure 3: Model Workflow

Applying the subcritical crack quantitative theory as described below, which was put forth by Charles and Hillig in 1962:

$$V = V_o \exp\left(\frac{-W_o + BK}{RT}\right) \qquad (1)$$

V = crack propagation velocity, R = gas constant, T = temperature, W_o = activation energy, K = stress intensity factor while B and V_o are constants.

Additionally, Wiederhorn and Bolz [10] provided a power law model that illustrates the relationship between the velocity of a crack and the degree of the intensity component.

$$V = V_o \left(\frac{K}{K_c}\right)^q \exp\left(\frac{-W_o}{RT}\right) \qquad (2)$$

Accordingly, the failure time of a rock can be calculated using the aforementioned formulae (Coleman, 1956). They can be applied in accordance with the correlation between the crack length, mean main stress, and intensity factor. Das and Scholz [7]; Wiederhorn and Bolz [10] state that an exponential relationship model was constructed between applied stress and average time.

$$t_f = t_o \exp\left(-b\frac{\sigma}{\sigma_0}\right) \qquad (3)$$

$t_f = failure\ time;\ \sigma = major\ stress;\ \sigma_0 = instantaneous\ strength$

Charles's power law was also used to calculate a failure time. As displayed below:

$$t_f = t'_o\left(\frac{\sigma}{\sigma_0}\right)^{-b'} \quad (4)$$

Assumptions used to generate the model:

1. Mechanical factors govern damage onset.
2. The horizontal stresses are anisotropic.
3. The rock is uniform with isotropic properties.
4. A vertical wellbore
5. The use of Mohr-Coulomb failure criterion
6. Homogeneous rock and brittle shear failure induced sanding.
7. Failure occurs when the crack velocity diverges or when it reaches a given (Wiederhorn et al.,1970)

The following represents the stress equations at the wall of a vertical borehole as provided by Shaima et al. [11]:

- Radial stress (σr):

  $\sigma r = Pw$ (5)
- Tangential stress ($\sigma\Theta$):

  $\sigma\Theta = \sigma H + \sigma h - 2(\sigma H - \sigma h)\cos 2\Theta - Pw + 2\eta(Pw - Pfo)$ (6)
- Axial stress (σz)

  $\sigma_z = \sigma_v - 2v(\sigma_H - \sigma_h)\cos 2\Theta + 2\eta(P_w - P_{fo})$ (7)

  $\tau_{\Theta z} = 0$

  $\tau_{r\Theta} = 0$

  $\tau_{rz} = 0$

For vertical well, Al-Ajmi, [12] stated that the maximum stress values will always be at $\Theta = \pm 90^{\circ}$.

$\sigma_r = P_w$

$\sigma_\Theta = 3\sigma_H - \sigma_h - P_w + 2\eta(P_f - P_{fo})$ (8)

$\sigma_z = \sigma_v - 2v(\sigma_H - \sigma_h)\cos 2\Theta + 2\eta(P_w - P_{fo})$

$\tau_{\Theta z} = \tau_{r\Theta} = \tau_{rz} = 0$

In a deviated well intended for production, the area of stress concentration surrounding the wellbore wall is Fjaer et al. [6]. The stresses at the borehole in a linear poro-elastic formation are as follows after simplification:

$$\sigma_r = P_w$$

$$\sigma_\theta = 3\sigma_H - \sigma_h - P_w + 2\eta\left(P_f - P_{fo}\right) \quad (9)$$

$$\sigma_z = \sigma_v + 2v(\sigma_H - \sigma_h) + 2\eta\left(P_f - P_{fo}\right)$$

$$\eta = \alpha\,\frac{1-2v}{2(1-v)}$$

Where,

$\sigma_r = radial\ stress \quad \sigma_\theta = tangential\ stress \quad \sigma_z = Axial\ stress \quad P_w = internal\ wellbore\ pressure\ P_f = pore\ pressure\ P_{fo} = far\ field\ pore\ pressure\ \eta = Poroelastic\ stress\ coefficient$

The simplest and easiest model to understand rock failure criterion is the Mohr-Coulomb criterion. It is said as follows:

$$\sigma_1 = 2\frac{C\cos\emptyset_f}{1-\sin\emptyset_f} + \sigma_3\frac{1+\sin\emptyset_f}{1-\sin\emptyset_f}$$

Where

$\sigma_1 = major\ stress\ on\ the\ element\ \emptyset_f = Internal\ friction\ Angle\ \sigma_3 = minor\ stress\ on\ the\ element\ c = Internal\ cohesion$

Failure time with Mohr-Coulomb failure criterion

$$t_f = t_o\exp\left(-b\frac{\sigma}{\sigma_0}\right)$$

$$\frac{t_f}{t_o} = \exp\left(-b\frac{\sigma}{\sigma_0}\right)$$

$$In\left(\frac{t_f}{t_o}\right) = -b\frac{\sigma}{\sigma_0}$$

$$\sigma = \left(\frac{\sigma_0}{-b}\right)In\left(\frac{t_f}{t_o}\right)$$

If the conventional effective stress concept is considered, the Mohr-Coulomb criterion becomes:

$$\sigma_1 - P_f = C_o + q(\sigma_3 - P_f)$$

$$\left(\frac{\sigma_0}{-b}\right) In\left(\frac{t_f}{t_o}\right) - P_f = C_o + q(\sigma_3 - P_f)$$

$$\left(\frac{\sigma_0}{-b}\right) In\left(\frac{t_f}{t_o}\right) - P_f = C_o + q\left(P_w - P_f\right)$$

$when\ P_f = P_w$

$$\left(\frac{\sigma_0}{-b}\right) In\left(\frac{t_f}{t_o}\right) - P_w = C_o + qP_w - qP_w$$

$$\left(\frac{\sigma_0}{-b}\right) In\left(\frac{t_f}{t_o}\right) - C_o = P_{wf} \qquad (10)$$

$$In\left(\frac{t_f}{t_o}\right) = P_{wf}\left(\frac{-b}{\sigma_0}\right) + C_o$$

$$t_f = t_o \exp P_{wf}\left(\frac{-b}{\sigma_0}\right) + C_o \qquad (11)$$

$t_f = failure\ time$ $P_{wf} = bottom\ hole\ flowing\ pressure$ $t_o = characteristic\ time$ $-b = exponent\ time\ to\ failure\ law$ $\sigma_0 = instantaenous\ strength$ $C_o = cohesive\ strength$

In this study, using a conceptual and theoretical model, the time of failure is investigated to maximize sand production while preserving wellbore stability. This analytical model may need to be validated by a thorough numerical analysis, field testing, and a fundamental understanding of sand production and wellbore stability requirements. The Mohr-Coulomb failure criterion is being solved numerically while integrating with stress-bound variables [13-15]. Because the Mohr-Coulomb criterion ignores the impact of intermediate main stress, it always calculates the minimum needed mud weight to be larger than the actual polyaxial failure requirement. The Mohr-Coulomb criterion results for the high strength formation approach upper bounds of results, suggesting that it may not be a reliable criterion for hard lithology failure analysis.

Conclusion

One of the difficult problems in the oil and gas sector is wellbore stability. The linear elastic-brittle hypothesis forms the basis of most analytical solutions to wellbore stability. According to this idea, borehole collapse happens when elastic forces exceed geomaterials' strength.

1. The research was aimed at developing a failure time dependent prediction model that takes into consideration the non-linear property of failure criterion.
2. Literature review showed that several sand prediction models have been built over the years, but most of the models neglected the non-linear effect of a failure criterion, which was considered in this study.
3. The prediction model was developed from first principle and then, MATLAB software was used to provide a solution to the equation.
4. The developed model was validated using two published case studies in literature.

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