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The Coefficient Estimates and Fekete-Szegö Problem for the Pseudo-Starlike and Pseudo-Convex Univalent Function Class

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Abstract: In this paper, we defined a new subclass of starlike and convex univalent functions and examined some geometric properties this function class. For this definition class, we gave some coefficient estimates and solve Fekete-Sezöge problem.

Keywords: Starlike function, convex function, univalent function, pseudo-starlike function, pseudo-convex function

1. Introduction

By $H(U)$, we will denote the class of analytic functions in the open unit disk $U=\{z\in\mathbb{C}:|z|<1\}$ of the complex plane $\mathbb{C}$. Let A be the class of the functions $f\in H(U)$ given by series expansions

$$f(z)=z+a_2z^2+a_3z^3+a_4z^4+\cdots+a_nz^n+\cdots=z+\sum_{n=2}^{\infty}a_nz^n,\ a_n\in\mathbb{C} \tag{1.1}$$

The subclass of A, which are univalent functions in U is denoted by S in the literature. The class S was introduced by Köebe [1] first time and has become the core ingredient of advanced research in this field. After a short time, in 1916 Bieberbach [2] published a paper in which the coefficient hypothesis was proposed. This hypothesis states that if $f\in S$ and has the series form (1.1), then $|a_n|\le n$ for each $n\ge 2$. There are many articles in the literature regarding to this hypothesis (see [3-14]).

It is well known that the starlike and convex function classes in the open unit disk U are defined analytically as follows and denoted by S^* and C, respectively

$$S^*=\left\{f\in S:\ \operatorname{Re}\left(\frac{zf'(z)}{f(z)}\right)>0,\ z\in U\right\}\text{ and }C=\left\{f\in S:\ \operatorname{Re}\left(\frac{(zf'(z))'}{f'(z)}\right)>0,\ z\in U\right\}.$$

Let's $f,g\in H(U)$, then it is said that f is subordinate to g and denoted by $f\prec g$, if there exists a Schwartz function ω, such that $f(z)=g(\omega(z))$.

In the past few years, numerous subclasses of the class S have been introduced as special choices of the classes S^* and C (see for example [4, 8-21]).

2. Materials and Methods

In this section, we will give some definitions and basic information.

Now, let's we define new subclass of univalent functions defined in the open unit disk U.

Definition 2.1. For $\beta\in[0,1]$ and $\lambda>\frac{1}{2}$ the function $f\in S$ is said to be in the class $\chi_{\sinh}(\beta,\lambda)$, if the following condition is satisfied

$$(1-\beta)\frac{z\left(f'(z)\right)^{\lambda}}{f(z)}+\beta\frac{\left[\left(zf'(z)\right)'\right]^{\lambda}}{f'(z)}\prec 1+\sinh z,\ z\in U.$$

In the special values of the parameters β and λ from the Definition 2.1, we have the following classes of univalent functions.

Definition 2.2. For $\lambda>\frac{1}{2}$ the function $f\in S$ is said to be in the class $S^{*}_{\sinh}(\lambda)$, if the following condition is satisfied

$$\frac{z\left(f'(z)\right)^{\lambda}}{f(z)}\prec 1+\sinh z,\ z\in U.$$

Definition 2.3. For $\lambda>\frac{1}{2}$ the function $f\in S$ is said to be in the class $C_{\sinh}(\lambda)$, if the following condition is satisfied

$$\frac{\left[\left(zf'(z)\right)'\right]^{\lambda}}{f'(z)}\prec 1+\sinh z,\ z\in U.$$

Definition 2.4. For $\beta\in[0,1]$ the function $f\in S$ is said to be in the class $\chi_{\sinh}(\beta)$, if the following condition is satisfied

$$(1-\beta)\frac{z\left(f'(z)\right)}{f(z)}+\beta\frac{\left(zf'(z)\right)'}{f'(z)}\prec 1+\sinh z,\ z\in U.$$

Let P be the class of analytic functions in U satisfied the conditions $p(0)=1$ and $\mathrm{Re}\left(p(z)\right)>0,\ z\in U.$ From these conditions, we have

$$p(z)=1+p_1z+p_2z^2+p_3z^3+\cdots=1+\sum_{n=1}^{\infty}p_nz^n,\ z\in U \qquad (2.1)$$

for $p\in\mathrm{P}$.

The class P defined above is known as the class Caratheodory functions [22] in the literature.

Now, let us give some necessary lemmas for the proof of our main results.

Lemma 2.1 ([23]). Let the function p belong to the class P. Then,

$|p_n|\le 2$ for each $n\in\mathbb{N}$, $|p_n-\nu p_kp_{n-k}|\le 2$ for $n,\ k\in\mathbb{N},\ n>k$ and $\nu\in[0,1]$.

The equalities hold for the function

$$p(z)=\frac{1+z}{1-z}.$$

Lemma 2.2 ([23]) Let the an analytic function p be of the form (2.1), then

$$2p_2=p_1^2+\left(4-p_1^2\right)x,$$

$$4p_3=p_1^3+2\left(4-p_1^2\right)p_1x-\left(4-p_1^2\right)p_1x^2+2\left(4-p_1^2\right)\left(1-|x|^2\right)y$$

for some $x,y\in\mathbb{C}$ with $|x|\le 1$ and $|y|\le 1$.

In this paper, we give some coefficient estimates and solve Fekete-Szegö problem for the class $\chi_{\sinh}(\beta,\lambda)$. Additionally, the results obtained in our study are compared with the results available in the literature.

3. Results & Discussion

In this section, we give some coefficient estimates for the functions belonging to the class $\chi_{\sinh}(\beta,\lambda)$ and solve Fekete-Szegö problem for this class.

Theorem 3.1. Let the function f given by series expansions (1.1) belong to the class $\chi_{\sinh}(\beta,\lambda)$. Then, we have the following estimates

$$|a_2| \le \frac{1}{(2\lambda-1)(1+\beta)} \text{ and}$$

$$|a_3| \le \frac{1}{(3\lambda-1)(1+2\beta)} \begin{cases} 1 \quad if \\ |2\lambda^2-4\lambda+1|(1+3\beta) \le (2\lambda-1)^2(1+\beta)^2, \\ \dfrac{|2\lambda^2-4\lambda+1|(1+3\beta)}{(2\lambda-1)^2(1+\beta)^2} \quad if \\ |2\lambda^2-4\lambda+1|(1+3\beta) \ge (2\lambda-1)^2(1+\beta)^2. \end{cases} \tag{3.1}$$

Proof. Let $f \in \chi_{\sinh}(\beta,\lambda)$, then exists a Schwartz function $\omega: U \to U$, such that

$$(1-\beta)\frac{z\left(f'(z)\right)^{\lambda}}{f(z)} + \beta\frac{\left[\left(zf'(z)\right)'\right]^{\lambda}}{f'(z)} = 1+\sinh\omega(z), z \in U. \tag{3.2}$$

Let's the function $p \in P$ defined as follows:

$$p(z) = \frac{1+\omega(z)}{1-\omega(z)} = 1+p_1z+p_2z^2+p_3z^3+\ldots = 1+\sum_{n=1}^{\infty} p_nz^n,\ z \in U. \tag{3.3}$$

It follows from that

$$\omega(z) = \frac{p(z)-1}{p(z)+1} = \frac{p_1}{2}z + \frac{1}{2}\left(p_2-\frac{p_1^2}{2}\right)z^2 + \frac{1}{2}\left(p_3-p_1p_2-\frac{p_1^3}{4}\right)z^3\ldots,\ z \in U. \tag{3.4}$$

If we perform the necessary operations in the left side of the equalities (3.2), take into account the expressions (3.4) and use the series expansion of the $\sinh$ function, we obtain the following equality

$$\begin{aligned} &(1-\beta)\left\{1+a_2(2\lambda-1)z+\left((3\lambda-1)a_3+\left(2\lambda^2-4\lambda+1\right)a_2^2\right)z^2+\cdots\right\} \\ &+\beta\left\{1+2a_2(2\lambda-1)z+\left(3(3\lambda-1)a_3+\left(8\lambda^2-16\lambda+4\right)a_2^2\right)z^2+\cdots\right\} \\ &= 1+\frac{p_1}{2}z+\left(\frac{p_2}{2}-\frac{p_1^2}{4}\right)z^2+\cdots,\ z \in U. \end{aligned} \tag{3.5}$$

Comparing the coefficients of the same degree terms on the right and left sides of the equalities (3.5), we obtain the following equalities

$$a_2 = \frac{p_1}{2(2\lambda-1)(1+\beta)},\quad (3\lambda-1)(1+2\beta)a_3+\left(2\lambda^2-4\lambda+1\right)(1+3\beta)a_2^2 = \frac{p_2}{2}-\frac{p_1^2}{4}. \tag{3.6}$$

Applying Lemma 2.1 to the first equality of (3.6), we obtain the first estimate of (3.1).
From the second equality of the equalities (3.6), we obtain

$$a_3 = \frac{1}{(3\lambda-1)(1+2\beta)}\left[\frac{p_2}{2}-\frac{p_1^2}{4}-\frac{\left(2\lambda^2-4\lambda+1\right)(1+3\beta)}{4(2\lambda-1)^2(1+\beta)^2}p_1^2\right].$$

Then, using the Lemma 2.2, we can write

$$a_3=\frac{1}{(3\lambda-1)(1+2\beta)}\left[\frac{4-p_1^2}{4}x-\frac{(2\lambda^2-4\lambda+1)(1+3\beta)}{4(2\lambda-1)^2(1+\beta)^2}p_1^2\right] \quad (3.7)$$

for some $x\in\mathbb{C}$ with $|x|\le 1$.

Applying triangle inequality to the last equality, we get

$$|a_3|\le\frac{1}{(3\lambda-1)(1+2\beta)}\left[\frac{4-t^2}{4}\xi+\frac{|2\lambda^2-4\lambda+1|(1+3\beta)}{4(2\lambda-1)^2(1+\beta)^2}t^2\right], \quad (3.8)$$

where $\xi=|x|$ and $t=|p_1|$.

From the inequality (3.8), can written

$$|a_3|\le\frac{1}{(3\lambda-1)(1+2\beta)}\left[\frac{|2\lambda^2-4\lambda+1|(1+3\beta)-(2\lambda-1)^2(1+\beta)^2}{4(2\lambda-1)^2(1+\beta)^2}t^2+1\right],\ t\in[0,2]. \quad (3.9)$$

Then, if we maximize the function

$$\chi(t)=\frac{|2\lambda^2-4\lambda+1|(1+3\beta)-(2\lambda-1)^2(1+\beta)^2}{4(2\lambda-1)^2(1+\beta)^2}t^2+1, t\in[0,2],$$

it can easily be seen that $\chi(t)\le 1$, if $|2\lambda^2-4\lambda+1|(1+3\beta)\le(2\lambda-1)^2(1+\beta)^2$ and

$$\chi(t)\le\frac{|2\lambda^2-4\lambda+1|(1+3\beta)}{(2\lambda-1)^2(1+\beta)^2}$$

if $|2\lambda^2-4\lambda+1|(1+3\beta)\ge(2\lambda-1)^2(1+\beta)^2$.

With this, the proof of the second estimate of the theorem is provided.

Thus, the proof of the theorem is completed.

In the cases $\beta=0$, $\beta=1$ and $\lambda=1$ from the Theorem 3.1, we obtain the following results.

Corollary 3.1. If $f\in S_{\sinh}^*(\lambda)$, then

$$|a_2|\le\frac{1}{2\lambda-1} \text{ and } |a_3|\le\frac{1}{3\lambda-1}\begin{cases}1 & if \quad |2\lambda^2-4\lambda+1|\le(2\lambda-1)^2,\\ \dfrac{|2\lambda^2-4\lambda+1|}{(2\lambda-1)^2} & if \quad |2\lambda^2-4\lambda+1|\ge(2\lambda-1)^2.\end{cases}$$

Corollary 3.2. If $f\in C_{\sinh}(\lambda)$, then

$$|a_2|\le\frac{1}{2(2\lambda-1)} \text{ and } |a_3|\le\frac{1}{3(3\lambda-1)}\begin{cases}1 & if \quad |2\lambda^2-4\lambda+1|\le(2\lambda-1)^2,\\ \dfrac{|2\lambda^2-4\lambda+1|}{(2\lambda-1)^2} & if \quad |2\lambda^2-4\lambda+1|\ge(2\lambda-1)^2.\end{cases}$$

Corollary 3.3. If $f\in\chi_{\sinh}(\beta)$, then

$$|a_2|\le\frac{1}{1+\beta} \text{ and } |a_3|\le\frac{1}{2(1+2\beta)}.$$

Now, we give the following theorem on the solution of the Fekete-Szegö problem for the class $\chi_{\sinh}(\beta,\lambda)$.

Theorem 3.2. Let $f\in\chi_{\sinh}(\beta,\lambda)$ and $\mu\in\mathbb{C}$, then

$$\left|a_3-\mu a_2^2\right| \le \frac{1}{(3\lambda-1)(1+2\beta)}\begin{cases} 1 & if \\ \left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right| \le (2\lambda-1)^2(1+\beta)^2, \\ \dfrac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right|}{(2\lambda-1)^2(1+\beta)^2} & if \\ \left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right| \ge (2\lambda-1)^2(1+\beta)^2. \end{cases} \tag{3.10}$$

Proof. Let $f \in \chi_{\sinh}(\beta,\lambda)$ and $\mu \in \mathbb{C}$, then from the first equality of the equalities (3.6) and (3.7), we can write

$$a_3-\mu a_2^2 = \frac{1}{(3\lambda-1)(1+2\beta)}\left\{\frac{4-p_1^2}{4}x - \frac{\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu}{4(2\lambda-1)^2(1+\beta)^2}p_1^{\,2}\right\} \tag{3.11}$$

for some $x \in \mathbb{C}$ with $|x| \le 1$.

Applying triangle inequality to the equality (3.11), we obtain

$$\left|a_3-\mu a_2^2\right| \le \frac{1}{(3\lambda-1)(1+2\beta)}\left\{\frac{4-t^2}{4}\xi + \frac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right|}{4(2\lambda-1)^2(1+\beta)^2}t^2\right\},$$

$$\xi \in [0,1],$$

where $\xi = |x|$ and $t = |p_1|$. From here easily can written

$$\left|a_3-\mu a_2^2\right| \le \frac{1}{(3\lambda-1)(1+2\beta)}\left\{\frac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right|-(2\lambda-1)^2(1+\beta)^2}{4(2\lambda-1)^2(1+\beta)^2}t^2+1\right\},$$

$$t \in [0,2]. \tag{3.12}$$

By maximizing the function $\varphi:[0,2]\to\mathbb{R}$ defined as follows

$$\varphi(t) = \frac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right|-(2\lambda-1)^2(1+\beta)^2}{4(2\lambda-1)^2(1+\beta)^2}t^2+1,\ t\in[0,2],$$

we can easily see that $\varphi(t) \le 1$ if $\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right| \le (2\lambda-1)^2(1+\beta)^2$ and

$$\varphi(t) \le \frac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right|}{(2\lambda-1)^2(1+\beta)^2}$$

if

$$\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right| \ge (2\lambda-1)^2(1+\beta)^2.$$

Thus, the proof of theorem is completed.

Taking $\beta = 0$, $\beta = 1$ and $\lambda = 1$ in the Theorem 3.2, we obtain the following results.

Corollary 3.4. If $S_{\sinh}^{*}(\lambda)$ and $\mu \in \mathbb{C}$, then

$$\left|a_3-\mu a_2^2\right| \leq \frac{1}{3\lambda-1}\begin{cases}1 & if \\ \left|2\lambda^2-4\lambda+1+(3\lambda-1)\mu\right| \leq (2\lambda-1)^2, \\ \dfrac{\left|2\lambda^2-4\lambda+1+(3\lambda-1)\mu\right|}{(2\lambda-1)^2} & if \\ \left|2\lambda^2-4\lambda+1+(3\lambda-1)\mu\right| \geq (2\lambda-1)^2.\end{cases}$$

Corollary 3.5. If $f \in C_{\sinh}(\lambda)$ and $\mu \in \mathbb{C}$, then

$$\left|a_3-\mu a_2^2\right| \leq \frac{1}{3(3\lambda-1)}\begin{cases}1 & if \\ \left|4\left(2\lambda^2-4\lambda+1\right)+3(3\lambda-1)\mu\right| \leq 4(2\lambda-1)^2, \\ \dfrac{\left|4\left(2\lambda^2-4\lambda+1\right)+3(3\lambda-1)\mu\right|}{4(2\lambda-1)^2} & if \\ \left|4\left(2\lambda^2-4\lambda+1\right)+3(3\lambda-1)\mu\right| \geq 4(2\lambda-1)^2.\end{cases}$$

Corollary 3.6. If $f \in \chi_{\sinh}(\beta)$ and $\mu \in \mathbb{C}$, then

$$\left|a_3-\mu a_2^2\right| \leq \frac{1}{2(1+2\beta)}\begin{cases}1 & if \ \left|1+3\beta-2(1+2\beta)\mu\right| \leq (1+\beta)^2, \\ \dfrac{\left|1+3\beta-2(1+2\beta)\mu\right|}{(1+\beta)^2} & if \ \left|1+3\beta-2(1+2\beta)\mu\right| \geq (1+\beta)^2.\end{cases}$$

Also, taking $\mu=0$ and $\mu=1$ in the Theorem 3.2, we obtain the following results.

Corollary 3.7. If $f \in \chi_{\sinh}(\beta,\lambda)$, then

$$\left|a_3\right| \leq \frac{1}{(3\lambda-1)(1+2\beta)}\begin{cases}1 & if \\ \left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)\right| \leq (2\lambda-1)^2(1+\beta)^2, \\ \dfrac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)\right|}{(2\lambda-1)^2(1+\beta)^2} & if \\ \left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)\right| \geq (2\lambda-1)^2(1+\beta)^2.\end{cases}$$

Corollary 3.8. If $f \in \chi_{\sinh}(\beta,\lambda)$, then

$$\left|a_3-a_2^2\right| \leq \frac{1}{(3\lambda-1)(1+2\beta)}\begin{cases}1 & if \\ \left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\right| \leq (2\lambda-1)^2(1+\beta)^2, \\ \dfrac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\right|}{(2\lambda-1)^2(1+\beta)^2} & if \\ \left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\right| \geq (2\lambda-1)^2(1+\beta)^2.\end{cases}.$$

Remark 3.1. We note that Corollary 3.7 confirms the second result of Theorem 3.1.

In the case $\mu \in \mathbb{R}$, we can prove the following theorem similarly to the proof of the Theorem 3.2.

Theorem 3.3. Let $f \in \chi_{\sinh}(\beta,\lambda)$ and $\mu \in \mathbb{R}$. Then,

$$\left|a_3-\mu a_2^2\right|\le\frac{1}{3\lambda-1}\begin{cases}1 & if \\ -\dfrac{(2\lambda-1)^2(1+\beta)^2+\left(2\lambda^2-4\lambda+1\right)(1+3\beta)}{(3\lambda-1)(1+2\beta)}\le\mu\le & \\ \dfrac{(2\lambda-1)^2(1+\beta)^2-\left(2\lambda^2-4\lambda+1\right)(1+3\beta)}{(3\lambda-1)(1+2\beta)}, & \\ \dfrac{\left|\left(2\lambda^2-4\lambda+1\right)(1+3\beta)+(3\lambda-1)(1+2\beta)\mu\right|}{(2\lambda-1)^2(1+\beta)^2} & if \\ \begin{cases}-\dfrac{(2\lambda-1)^2(1+\beta)^2+\left(2\lambda^2-4\lambda+1\right)(1+3\beta)}{(3\lambda-1)(1+2\beta)}\le\mu\le\mu \quad or \\ \mu\le\dfrac{(2\lambda-1)^2(1+\beta)^2-\left(2\lambda^2-4\lambda+1\right)(1+3\beta)}{(3\lambda-1)(1+2\beta)}.\end{cases} & \end{cases}$$

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