Available online www.jsaer.com

Journal of Scientific and Engineering Research, **2024, 11(11):65-72**

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

On the Pseudo-Starlike and Pseudo-Convex Univalent Function Classes of Complex Order

Nizami MUSTAFA*, Davut MİNGSAR

*Kafkas University, Faculty of Science and Letters, Department of Mathematics, Kars, Turkey

Abstract: In this paper, we defined a new subclass of starlike bi-univalent functions and examine some geometric properties this function class. For this definition class, we gave some coefficient estimates and solve Fekete-Sezöge problem.

Keywords: Starlike function, convex function, pseudo-starlike function, pseudo-convex function

1. Introduction

Let $H(U)$ be the class of analytic functions on the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$ in the complex plane $\mathbb{C}$. By A, we will denote the class of the functions $f \in H(U)$ given by the following series expansions

$$f(z) = z + a_2 z^2 + a_3 z^3 + a_4 z^4 + \cdots + a_n z^n + \cdots = z + \sum_{n=2}^{\infty} a_n z^n,\ a_n \in \mathbb{C}\ . \qquad (1.1)$$

The subclass of univalent functions of A is denoted by S in the literature. This class was first introduced by Köebe [1] and has become the core ingredient of advanced research in this field. Within a short period, in 1916 Bieberbach [2] published a paper in which the famous coefficient hypothesis was proposed. This hypothesis states that if $f \in S$ and has the series form (1.1), then $|a_n| \le n$ for each $n \ge 2$. In 1985, it was de-Branges [3], who settled this long-lasting conjecture. There were a lot of papers devoted to this conjecture and its related coefficient problems (see [4-12]).

It is well known that the starlike and convex function classes defined on the open unit disk U are defined analytically as follows

$$S^* = \left\{ f \in S:\ \mathrm{Re}\left(\frac{zf'(z)}{f(z)} \right) > 0,\ z \in U \right\},\ C = \left\{ f \in S:\ \mathrm{Re}\left(\frac{(zf'(z))'}{f'(z)} \right) > 0,\ z \in U \right\}.$$

As is known that an analytical function ω satisfying the conditions $\omega(0) = 0$ and $|\omega(z)| < 1$ is called Schwartz function. Let's $f, g \in H(U)$, then it is said that f is subordinate to g and denoted by $f \prec g$, if there exists a Schwartz function ω, such that $f(z) = g(\omega(z))$.

In the past few years, numerous subclasses of the collection S have been introduced as special choices of the classes S^* and C (see for example [5-20]).

2. Materials and Methods

Now, let's define some new subclass of univalent functions in the open unit disk U.

Definition 2.1. For $\beta \in [0,1]$, $\lambda > \frac{1}{2}$ and $\tau \in \mathbb{C} - \{0\}$ the function $f \in S$ is said to be in the class $\chi_{\sinh}(\beta, \lambda, \tau)$, if the following conditions are satisfied

$$(1-\beta)\left\{1+\frac{1}{\tau}\left[\frac{z\left(f'(z)\right)^{\lambda}}{f(z)}-1\right]\right\}+\beta\left\{1+\frac{1}{\tau}\left[\frac{\left[\left(zf'(z)\right)'\right]^{\lambda}}{f'(z)}-1\right]\right\} \prec 1+\sinh z,\ z \in U .$$

From the Definition 1.1, in the special values of the parameters, we obtain the following function classes.

Definition 2.2. For $\lambda > \frac{1}{2}$ and $\tau \in \mathbb{C} - \{0\}$ the function $f \in \Sigma$ is said to be in the class $S^{*}_{\sinh}(\lambda, \tau)$, if the following conditions are satisfied

$$\left\{1+\frac{1}{\tau}\left[\frac{z\left(f'(z)\right)^{\lambda}}{f(z)}-1\right]\right\} \prec 1+\sinh z,\ z \in U .$$

Definition 2.2.1. For $\tau \in \mathbb{C} - \{0\}$ the function $f \in S$ is said to be in the class $S^{*}_{\sinh}(\tau)$, if the following conditions are satisfied

$$\left\{1+\frac{1}{\tau}\left[\frac{zf'(z)}{f(z)}-1\right]\right\} \prec 1+\sinh z,\ z \in U .$$

Definition 2.2.2. For $\lambda > \frac{1}{2}$ the function $f \in S$ is said to be in the class $S^{*}_{\sinh}(\lambda)$, if the following conditions are satisfied

$$\frac{z\left(f'(z)\right)^{\lambda}}{f(z)} \prec 1+\sinh z,\ z \in U .$$

Definition 2.2.3. For the function $f \in S$ is said to be in the class $S^{*}_{\sinh}$, if the following conditions are satisfied

$$\frac{zf'(z)}{f(z)} \prec 1+\sinh z,\ z \in U .$$

Definition 2.3. For $\lambda > \frac{1}{2}$ and $\tau \in \mathbb{C} - \{0\}$ the function $f \in S$ is said to be in the class $C_{\sinh}(\lambda, \tau)$, if the following conditions are satisfied

$$\left\{1+\frac{1}{\tau}\left[\frac{\left[\left(zf'(z)\right)'\right]^{\lambda}}{f'(z)}-1\right]\right\} \prec 1+\sinh z,\ z \in U .$$

Definition 2.3.1. For $\tau \in \mathbb{C} - \{0\}$ the function $f \in S$ is said to be in the class $C_{\sinh}(\tau)$, if the following conditions are satisfied

$$\left\{1+\frac{1}{\tau}\left[\frac{\left(zf'(z)\right)'}{f'(z)}-1\right]\right\}\prec 1+\sinh z,\ z\in U .$$

Definition 2.3.2. For $\lambda>\frac{1}{2}$ the function $f\in S$ is said to be in the class $C_{\sinh}(\lambda)$, if the following conditions are satisfied

$$\frac{\left[\left(zf'(z)\right)'\right]^{\lambda}}{f'(z)}\prec 1+\sinh z,\ z\in U .$$

Definition 2.3.3. For the function $f\in S$ is said to be in the class $C_{\sinh}$, if the following conditions are satisfied

$$\frac{\left(zf'(z)\right)'}{f'(z)}\prec 1+\sinh z,\ z\in U .$$

Definition 2.4. For $\beta\in[0,1]$ and $\tau\in\mathbb{C}-\{0\}$ the function $f\in S$ is said to be in the class $\chi_{\sinh}(\beta,\tau)$, if the following condition is satisfied

$$(1-\beta)\left\{1+\frac{1}{\tau}\left[\frac{zf'(z)}{f(z)}-1\right]\right\}+\beta\left\{1+\frac{1}{\tau}\left[\frac{\left(zf'(z)\right)'}{f'(z)}-1\right]\right\}\prec 1+\sinh z,\ z\in U .$$

Definition 2.4.1. For $\beta\in[0,1]$ the function $f\in S$ is said to be in the class $\chi_{\sinh}(\beta)$, if the following conditions are satisfied

$$(1-\beta)\frac{zf'(z)}{f(z)}+\beta\frac{\left(zf'(z)\right)'}{f'(z)}\prec 1+\sinh z,\ z\in U .$$

Definition 2.5. For $\beta\in[0,1]$ and $\lambda>\frac{1}{2}$ the function $f\in S$ is said to be in the class $\chi_{\sinh}(\beta,\lambda)$, if the following condition is satisfied

$$(1-\beta)\frac{z\left(f'(z)\right)^{\lambda}}{f(z)}+\beta\frac{\left[\left(zf'(z)\right)'\right]^{\lambda}}{f'(z)}\prec 1+\sinh z,\ z\in U .$$

Let P be the class of analytic functions in U satisfied the conditions $p(0)=1$ and $\mathrm{Re}\left(p(z)\right)>0$, $z\in U$, which from the subordination principle easily can written

$$\mathrm{P}=\left\{p\in A:\ p(z)\prec\frac{1+z}{1-z},z\in U\right\},$$

where $p(z)$ has the series expansion of the form

$$p(z)=1+p_1z+p_2z^2+p_3z^3+\cdots=1+\sum_{n=1}^{\infty}p_nz^n,\ z\in U . \qquad (2.1)$$

Now, let us present some necessary lemmas known in the literature for the proof of our main results.

Lemma 2.1 ([21]). Let the function $p(z)$ belong in the class P. Then,

$$|p_n| \le 2 \text{ for each } n \in \mathbb{N} \text{ and } |p_n - \lambda p_k p_{n-k}| \le 2 \text{ for } n,k \in \mathbb{N},\ n > k \text{ and } \lambda \in [0,1].$$

The equalities hold for

$$p(z) = \frac{1+z}{1-z}.$$

Lemma 2.2 ([21]) Let an analytic function $p(z)$ be of the form (2.1), then

$$2p_2 = p_1^2 + (4 - p_1^2)x,$$

$$4p_3 = p_1^3 + 2(4-p_1^2)p_1 x - (4-p_1^2)p_1 x^2 + 2(4-p_1^2)(1-|x|^2)y$$

for $x, y \in \mathbb{C}$ with $|x| \le 1$ and $|y| \le 1$.

In this paper, we give some coefficient estimates and examine Fekete-Szegö problem for the class $\chi_{\sinh}(\beta,\lambda,\tau)$. Additionally, the results obtained in our study are compared with the results available in the literature.

3. Results & Discussion

In this section, we examine the coefficient estimates problem for the function class $\chi_{\sinh}(\beta,\lambda,\tau)$ and solve Fekete-Szegö problem for this class.

Firstly, we give the following theorem on coefficient evaluation.

Theorem 3.1. If $f \in \chi_{\Sigma,\sinh}(\beta,\lambda,\tau)$, then are provided the following inequalities

$$|a_2| \le \frac{|\tau|}{(2\lambda-1)(1+\beta)} \quad \text{and}$$

$$|a_3| \le \frac{|\tau|}{(3\lambda-1)(1+2\beta)} \begin{cases} 1 & \text{if} \quad a(\tau,\lambda,\beta) \le 0, \\ \dfrac{|2\lambda^2-4\lambda+1|(1+3\beta)|\tau|}{(2\lambda-1)^2(1+\beta)^2} & \text{if} \quad a(\tau,\lambda,\beta) \ge 0, \end{cases} \tag{3.1}$$

where $a(\tau,\lambda,\beta) = \dfrac{|2\lambda^2-4\lambda+1|(1+3\beta)|\tau| - (2\lambda-1)^2(1+\beta)^2}{4(2\lambda-1)^2(1+\beta)^2}$.

Proof. Let $f \in \chi_{\sinh}(\beta,\lambda,\tau)$. Then, are a Schwartz function $\omega: U \to U$, such that

$$(1-\beta)\left\{1+\frac{1}{\tau}\left[\frac{z(f'(z))^{\lambda}}{f(z)}-1\right]\right\} + \beta\left\{1+\frac{1}{\tau}\left[\frac{\left[(zf'(z))'\right]^{\lambda}}{f'(z)}-1\right]\right\} = 1+\sinh\omega(z),\ z\in U. \tag{3.2}$$

Let's the function $p \in P$ defined as follows

$$p(z) = \frac{1+\omega(z)}{1-\omega(z)} = 1 + p_1 z + p_2 z^2 + p_3 z^3 + \ldots = 1 + \sum_{n=1}^{\infty} p_n z^n,\ z \in U. \tag{3.3}$$

From these equality, we can write

$$\omega(z) = \frac{p(z)-1}{p(z)+1} = \frac{p_1}{2}z + \frac{1}{2}\left(p_2 - \frac{p_1^2}{2}\right)z^2 + \frac{1}{2}\left(p_3 - p_1 p_2 - \frac{p_1^3}{4}\right)z^3 \ldots,\ z \in U. \tag{3.4}$$

From the (3.2), (3.4) can written

$$
\begin{aligned}
&(1-\beta)\left\{1+\frac{1}{\tau}\left[a_2(2\lambda-1)z+\left((3\lambda-1)a_3+\left(2\lambda^2-4\lambda+1\right)a_2^2\right)z^2+\ldots\right]\right\}\\
&+\beta\left\{1+\frac{1}{\tau}\left[2a_2(2\lambda-1)z+\left(3(3\lambda-1)a_3+\left(8\lambda^2-16\lambda+4\right)a_2^2\right)z^2+\ldots\right]\right\} \qquad (3.5)\\
&=1+\frac{p_1}{2}z+\left(\frac{p_2}{2}-\frac{p_1^2}{4}\right)z^2+\ldots,\ z\in U,
\end{aligned}
$$

Comparing the coefficients of the same degree terms on the right and left sides of the equality (3.5), we obtain the following equalities for the coefficients a_2 and a_3

$$a_2=\frac{\tau p_1}{2(2\lambda-1)(1+\beta)}, \qquad (3.6)$$

$$(3\lambda-1)(1+2\beta)a_3+\left(2\lambda^2-4\lambda+1\right)(1+3\beta)a_2^2=\left(\frac{p_2}{2}-\frac{p_1^2}{4}\right)\tau, \qquad (3.7)$$

According to the Lemma 2.1, from the equality (3.6) we obtain the first result of theorem.

From the equalities (3.7) and (3.6) we obtain the following equality for a_3

$$a_3=\frac{\tau}{(3\lambda-1)(1+2\beta)}\left\{\frac{p_2}{2}-\frac{p_1^2}{4}-\frac{\left(2\lambda^2-4\lambda+1\right)(1+3\beta)\tau}{4(2\lambda-1)^2(1+\beta)^2}p_1^2\right\}. \qquad (3.8)$$

Since $\frac{p_2}{2}-\frac{p_1^2}{4}=\frac{4-p_1^2}{4}x$, we can write

$$a_3=\frac{\tau}{(3\lambda-1)(1+2\beta)}\left\{\frac{4-p_1^2}{4}x-\frac{\left(2\lambda^2-4\lambda+1\right)(1+3\beta)\tau}{4(2\lambda-1)^2(1+\beta)^2}p_1^2\right\}$$

for some $x\in\mathbb{C}$ with $|x|\le 1$.

Applying triangle inequality to the last equality, we obtain

$$|a_3|\le\frac{|\tau|}{(3\lambda-1)(1+2\beta)}\left\{\frac{4-t^2}{4}\xi+\frac{\left|2\lambda^2-4\lambda+1\right|(1+3\beta)|\tau|}{4(2\lambda-1)^2(1+\beta)^2}t^2\right\},\ \xi\in[0,1], \qquad (3.9)$$

where $\xi=|x|$ and $t=|p_1|$.

From the inequality (3.9), we can write

$$|a_3|\le\frac{|\tau|}{(3\lambda-1)(1+2\beta)}\left\{a(\tau,\lambda,\beta)t^2+1\right\},\ t\in[0,2], \qquad (3.10)$$

where $a(\tau,\lambda,\beta)=\dfrac{\left|2\lambda^2-4\lambda+1\right|(1+3\beta)|\tau|-(2\lambda-1)^2(1+\beta)^2}{4(2\lambda-1)^2(1+\beta)^2}$.

Then, maximizing the function $\chi(t)=a(\tau,\lambda,\beta)t^2+1$, we obtain the second result of theorem.

With this the proof of theorem is completed.

Taking $\beta=0$ and $\beta=1$ in the Theorem 3.1, we obtain the following results, respectively.

Corollary 3.1. If $f\in S_{\sinh}^{*}(\lambda,\tau)$, then

$$|a_2| \le \frac{|\tau|}{2\lambda-1} \text{ and } |a_3| \le \frac{|\tau|}{3\lambda-1}\begin{cases} 1 & \text{if } \left|2\lambda^2-4\lambda+1\right||\tau| \le (2\lambda-1)^2, \\ \dfrac{\left|2\lambda^2-4\lambda+1\right||\tau|}{(2\lambda-1)^2} & \text{if } \left|2\lambda^2-4\lambda+1\right||\tau| \ge (2\lambda-1)^2. \end{cases}$$

Corollary 3.2. If $f \in C_{\sinh}(\lambda,\tau)$, then

$$|a_2| \le \frac{|\tau|}{2(2\lambda-1)} \text{ and } |a_3| \le \frac{|\tau|}{3(3\lambda-1)}\begin{cases} 1 & \text{if } \left|2\lambda^2-4\lambda+1\right||\tau| \le (2\lambda-1)^2, \\ \dfrac{\left|2\lambda^2-4\lambda+1\right||\tau|}{(2\lambda-1)^2} & \text{if } \left|2\lambda^2-4\lambda+1\right||\tau| \ge (2\lambda-1)^2, \end{cases} \tag{3.11}$$

Note: 3.1. In the special values of the parameters λ and τ from Corollary 3.1 and Corollary 3.2, we obtain the results for the classes $S^*_{\sinh}(\tau)$, $S^*_{\sinh}(\lambda)$, $S^*_{\sinh}$ and $C_{\sinh}(\tau)$, $C_{\sinh}(\lambda)$, $C_{\sinh}$, respectively.

Now, we focused on the solution of the Fekete-Szegö problem for the class $\chi_{\sinh}(\beta,\lambda,\tau)$.

Theorem 3.2. Let $f \in \chi_{\sinh}(\beta,\lambda,\tau)$ and $\mu \in \mathbb{C}$. Then,

$$\left|a_3-\mu a_2^2\right| \le \frac{|\tau|}{(3\lambda-1)(1+2\beta)}\begin{cases} 1 & \text{if } |1-\mu||\tau| \le l(\tau,\lambda,\beta), \\ l(\tau,\lambda,\beta)+1 & \text{if } |1-\mu||\tau| \ge l(\tau,\lambda,\beta), \end{cases}$$

where

$$l(\tau,\lambda,\beta) = \frac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right||\tau|-(2\lambda-1)^2(1+\beta)^2}{(2\lambda-1)^2(1+\beta)^2}.$$

Proof. Let $f \in \chi_{\sinh}(\beta,\lambda,\tau)$ and $\mu \in \mathbb{C}$. From the equalities (2.6) and (2.8), we can write

$$a_3-\mu a_2^2 = \frac{\tau}{4(3\lambda-1)(1+2\beta)}\left[\left(4-p_1^2\right)x - \frac{\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu}{(2\lambda-1)^2(1+\beta)^2}\tau p_1^2\right] \tag{3.12}$$

for some $x \in \mathbb{C}$ with $|x| \le 1$.

Applying triangle inequality to the equality (3.12), we obtain

$$\left|a_3-\mu a_2^2\right| \le \frac{|\tau|}{4(3\lambda-1)(1+2\beta)}\left[\left(4-t^2\right)\xi + \frac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right|}{(2\lambda-1)^2(1+\beta)^2}|\tau|t^2\right],$$

$\xi \in [0,1]$,

where $\xi = |x|$ and $t = |p_1|$.

From the last inequality, we can write

$$\left|a_3-\mu a_2^2\right| \le \frac{|\tau|}{4(3\lambda-1)(1+2\beta)}\left[l(\tau,\lambda,\beta)t^2+4\right],\ t \in [0,2], \tag{3.13}$$

where

$$l(\tau,\lambda,\beta) = \frac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right||\tau|-(2\lambda-1)^2(1+\beta)^2}{(2\lambda-1)^2(1+\beta)^2}.$$

Maximizing the expression on the right hand side of the inequality (3.13) according to the parameter t, we have result of theorem.
Thus, the proof of theorem is completed.
If we take $\beta = 0$ and $\beta = 1$ in Theorem 3.2, we obtain the following results, respectively.

Corollary 3.3. If $f \in S^*_{\sinh}(\lambda, \tau)$, then

$$\left|a_3 - \mu a_2^2\right| \le \frac{|\tau|}{3\lambda - 1} \begin{cases} 1 & if \ |1-\mu||\tau| \le l_1(\tau,\lambda,\beta), \\ l_1(\tau,\lambda,\beta) + 1 & if \ |1-\mu||\tau| \ge l_1(\tau,\lambda,\beta), \end{cases}$$

Where $l_1(\tau,\lambda) = \dfrac{\left|\left(2\lambda^2 - 4\lambda + 1\right) + (3\lambda - 1)\mu\right||\tau| - (2\lambda - 1)^2}{(2\lambda - 1)^2}$

Corollary 3.4. If $f \in C_{\sinh}(\lambda, \tau)$, then

$$\left|a_3 - \mu a_2^2\right| \le \frac{|\tau|}{3(3\lambda - 1)} \begin{cases} 1 & if \ |1-\mu||\tau| \le l_2(\tau,\lambda,\beta), \\ l_2(\tau,\lambda,\beta) + 1 & if \ |1-\mu||\tau| \ge l_2(\tau,\lambda,\beta), \end{cases}$$

where $l(\tau,\lambda,\beta) = \dfrac{\left|\left(2\lambda^2 - 4\lambda + 1\right) + 3(3\lambda - 1)\mu\right||\tau| - (2\lambda - 1)^2}{(2\lambda - 1)^2}$.

Note: 3.2. In the special values of the parameters λ and τ from Corollary 3.3 and Corollary 3.4, we obtain the results for the classes $S^*_{\sinh}(\tau)$, $S^*_{\sinh}(\lambda)$, $S^*_{\sinh}$ and $C_{\sinh}(\tau)$, $C_{\sinh}(\lambda)$, $C_{\sinh}$, respectively.

References

[1]. Köebe, P. (1909). Über die Uniformisierrung der algebraishen Kurven, durch automorpher Funktionen mit imaginarer Substitutions gruppe. Nachr. Akad. Wiss. Göttingen Math. -Phys. 68-76.

[2]. Bieberbach, L. (1916). Über die Koeffizienten derjenigen Potenzreihen welche eine schlichte Abbidung des Einbeiskreises vermitteln. Sitzungsberichte Press. Akad. Der Wiss. 138, 940-955.

[3]. De Branges, L. (1985). A proof of the Bieberbach conjecture. Acta Math. 154, 137-152.

[4]. Janowski, W. (1970). Extremal problems for a family of functions with positive real part and for some related families. Ann. Pol. Math. 23, 159-177.

[5]. Sharma, K., Jain, N. K., Ravichandran, V. (2016). Starlike function associated with a cardioid. Afr. Math. 27, 923-939.

[6]. Kumar, S. S., Arora, K. (2020). Starlike functions associated with a petal shaped domain. ArXiv 2020, arXiv: 2010.10072.

[7]. Mendiratta, R., Nagpal, S., Ravichandran, V. (2015). On a subclass of strongly starlike functions associated with exponential function. Bull. Malays. Math. Soc. 38, 365-386.

[8]. Bano, K., Raza, M. (2020). Starlike functions associated with cosine function. Bull. Iran. Math. Soc. 47, 1513-1532.

[9]. Alotaibi, A., Arif, M., Alghamdi, M. A., Hussain, S. (2020). Stalkiness associated with cosine hyperbolic function. Mathematics 8, 1118.

[10]. Mustafa, N., Nezir, V., Kankılıç. A. (2023). Coefficient estimates for certain subclass of analytic and univalent functions associated with sine hyperbolic function. 13th International Istanbul Scientific Research Congress on Life, Engineering and Applied Sciences on April 29-30, 234-241, Istanbul, Turkey.

[11]. Frasin, B. And Aouf, M. K. (2011). New Classes of Bi-univalent Functions. Applied Mathematics Letter, 24(9), 1569-1573.

[12]. Sharma, K., Jain, N. K., Ravichandran, V. (2016). Starlike function associated with a cardioid. Afr. Math. 2016, 27, 923-939.

[13]. Mustafa, N., Nezir, V., Kankılıç. A. (2023). The Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine hyperbolic function. 13th International Istanbul Scientific Research Congress on Life, Engineering and Applied Sciences on April 29-30, 242-249, Istanbul, Turkey.

[14]. Mustafa, N., Nezir, V. (2023). Coefficient estimates and Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine hyperbolic function. 13th International Istanbul Scientific Research Congress on Life, Engineering and Applied Sciences on May 1-2, 475-481, Istanbul, Türkiye.

[15]. Mustafa, N., Demir, H. A. (2023). Coefficient estimates for certain subclass of analytic and univalent functions with associated with sine and cosine functions. 4th International Black Sea Congress on Modern Scientific Research on June 6-8, 2555-2563, Rize Turkey.

[16]. Mustafa, N., Demir, H. A. (2023). Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine and cosine functions. 4th International Black Sea Congress on Modern Scientific Research on June 6-8, 2564-2572, Rize Turkey.

[17]. Mustafa, N., Nezir, V., Kankılıç. A. (2023). Coefficient estimates for certain subclass of analytic and univalent functions associated with sine hyperbolic function with complex order. Journal of Scientific and Engineering Research, 10(6), 18-25.

[18]. Mustafa, N., Nezir, V., Kankılıç. A. (2023). The Fekete-Szegö problem for certain subclass of analytic and univalent functions associated with sine hyperbolic function with complex order. Eastern Anatolian Journal of Science, 9(1), 1-6.

[19]. Mustafa, N., Demir, H. A. (2023). Coefficient estimates for certain subclass of analytic and univalent functions with associated with sine and cosine functions with complex order. Journal of Scientific and Engineering Research, 10(6), 131-140.

[20]. Kankılıç. A., Mustafa, N. (2024). On the Pseudo Starlike Bi-univalent Function Class of Complex Order. Journal of Scientific and Engineering Research, 15(5), 26-34.

[21]. Duren, P. L. (1983). Univalent Functions. In Grundlehren der Mathematischen Wissenschaften, New York, Berlin, Heidelberg and Tokyo, Springer-Verlag, Volume 259.